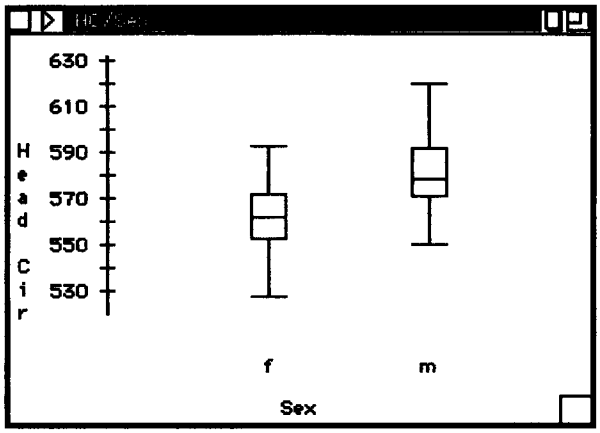


Fig. 10 Box plots for head circumferences of females and males

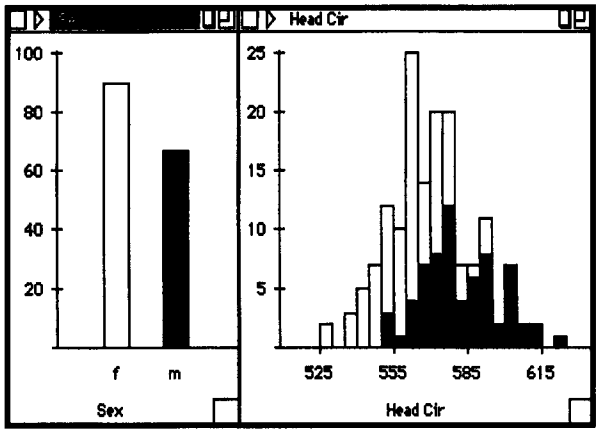


Fig. 11 Male/female bar graph and head circumference histogram

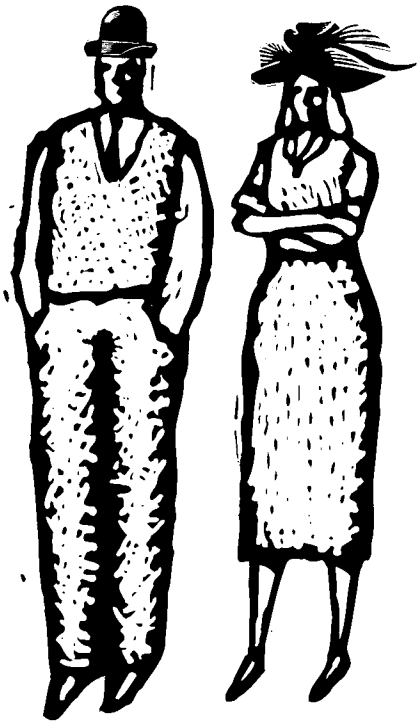
ments of 10. This scaling often results in relatively "flat" histograms, and any patterns in the distribution may be difficult to determine. Another shortcoming of both Data Insights and Statistics Workshop is that neither supplies the frequency or the relative-frequency tables associated with a particular histogram. The only options are to read—approximate—the frequency of each interval group from the histogram or to sort—order from smallest to largest—the values and print these ordered values. Sorting the data using Statistics Workshop yielded the frequency–relative-frequency table shown in **table 2.**

Communication

From our investigation of head circumferences, it is clear that some head sizes are more common than others. What does this information tell us about the original problem of ordering hats? It seems reasonable that a hat should be slightly larger than the wearer's head circumference. The smallest hat size available from L. L. Bean corresponds to a head circumference of approximately 520 mm. So anyone with a head size smaller than 520 mm would order this size hat. Using increments of 10 mm yields hat sizes of 530, 540, and so on.

The data-collection method employed in this investigation was not random and may not reflect the true nature of the distribution of all head circumferences. However, the results from our study suggest approximately how many hats of each size should be ordered. Suppose hats must be ordered in groups of 1000 hats—how many of each size should be ordered? The number of hats that should be ordered for each size, on the basis of the results of our investigation, is shown in **table 3.**

Are the distributions of head sizes different for men and women?



