

Session 10, Grades 3–5

Classroom Case Studies

This is the final session of the *Measurement* course! In this session, we will examine how measurement concepts from the previous nine sessions might look when applied to situations in your own classroom. This session is customized for three grade levels. Select the grade level most relevant to your teaching.

The session for grades 3–5 begins below. Go to page 195 for grades K–2 and page 217 for grades 6–8.

Key Terms in This Session

Previously Introduced

- area

Introduction

In the previous sessions, we explored many different topics related to measurement. You put yourself in the position of a mathematics learner, both to analyze your individual approach to solving problems and to get some insights into your own understanding of measurement topics. It may have been difficult to separate your thinking as a mathematics learner from your thinking as a mathematics teacher. Not surprisingly, this is often the case! In this session, however, we will shift the focus to your own classroom and to the approaches your students might take with mathematical tasks involving measurement. **[See Note 1]**

For the list of materials that are required and/or optional in this session, **see Note 2.**

Learning Objectives

In this session, you will do the following:

- Explore the development of the concept of area in the elementary grades
- Examine students' understanding of area and perimeter relationships
- Investigate instructional tasks on area that are developmentally appropriate for elementary school students
- Explore how you might teach related measurement topics

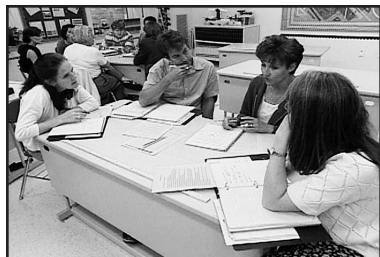
Note 1. This session uses classroom case studies to examine how children in grades 3–5 think about and work with measurement concepts. If you are taking this course on your own, you may want to share your observations on Channel Talk or ask some of your colleagues for their input. Using the classrooms of fellow teachers as well as your own as case studies will allow you to make additional observations.

Note 2. Materials Needed:

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| • Graph paper (optional) | • Square tiles (optional) |
| • Centimeter grid paper (optional) | • Geometry software, such as Geometer's Sketchpad (optional) |
| • Pictures or models of different types of triangles (optional) | • Protractor (optional) |

Part A: The Concept of Area (25 min.)

To begin to explore what the teaching of measurement might look like in the classroom, participants in the *Measurement* course first revisited a problem on area presented during Session 6. The participants then considered how children make sense of these ideas and discussed ways to present area concepts to elementary school students.



Video Segment (approximate time: 2:20-5:38): You can find this segment on the session video 2 minutes and 20 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, four teachers discuss some of the important concepts involving area that are encountered by students in grades 3-5. When planning instructional sequences, teachers need to consider what mathematical skills and concepts students need to understand and what activities will help them develop that understanding.

Problem A1. Answer the questions based on what you saw in the video:

- What concepts and skills did the teachers mention as being important for students to understand?
- What types of activities might be used to help students make sense of these concepts and skills?
- Are there related concepts or skills that will affect whether or not students can understand and use these ideas?
- Thinking back to the big ideas of this course, what are some other ideas that students should encounter to help extend and deepen their understanding of area?

Problem A2. Choose one of the concepts that you listed for Problem A1 and describe an instructional activity that you might use to help students grasp that concept.

Problem A3. What role do manipulative materials play in making sense of these mathematical ideas? Do they support or hinder students' mathematical understanding of conservation of area?

Join the Discussion! www.learner.org

Post your answer to Problem A3 on an email discussion list; then read and respond to answers posted by others. Go to the *Measurement* Web site at www.learner.org/learningmath and find Channel Talk.

Part B: Reasoning

About Measurement (40 min.)

