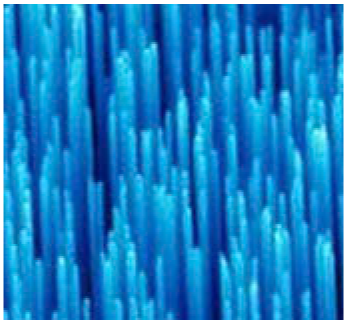
Physics for the 21st Century

Unit 8: Emergent Behavior in Quantum Matter



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Unit Overview

This unit takes an approach to physics that differs markedly from much of what we have encountered in previous units. Rather than cataloging the elementary components of matter, we look at what happens at the macroscopic scale when the interactions of these components with one another and their environment lead to entirely new—emergent—behavior. After introducing the concept of emergence, the unit examines emergent behavior in solid matter, quantum plasmas, and the very different behavior of the liquid forms of two different isotopes of helium (He). The next two sections cover the search for a microscopic theory of superconductivity and its culmination in Bardeen-Cooper-Schrieffer (BCS) theory, which triumphantly accounted for the emergent properties of conventional superconductors. The final three sections focus on efforts to understand emergence in new and different contexts, from freshly discovered forms of superconductivity on Earth to the cosmic superfluidity observed in pulsars—rotating stars made up primarily of neutrons.

Content for This Unit

Sections:

Section 1: Introduction

The term emergent behavior refers to the collective phenomena observed in macroscopic systems that are distinct from their microscopic constituents. It is brought about by the interaction of the microscopic constituents with one another and with their environment. Whereas the Standard Model of particle physics described in Units 1 and 2 has enjoyed great success by building up systems of particles and interactions from the ground up, nearly all complex and beautiful phenomena observed in the laboratory or in nature defy this type of reductionist explanation. Life is perhaps the ultimate example. In this unit, we explore the physics of emergent phenomena and learn a different approach to problem solving that helps scientists understand these systems.

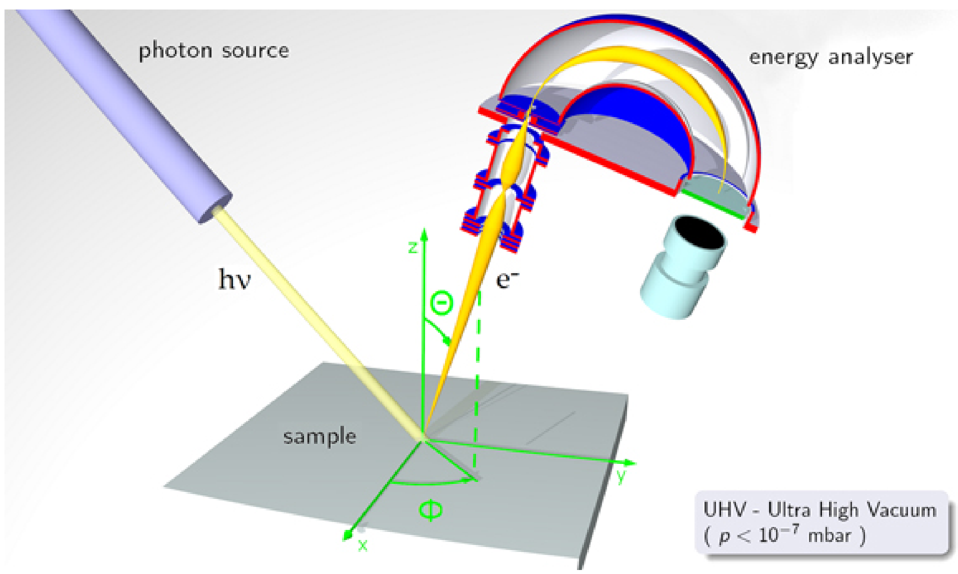

Figure 1: New experimental probes, such as angle-resolved photoemission spectroscopy (ARPES) enable us to study emergent behavior in quantum matter at an unprecedented level of detail. ARPES is an experimental technique to observe the distribution of the electrons (more precisely, the density of single-particle electronic excitations) in the reciprocal space of solids.
Source: © Wikimedia Commons, Public Domain. Author: Saiht, 15 June 2009.

