From *A History of the Circle* by Ernest Zebrowski:

In seeking to make sense of the physical world, we encounter a whole series of dilemmas. Not only are our observations inexact, but the laws of mathematics themselves may not apply exactly in the realm of the physically observable (e.g., there is no such thing as a true circle in the physical world). Still, we needn't throw up our hands in despair and abandon mathematical reasoning. Suppose, for instance, that a wheel's diameter is 28.5 inches give or take 0.5 inch. The half-inch is the uncertainty in the diameter, and it may reflect the fact that we've measured slightly different diameters from top to bottom versus side to side, or it may reflect the fact that this is the limit of precision of our measuring instrument. Either way, the uncertainty in diameter will lead to an uncertainty in circumference. Does it make sense to ask how much uncertainty?

It does, as long as we keep in mind that we are not dealing with exact numbers in any case. One way to find the uncertainty in this wheel's circumference is to do three calculations:

upper limit on C = π • (upper limit on d) = π • 29.0 in. = 91.1 in.
lower limit on C = π • lower limit on d) = π • 28.0 in. = 88.0 in.
best value for C = π • (best value for d) = π • 28.5 in. = 89.5 in.

In examining the answers, we see that the circumference can be stated as 89.5 in., give or take about 1.6 in. This ±1.6 inches is therefore the *uncertainty* in circumference.

Now, if the wheel in this example were a perfect Euclidean circle whose diameter was exactly 28.5 inches, and it rolled through 1,000 revolutions, it would travel forward a distance of 89,535.39063... inches, or 7,461.282552... feet. Because the wheel cannot be perfect, however, it makes no sense to retain all of these digits. An uncertainty of 1.6 inches in circumference, adding up (in the worst case) over the course of 1,000 revolutions, leads to a total uncertainty of 1,600 inches or 133 feet in the total distance traveled. The uncertainty in diameter, in other words, propagates through our calculation and reveals itself as an uncertainty in the total distance the wheel rolls.

If we quote this particular computational result as 7,461.282552... feet, we mislead anyone who reads it, for we can be quite sure that if we actually rolled the wheel, we would never measure this exact number. A better approach is to state

distance = 7,461 feet ± 133 feet

Unfortunately, this notation can get tiresome if we do it with every computational result. Instead, as a matter of style and expediency, it's more common to simply round off the answer, so it doesn't misrepresent the precision too terribly. In this example, we might report the result as simply 7,500 feet. The thoughtful reader will take this to mean an answer accurate to within 50 or 100 feet.

People often balk at rounding off numbers this way, particularly when our modern calculators give us all those wonderful digits. The thing to remember is that those digits are based on a purely mathematical logic, while the rounded-off result is probably intended to describe something quite real. In linking the mathematical ideal with messy reality, we always make a big leap. A purely mathematical calculation can never make our wheel rounder, our initial measurements more precise, or the physical universe different from what it is.

Zebrowski, Ernest (1999). *A History of the Circle* (pp. 48-49). Piscataway, N.J.: Rutgers University Press.