Exploring Standards

The National Council of Teachers of Mathematics (NCTM, 2000) has identified measurement as a strand in its *Principles and Standards for School Mathematics*. In grades pre-K–12, instructional programs should enable all students to do the following:

- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements

In grades 3–5 classrooms, students are expected to do the following:

- Understand such attributes as length, area, weight, volume, and size of angle, and select the appropriate type of unit for measuring each attribute
- Understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems
- Carry out simple unit conversions, such as from centimeters to meters, within a system of measurement
- Understand that measurements are approximations and understand how differences in units affect precision
- Explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way
- Develop strategies for estimating the perimeters, areas, and volumes of irregular shapes
- Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles
- Select and use benchmarks to estimate measurements
- Develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms
- Develop strategies to determine the surface areas and volumes of rectangular solids

The NCTM (2000) Measurement Standards suggest that “students in grades 3–5 should explore how measurements are affected when one attribute to be measured is held constant and the other is changed. For example, consider the area of four tiles joined together along adjacent sides. The area of each tile is a square unit. When joined, the area of the resulting polygon is always four square units, but the perimeter varies from eight to ten units, depending on how the tiles are arranged. ... This activity provides an opportunity to discuss the relationship of area to perimeter. It also highlights the importance of organizing solutions systematically” (NCTM, 2000, p. 173).

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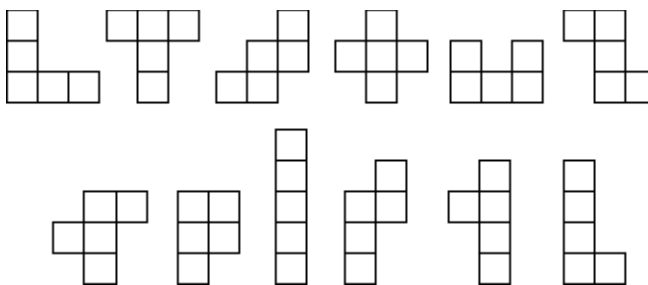
Part B, cont'd.

Analyzing a Case Study

To begin the exploration of what measurement topics look like in a classroom at your grade level, you will watch a video segment of a teacher who took the *Measurement* course and then adapted the mathematics to his own teaching situation. We will begin by looking at some of the content addressed in the videotaped lesson. [See Note 3]

Mr. Belber uses pentominoes to explore area and perimeter relationships with his class. A pentomino is made from five squares arranged so that each square shares at least one adjacent side with at least one other square. There are 12 unique pentominoes.

On graph paper, draw the 12 pentominoes, or print out the prepared set on page 211. Cut them out so that you can use the 12 pentominoes in the next problem.



Problem B1. Put any two pentominoes together and determine both the perimeter and the area of the new shape. What is the largest perimeter possible, and what is the smallest perimeter possible?



Video Segment (approximate time: 11:08-16:18): You can find this segment on the session video 11 minutes and 8 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

Watch the following segment from Mr. Belber's class. As you watch, think about how both the lesson and the teacher are assisting students in making sense of the relationships between area and perimeter described in the NCTM Standards.

Problem B2. As students discover different perimeters, Mr. Belber has them record their findings on grid paper. What are the advantages and disadvantages of this recording scheme? What problem-solving strategies are students using to find the possible perimeters?

Problem B3. Mr. Belber wanted his students not only to use a guess-and-check strategy to find the possible perimeters, but also to analyze how the placement of the pentominoes next to each other affected the perimeter. How can the perimeter of a new shape made from two pentominoes be determined without counting? How did Mr. Belber help students understand this analytic method? What additional questions could you ask to confirm that the students understand the method?

Problem B4. This lesson focuses on the fact that if area remains constant (in this case, 10 in^2), the perimeter of shapes constructed with that area can vary. Do you think the mathematical purpose of the lesson is clear? What other factors make a lesson successful?

Join the Discussion!

www.learner.org

Post your answer to Problem B4 on an email discussion list; then read and respond to answers posted by others. Go to the *Measurement* Web site at www.learner.org/learningmath and find Channel Talk.

Problem B5. Sometimes we want students to generalize what they have learned. How did Mr. Belber extend the learning from this lesson? What generalizations might he expect students to mention?

Note 3. The purpose of the video segments is not to reflect on the teaching style of the teacher portrayed. Instead, look closely at the methods the teacher uses to bring out the ideas of measurement while engaging his students in activities.