Understanding emergent behavior requires a change of focus. Instead of adopting the traditional reductionist approach that begins by identifying the individual constituents and interactions of a system and then uses them as the basic building blocks for creating a model of a system's behavior, we must focus on identifying the origins of the emergent collective behavior characteristic of the system. Thus, in creating models of quantum matter, we use the organizing principles and concepts responsible for emergent quantum behavior as our basic building blocks. These new building blocks, the collective organizing principles, represent gateways to emergence.

Certain aspects of emergent behavior are considered "protected," in the sense that they are insensitive to the details of the underlying microscopic physical processes.

Much of this unit deals with the phenomenon of superconductivity, where electrical current can flow with absolutely no resistance whatsoever. Unraveling the mystery of how electrons (which experience a repulsive electrical interaction) can cooperate and produce coherent collective phenomena is a detective story that is still being written.

To build a little intuition about the subtle interactions that give rise to superconductivity (and the related phenomenon of superfluidity), and to set the stage for what follows, imagine two electrically charged bowling balls placed on a spring mattress. They experience a repulsive electrostatic interaction, but the indentation in the mattress made by one of the bowling balls can influence the other one, producing a net attractive force between them. This is crudely analogous to the direct interactions between electrons and their coupling to the excitations of the crystal lattice of a superconducting metal: an attractive interaction between electrons can result if the interaction between each electron and the excitations of the embedding lattice is strong enough to overcome the electrostatic repulsion and this interaction can give rise to the pairing condensate that characterizes the superconducting state.

Another concept that will arise in this unit is the notion of a "quasiparticle." The concept of a quasiparticle arises in a simplifying framework that ascribes the combined properties of an electron and its modified surroundings into a "virtual" equivalent composite object that we can treat as if it were a single particle. This allows us to use the theoretical toolkit that was built up for the analysis of single particles.

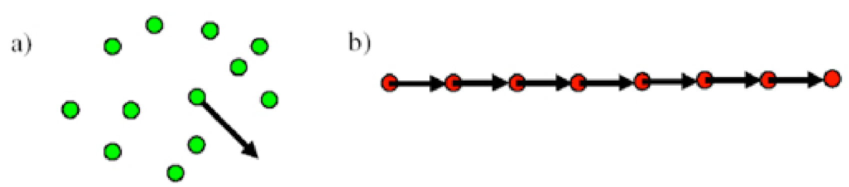

Figure 2: As shown in the figure, dimensionality can influence dramatically the behavior of quasiparticles in metals.
Source: © Joseph Sulpizio and David Goldhaber-Gordon, Stanford University.

As we will see below, both the superfluid state of ^{3}He and the superconducting states of metals come about because of quasiparticle pairing processes that transform a collection of fermions (the nuclei in the case of superfluid ^{3}He, and electrons in the case of superconductivity) into a collective, coherent single quantum state, the superfluid condensate. This exhibits the macroscopic quantum mechanical effects of a superfluid flowing with no dissipation, and of the flow of electrical current with literally zero resistance, in a superconductor.

Understanding the subtle interactions that give rise to the pairing of fermions requires looking at the fluids and materials in a different way.

Our change in focus means, in general, that instead of following the motion of single particles in a material, we will focus on the behavior of the material as a whole—for example, the density fluctuations found in bulk matter. A simple example of a density fluctuation is a sound wave traveling through a medium such as air, water, or a solid crystal. Just as light can equally well be described as fluctuations of the electromagnetic field whose quantized particles are called "photons," the collective density fluctuations in crystalline solids can be described by quantized particles called phonons. Analogously to the particle interactions described in Unit 2, the electronic density fluctuations can couple to phonons and other fields, and the interaction between the density wave and various fields can represent both the influence of particle interactions and the external probes used to measure systems' behavior. We will also be interested in the spin fluctuations of fermions, which are particles with half-integer spin.

