Mental Random Numbers: Perceived and Real Randomness

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Two ideas involving probability are essential for an understanding of statistical inference: the frequency interpretation of probability and the concept of randomness. In general, students do not have much difficulty with the frequency interpretation, particularly if asked to perform and keep track of computer experiments that simulate a few hundred coin tosses or rolls of a die. The idea of randomness is much more complicated. Our intuitive understanding of randomness does not agree with the probabilistic concept of randomness that underlies much of statistical inference. The problem is discussed extensively in the literature (see, for example, Wagenaar, 1972). The present paper discusses an experiment, easily carried out in the classroom, that reveals the difficulties encountered by the human mind, when asked to imitate random behaviour in the probabilistic sense.

In the USA, a favourite method for losing money is to play one of the many official (and unofficial) numbers games. The *Connecticut Daily Number*, which can be played every day of the week except Sundays, is typical of many of these games. The player selects a three-digit number between 000 and 999. If the number is drawn, the player wins $500 for every dollar bet.

Most students have seen on television how the winning number is determined. In each of three separate glass urns, ten ping-pong balls labelled 0 through 9 are agitated by streams of air, until one is caught in the narrow neck at the top. The three digits selected constitute the number of the day. According to the frequency interpretation of probability, each of the ten digits between 0 and 9 appears with long run frequency one in ten at each of the three positions. In addition, since three separate urns are being used, what happens in one urn does not affect what happens in the other urns. In the long run, each of the 1000 possible combinations of three digits appears equally frequently, that is, with probability $\frac{1}{1000} = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$. This is an illustration of the multiplication rule of probabilities for independent events.

In order to show students that the human mind may perceive randomness according to different laws, I ask them "to mentally draw a sequence of ping-pong balls". Mental selections involving ten digits are rather confusing. Experiments involving three imaginary ping-pong balls with labels 1, 2 and 3 are simpler. Specifically, I ask students to make five successive random selections from among the digits 1, 2 and 3, and to write them down on a piece of paper. To set the pace, I tap a pencil five times at approximately one-second intervals. At each tap, the student is expected to write down a one, or a two, or a three.

When analysing the resulting data, I only use the last two digits written down. The first three digits merely help the student to get into the swing of things. Table 1 gives the frequencies of the nine possible combinations of the last two digits written down by 450 students who participated in the experiment.

Table 1. *Frequencies of mental digit pairs*

		Second digit			
		1	*2*	*3*	*Totals*
First digit	1	31	72	60	163
	2	57	27	63	147
	3	53	58	29	140
	Totals	141	157	152	450

Under conditions of true random selection, the nine possible combinations 11, 12, 13, 21, 22, 23, 31, 32, 33 are equally likely, each having probability:

$$\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}.$$

Expected frequencies for each cell then equal 50. Mere visual inspection reveals that the actually observed frequencies for the identical digit pairs 11, 22, 33 are much smaller than can be expected under conditions of true randomness, while the frequencies for the non-identical digit pairs are too large. Under conditions of true randomness, successive selections are independent, so that an identical digit pair like 22 has the same probability as a mixed digit pair like 21. But when making mental selections, students apparently remember the digit they have just written down and then avoid it as their next selection, thus introducing an element of dependence into what is supposed to be a sequence of independent selections.

It is interesting to note that in spite of the lack of independence of successive digit selection, the marginal totals for both the first and the second digits do not exhibit any significant deviations from theoretical frequencies. These visual conclusions can be confirmed by appropriate chi-square tests.

For comparison, the frequencies in Table 2 were obtained with the help of a table of random numbers.

Table 2. *Frequencies of random digit pairs*

		Second digit			
		1	*2*	*3*	*Totals*
First digit	1	44	58	50	152
	2	43	48	53	144
	3	55	54	45	154
	Totals	142	160	148	450

Reference

Wagenaar, W.A. (1972). Generation of random sequences by human subjects: a critical survey of literature, *Psychological Bulletin*, **77**, 65-72.

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