Part C: Problems That Illustrate Measurement Reasoning (55 min.)

In this part, you'll look at several problems that are appropriate for students in grades 3–5. For each problem, answer the questions below. If time allows, obtain the necessary materials and solve the problems.

- What is the measurement content in the problem? What are the big ideas that you want students to consider and understand?
- What prior knowledge is required? What later content does it prepare students for?
- How does the content in this problem relate to the mathematical ideas in this course?
- What other questions might extend students' thinking about the problem?
- What other instructional activities or problems might you use in conjunction with this one to further your content goals?

Problem C1. Take 24 square tiles. Make all the rectangles that have an area of 24 square units. Record the dimensions of each rectangle. What do you notice about the relationship of the length and width of a rectangle to its area? How are the dimensions of the rectangles related to their areas? Write a rule for finding the area of any rectangle given its length and width.

Problem C2. Examine the following measurements collected by students:

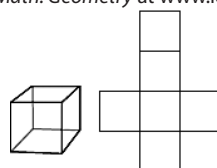
Names	Circumference of Flagpole	Length of Math Book
Carlos and Pam	58 cm	29 cm
Linda and Jen	56 cm	29.5 cm
Yoji and Pete	57.5 cm	28.8 cm

Why aren't the measurements the same? What affects precision?

Problem C3. Cut out the net of a box with dimensions of 4 by 6 by 2 from centimeter grid paper. Without folding the net into a box, predict how many centimeter cubes will fit into the box when folded. Then connect two centimeter cubes together to form a 1-by-1-by-2 package. How many 1-by-1-by-2 packages fit into the box? What strategies did you use to predict how many centimeter cubes and how many 1-by-1-by-2 packages fit into the box? Check your answers by filling the box with cubes. Finally, generalize an approach to determining the number of cubes in a box. [See Note 4]

Note 4. A net is a two-dimensional representation of a three-dimensional object, like a cube. It shows all the faces of the three-dimensional object and is connected in such a way that it lies flat, but can be folded up to form the three-dimensional object. For more information about nets, go to *Learning Math: Geometry* at www.learner.org/learningmath and find Session 9.

Here is a sample net:



Part C, cont'd.

Problem C4. Use geometry software such as Geometer's Sketchpad for this problem. If geometry software is not available, collect pictures or models of the different types of triangles and measure the angles with a protractor.

Use the software to measure the angles in each of the triangles in the table. Find the sum of the angles of the triangle. Record your findings.

Type of Triangle	Measure $\angle A$	Measure $\angle B$	Measure $\angle C$	Sum
Right Triangle				
Equilateral Triangle				
Isosceles Triangle				
Acute Triangle				
Obtuse Triangle				

What patterns do you notice about the sum of the angles in a triangle? Create a few more triangles and find the angle sum for each of them. Do you think these patterns will hold for all triangles? Why or why not?