This unit will introduce the frontiers of research in the study of emergent behavior in quantum matter and call attention to the applicability of some key organizing principles to other subfields in physics. Sections 2 through 5 present what we might call "old wine in a new bottle"—an emergent perspective on subject matter described in many existing texts on quantum matter, while the last three sections highlight the frontiers of research in the field.

In the two sections devoted to superconductivity, I have gone to some length to sketch the immediate emergent developments that led up to the historic paper in which Bardeen, Cooper, and Schrieffer described their microscopic theory known as BCS. I have done so, in part, because I can write about these from personal experience. But I also believe that learning how a major problem in physics was finally solved after 45 years of trying might help nonscientists and would-be scientists appreciate the complex process of discovery and provide encouragement for young researchers seeking to solve some of the most challenging problems that our community faces today.

Section 2: Emergent Behavior in Crystalline Solids

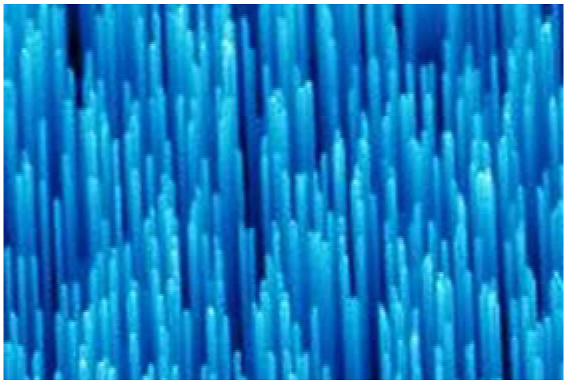

Figure 3: Nanowires are crystalline fibers with emergent behaviors expected to be used for nanoscale applications.
Source: © Deli Wang Laboratory at UCSD.

Crystalline solids provide familiar examples of emergent behavior. This section will outline the theoretical steps that have revealed the fundamental nature of solids and the ways in which such critical ideas as quantum statistics, excitations, energy bands, and collective modes have enabled theorists to understand how solids exhibit emergent behavior.

At high enough temperatures, any form of quantum electronic matter becomes a plasma—a gas of ions and electrons linked via their mutual electromagnetic interaction. As it cools down, a plasma will first become liquid and then, as the temperature falls further, a crystalline solid. For metals, that solid will contain a stable periodic array of ions along with electrons that are comparatively free to move under the application of external electric and magnetic fields.

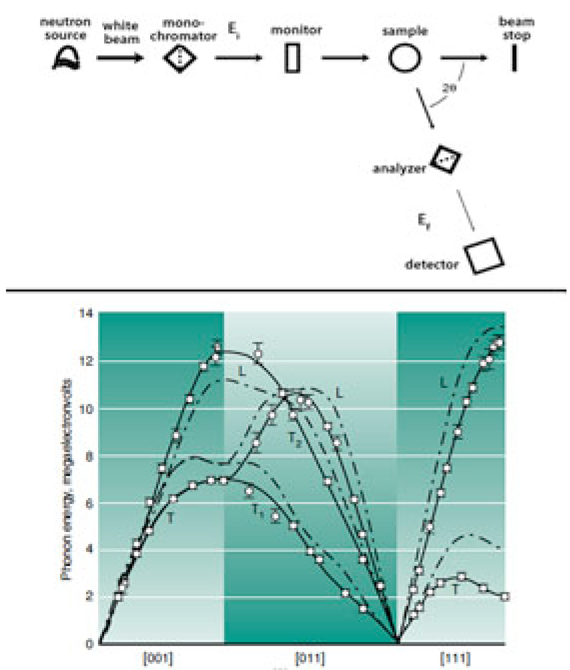

Figure 4: Left: Diagram showing experimental set-up for measurement of the energy loss spectrum of neutrons that are inelastically scattered by a crystal. Right: A typical phonon spectrum obtained through an inelastic neutron scattering (INS) experiment.
Source: © Top: Wikimedia Commons, Public Domain. Author: Joachim Wuttke, 22 February 2007. Bottom: Courtesy of Joe Wong, Lawrence Livermore National Laboratory.