The histogram shown in **figure 4** suggests that head circumferences may follow a normal distribution. In a more advanced setting, our study of hat sizes might include the use of the normal distribution as a model for describing the distribution of head circumferences. Using properties of the normal distribution, we could determine the expected number of hats for each hat size. These expected values could then be compared with the number of hats observed from our investigation.

Additional Studies

UP TO THIS POINT IN OUR INVESTIGATION, no attention has been paid to the sex of each individual in the study. As long as a particular style of hat is appropriate for either males or females, the data and the analysis from the previous study are useful for making a decision about how many hats to order. Should these same results be used when ordering hats that are styled specifically for women or specifically for men? Essentially, we have come full circle in the statistical problem-solving process. That is,

our analysis and interpretation have led us to another question: Are the distributions of head sizes different for men and women?

Data collection, analysis, and interpretation

When the data on head circumferences were collected, each student also recorded her or his sex. A graph that is especially useful for comparing two or more groups of data is the *box plot.* A box plot gives a nondetailed display of the range of each quarter of the data. The five-number summary and the box plot, along with outliers, produced by Data Desk for the entire data set is shown in **figure 9.**

Distributions for male and female head circumferences are different

A unique feature of Data Desk is the ability to select a specific point on a graph and to identify the case value of any associated variable. For example, in **figure 9,** when the most extreme outlier is selected, the value of the variable "Name" associated with this point is shown to be "m perry." The male and female box plots are shown in **figure 10.** These plots indicate that—

- the median male head circumference is somewhat larger—more than 10 mm—than the median female head circumference;
- the amount of variation in head circumferences for males and females is about the same;
- approximately 25 percent of males have larger head circumferences than the largest female head circumference; and
- approximately 25 percent of females have smaller head circumferences that the smallest male head circumference.

These results suggest that an analysis similar to that done on the combined data should be done for the data separated by sex. Another unique feature of Data Desk is its ability to "link" different graphs. **Figure 11** shows the bar graph on sex and the histogram on head circumference for the entire data set. By selecting the male bar in the sex bar graph, the corresponding males' head circumferences are highlighted in the head-circumference histogram. It is clear from the region highlighted in this histogram that the male head circumferences dominate the upper range of head circumferences and the female head circumferences dominate the lower range. Consequently, the distributions for male and female head circumferences are different.

Summary and Conclusions

A STATISTICAL INVESTIGATION WAS UNDERTAKEN TO study the problem of ordering hats. The investigation modeled a five-component process and used statistical software in the analysis component of this process. The data representations presented for solving this problem are appropriate for students in the middles grades or higher. The investigation demonstrates that with appropriate technology, statistical concepts and problem-solving strategies can be developed without employing complicated statistical techniques or formulas.

References

BBN Laboratories. Statistics Workshop. Wings for Learning/Sunburst, Scotts Valley, CA 95067-0002. Software.

Biehler, Rolf. "Software Tools and Mathematics Education: The Case of Statistics." In *Learning from Computers: Mathematics Education and Technology,* edited by C. Keitel and K. Ruthven, pp. 68–100. Berlin: Springer Publishing, 1993.

Edwards, Lois A. Data Insights. Wings for Learning/Sunburst, Scotts Valley, CA 95067-0002. Software.

Graham, Alan. *Statistical Investigations in the Secondary School.* London: Cambridge University Press, 1987.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: The Council, 1989.

Perry, Mike, and Gary Kader. "STAT-LINC and STATMAPS: Teacher Education and Curriculum Design." *American Statistical Association 1992 Proceedings of the Section on Statistical Education,* 40–46. Alexandria, Va.: The Association, 1992.

Velleman, Paul. Data Desk. New York: W. H. Freeman & Co., 1973. Software.

Kader, Gary and Perry, Mike (September–October, 1994). Learning Statistics with Technology. *Mathematics Teaching in the Middle School, 1* (2), 130–136.