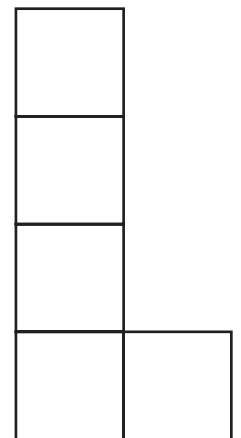
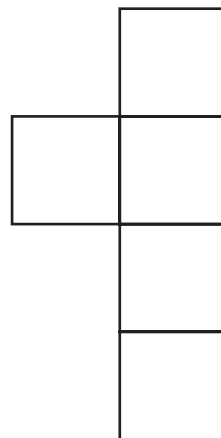
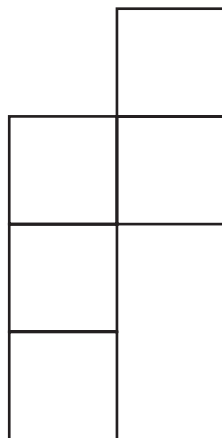
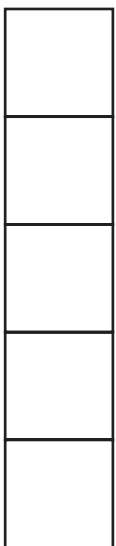
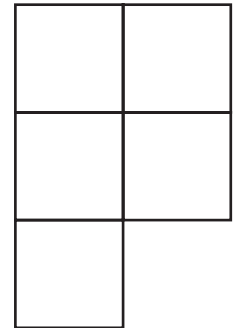
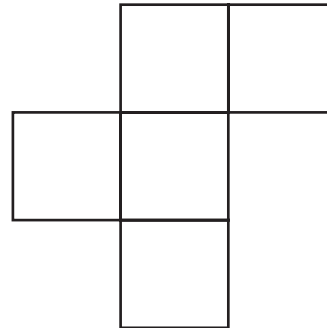
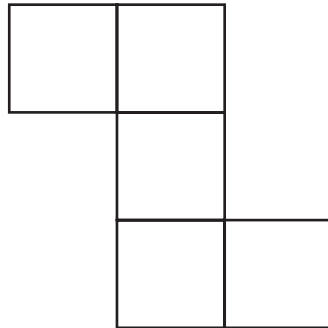
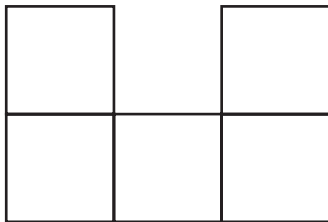
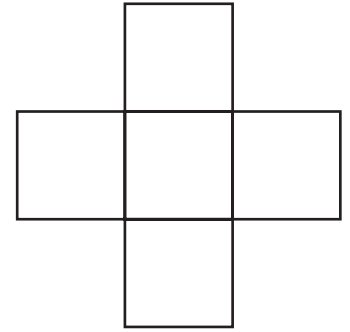
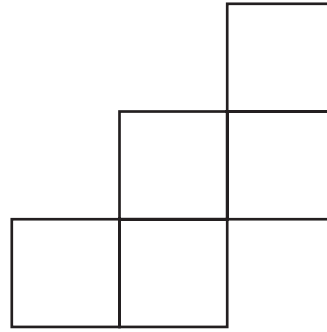
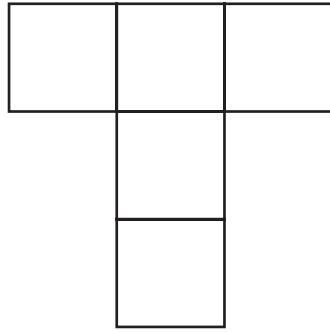
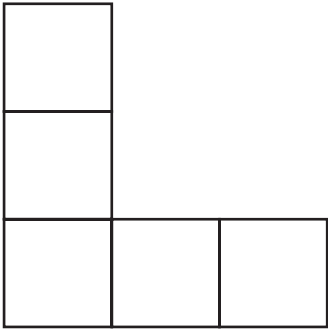
Homework

Solutions are not provided for these homework problems, since answers will vary depending on individual experiences.

Problem H1. Assume that you need to report back to your grade-level team or to the entire school staff at a faculty meeting about your experiences and learning in this course. What are the main messages about the teaching of measurement you would share with your colleagues? Prepare a one-page handout or an overhead or slide that could be distributed or shown at the meeting.

Problem H2. Look at a lesson or activity in your own mathematics program for your grade level that you think has potential for developing students' reasoning about measurement. If you were to use this lesson or activity now, after taking this course, how might you modify or extend it to bring out more of the important concepts about measurement?

Pentominoes for Problem B1



Solutions

Part A: The Concept of Area

Problem A1.

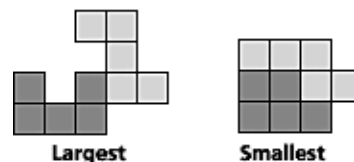
- Teachers talked about the importance of students understanding that area is the measure of the amount of space covered as well as what a square unit is. They talked about how, when asked to build a 3-by-5 rectangle, students will often create just a border and not cover the middle of the rectangle. The teachers would like for students to see the connection between area and multiplication as well as count the perimeter accurately.
- Having hands-on experiences where students are building rectangles with manipulatives such as tiles helps them to see and understand area. They are physically covering the rectangle and counting square units to determine area. Drawing on grid paper also helps students to visualize the concept of area. Either way, students can see the rectangular arrays that are formed and connect area to multiplication. For example, in a 6-by-8 rectangle, students can see six rows of eight tiles or eight rows of six tiles, and the concept of multiplication is brought to the forefront for them.
- As mentioned above, learning about area provides students with an opportunity to deepen their understanding of multiplication. Determining the area of a rectangle becomes a context in which students can “see” multiplication. Looking at it another way, having a firm understanding of multiplication will also help students in their study of area. When considering the interpretation of the meaning of multiplication as an array of length times width, it is clear that students’ knowledge of both area and multiplication are developing at the same time.
- Students should also look at conservation of area, appropriate type and size of units of measurement, relationships between perimeter and area, strategies for determining areas of irregular shapes, and surface area.

Problem A2. Answers will vary. One way to allow students to explore area as a covering is to have them find the area of an irregular shape (for example, the outline of a pair of scissors) on grid paper. They can then determine the number of square units that it covers.

Problem A3. Using manipulative materials is essential for giving students the opportunity to see and feel area. Understanding that area is the measure of the amount of surface covered is much easier when students are actually covering rectangular surfaces with square unit tiles. The idea of conservation is also made concrete to students when they can actually hold the amount of area in their hands by using manipulative materials. If students are given 12 tiles to make various rectangles, they can convince themselves that all the rectangles will have the same area—even though some may look bigger or smaller—because they were all made with the same number of tiles. Manipulative materials are an essential tool for learning about area.

Part B: Reasoning About Measurement

Problem B1. The largest possible perimeter is 22 units; the smallest is 14 units. This is because all but one pentomino have a perimeter of 12 units. One pentomino has a perimeter of 10 units. A method for determining perimeter when combining two pentominoes is to add the perimeters of the two pentominoes and then subtract the number of sides touching when the pentominoes are combined. To get the largest possible perimeter, combine two 12-unit pentominoes so that the least number of sides touch, which is two. So $12 + 12 - 2 = 22$. To get the smallest perimeter, combine one 12-unit pentomino with the 10-unit pentomino so that as many sides as possible are touching. The maximum number of sides touching is eight, making the smallest possible combined perimeter $12 + 10 - 8$, or 14 units.



Solutions, cont'd.

Problem B2. One advantage of recording on grid paper is that it allows students to trace the perimeters of the shapes they've formed. Recording on grid paper also enables students to keep track of the different shapes. Recording all of their findings also helps them notice patterns. When tracing their combined shapes, however, if students do not also include the detail of all the squares in the pentomino, they will not be able to see the number of sides that are touching to make their new combined shape. This will lessen the power of recording their findings.

Some of the problem-solving strategies students used included the guess-and-check strategy, as well as analyzing shapes and drawings for patterns (e.g., all perimeters must be even, and the combined perimeter will be the sum of the perimeters of both pentominoes less the total number of sides touching).

Problem B3. To determine the perimeter without counting, the students first added the perimeters of the two pentominoes being combined for the new shape and then subtracted the number of sides touching between the two pentominoes. To help students see this analytic method, Mr. Belber asked them questions that focused on the number of sides that touched in the new shape and how that number decreased the sum of the two pentomino perimeters.

Additionally, you could ask students the following questions to confirm that they understand the concept: What patterns do you notice in the perimeters of the combined figure when the two pentominoes have two sides touching? Four sides touching? etc. Does your answer depend on which two pentominoes you choose? Can you explain why this pattern makes sense, or why increasing the number of sides touching makes the perimeter smaller?

Problem B4. This lesson has a clear mathematical purpose, which is the understanding that if area remains constant, the perimeter of shapes constructed having that area can vary. Using manipulatives helps students see that area stays constant. The tiles also allow students to see and feel the number of sides touching, and how that number affects the perimeter.

Problem B5. Mr. Belber extended learning by asking students to explain why they knew they had found the smallest or largest perimeter, how they knew they had found all possible perimeters, and why they got only even perimeters. Some generalizations he might expect students to make include the fact that only even perimeters can be made and that a rule for determining perimeter is that the perimeters of two pentominoes, minus the number of sides touching in the combined new shape, equals the perimeter of the new shape. Mr. Belber's homework assignment of repeating the task with three pentominoes provides additional generalizing opportunities and could ultimately lead to a rule for determining the maximum and minimum perimeters for any number of pentominoes.

Solutions, cont'd.

Part C: Problems That Illustrate Measurement Reasoning

Problem C1.

Solution:

Using 24 square tiles, you can make four distinct rectangles: 1 by 24; 2 by 12; 3 by 8; and 4 by 6. Even though the lengths and widths differ from rectangle to rectangle, their product is always the same, and equal to the area of the rectangles—24. Each of these rectangles represents an array of tiles with particular dimensions. If we count the tiles in each array, we get the area of the rectangle. The rule for finding the area of any rectangle, given its length and width, is length multiplied by width.

Answers to Questions:

- This problem prompts students to derive the formula for area of a rectangle. It also delves into their understanding of multiplication by having them explore the relationship between multiplication and determining area.
- This problem builds on students' prior experiences with multiplication, area, and patterns. It also lays the groundwork for the kind of pattern recognition and generalization students will use in algebra.
- The concept of area and the derivation of its formula were extensively explored in the course. Different arrangements of tiles also draw upon ideas such as conservation of area.
- How do you know you've found all the combinations? Given an area and one dimension of a rectangle, find the missing dimension.
- Use this pattern approach for deriving the area formula for parallelograms. In this case, it would be helpful to have students work on grid or graph paper so that they could count side length and area. Students could also work on finding other "formulas" that involve multiplication; for example, finding the total number of students when given a class size and number of classes, or finding the number of feet or toes when given a set number of people.

Problem C2.

Solution:

The measurements aren't the same because the students may have used different measuring tools and techniques. Also, physically measuring an object is likely to produce some degree of measurement error. The measurement process is, by its nature, never exact. Precision is affected by the measuring tool. The smaller the unit on a measuring tool, the more precise it is.

Answers to Questions:

- The measurement content of this lesson is the idea that measurements are approximations and that differences in units affect precision.
- The idea of measurement error forms the basis for the study of standard deviation. This problem builds on students' prior experiences with measurement units, measuring tools, and decimals.
- This problem relates to one of the big ideas of the course, namely that measurement is an approximation. Also, concepts such as measurement error, precision, and accuracy are evident in this type of problem.
- How do you decide at what point a measurement is inaccurate? In other words, how much error is acceptable? How important is measurement precision in different contexts (i.e., building a bridge or cutting a piece of wrapping paper to wrap a box)?

Solutions, cont'd.

Problem C2, cont'd.

- e. Have students measure time and discuss the accuracy of the measurements. Using stopwatches or wrist-watches with a stopwatch function, have students try to record a set length of time. Using another watch or clock to track the time, tell the students to “start” their stopwatches. Fifteen seconds later, say “stop” to have the students stop their watches. Students should then write down the time, as precisely as possible, on their watch (e.g., 00:15:09 or 00:15:13 or 00:14:97). Theoretically, all the students should have recorded the same length of time. Their times, however, will likely vary because measurement is an estimate. Discuss why measurements aren’t the same, if they are accurate, and what affects precision.

Problem C3.

Solution:

The number of centimeter cubes that will fit into a 4-by-6-by-2 box is 48. The number of 1-by-2-by-2 cubes that will fit into this box is 24. Answers will vary for the type of strategy that can be used to predict how many packages of a particular size will fit into the box. One strategy is to first see how many centimeter cubes are needed to cover the base of the box (4-by-6 rectangle), which is 24. Then, because the height of the box will be 2, multiply the number of cubes in the base by 2 to get 48. Similarly, you can find out how many 1-by-1-by-2 cubes will cover the base of the box. Since the height of the box is the same as one dimension of the 1-by-1-by-2 cube, you can place the cubes vertically to determine how many will cover the base of the larger box, and in the process, entirely fill the space that will be encompassed by the folded box. Answers will vary on what kinds of approaches help determine the number of cubes in a box. One method is to multiply all three dimensions of a box to determine the number of centimeter cubes that will fit in that box.

Answers to Questions:

- a. The measurement content of this problem is the determination of the volume of a rectangular prism. The big ideas include understanding volume, cubic units, moving from two dimensions to three dimensions, and developing a formula for finding the volume of a rectangular prism.
- b. This problem builds on students’ prior experiences with three-dimensional figures and discovering formulas (as in Problem C1). It also leads to a whole host of packing problems. Filling a box with packages of different dimensions (for example, 1 by 1 by 2) is harder than filling it with unit cubes, because you may not be able to fill the box completely, depending on how you arrange the packages. This can help students develop number sense. Students can also try packing more complicated shapes, such as spheres, into rectangular boxes. Now that they cannot fill the space completely, what is optimal?
- c. The concepts of volume and the derivation of its formula were covered in this course. Understanding the relationship between two-dimensional representations and their three-dimensional counterparts was also a part of this course.
- d. Many students will not “see” the formula with only two examples. This activity may instead lead them to compare volume and surface area of a solid. Most students will need many more exercises just like these (giving dimensions, cutting out a net, predicting volume, and checking volume based on given dimensions and a net, etc.) before they can generalize a formula. As they attempt more examples, it may be helpful for students to keep their data organized in a table. For students who already have a good grasp of volume of a solid, consider the following questions: Why does a 1-by-1-by-2 package completely fill the box? Are there other smaller-sized packages that will completely fill the box? Why or why not?
- e. A certain toy company makes sets of children’s blocks. The blocks are 1 in. cubes. The company is looking for a rectangular box that will hold a set of 64 blocks with no leftover space. Design a box for the company. Explain why yours is the best design. Additionally, students could explore the relationship between surface area and volume of a rectangular prism by generating all possible rectangular prisms with a given volume.

Solutions, cont'd.

Problem C4.

Solution:

Depending on the triangles drawn, the measurements of the particular angles will vary (with the exception of the equilateral triangle, whose angles will all measure 60 degrees). The sum of the angles for every triangle is 180 degrees, and this will remain true for any triangle. Testing more triangles will demonstrate this further, but it is not a proof.

Answers to Questions:

- a. The measurement content of this problem is the discovery of the fact that the sum of the interior angles of a triangle is 180 degrees.
- b. This problem builds on students' prior experiences finding patterns, identifying various types of triangles, and measuring angles. It lays the groundwork for using this (sum of interior angles equals 180 degrees) and other attributes of triangles, as well as angle relationships, to determine angle measures without using measuring tools such as protractors. The patterning nature of this problem continues to build a strong generalizing foundation.
- c. The concepts of measurement of angles and the sum of angles in a triangle were discussed in this course.
- d. Students could apply what they now know about the sum of the angles in a triangle to find missing angle measures without measuring. For example, given two angle measures in a triangle, find the measure of the third angle.
- e. Have students use geometry software or pattern blocks to determine the sum of interior angles of other regular polygons, such as squares and hexagons. Students can cover a new shape with triangles and use the fact that the sum of interior angles in a triangle is always 180 degrees to determine the sum of the angles in the new shape. If the students use pattern blocks, they can also find each angle of a regular polygon by placing several of that same type of polygon side by side around a point until the blocks fill up 360 degrees. The students can then divide 360 by the number of polygons around that point to obtain the measure of each angle. For example, it takes three hexagons meeting at a single point to complete 360 degrees. When you divide 360 degrees by 3, you get 120 degrees, which is the measure of each interior angle of a regular hexagon.