The crystallization process has broken a basic symmetry of the plasma: there are no preferred directions for the motion of its electrons. Broken symmetry is a key organizing concept in our understanding of quantum matter. The crystalline confinement of the ions to regular positions in the crystal leads to a quantum description of their behavior in terms of the ionic elementary excitations, called "phonons," which describe their vibrations about these equilibrium positions. Physicists can study phonons in detail through inelastic neutron scattering experiments (Figure 4) that fire neutrons at solid samples and measure the neutrons' energies after their collisions with the vibrating ions in the solid. Moreover, the solids' low-energy, long-wavelength behavior is protected: It is independent of details and describable in terms of a small number of parameters—in this case, the longitudinal and transverse sound velocities of their collective excitations, the quantized phonons.

Independent electrons in solids

What of the electrons? The interactions between the closely packed atoms in a periodic array in a crystal cause their outermost electrons to form the energy bands depicted in Figure 5. Here, the behavior of the electrons is characterized by both a momentum and a band index, and their corresponding physical state depends to a first approximation on the valency of the ions and may be characterized by the material's response to an external electric field. Thus, the solid can take any one of three forms. It may be a metal in which the electrons can move in response to an external electric field; an insulator in which the electrons are localized and no current arises in response to the applied electric field; or a semi-conductor in which the valence band is sufficiently close to a conduction band that a small group of electrons near the top of the band are easily excited by heating the material and can move in response to an applied electric field.

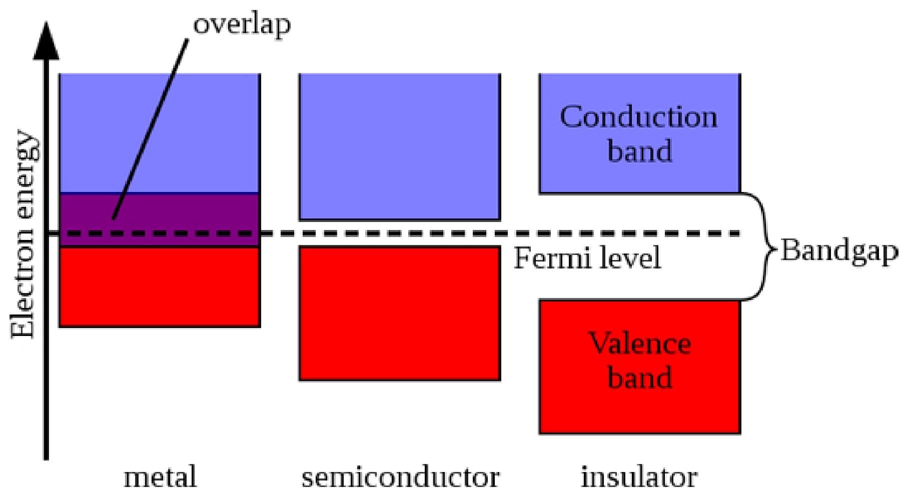

Figure 5: Comparison of the electronic band structures of metals, semiconductors, and insulators.
Source: © Wikimedia Commons, Public Domain. Author: Pieter Kuiper, 6 June 2007.

Physicists for many years used a simple model to describe conduction electrons in metals: the free electron gas or, taking the periodic field of ions into account through band theory, the independent electron model. The model is based on Wolfgang Pauli's exclusion principle that we met in Unit 2. Because electrons have an intrinsic spin of 1/2, no two can occupy the same quantum state. In the absence of the periodic field of ions, the quantum states of each electron can be labeled by its momentum, p, and its spin.

Specific Heat

The temperature dependence of the electronic specific heat in simple metals is easily explained; because of Pauli's principle that no two electrons can occupy the same quantum state, at a temperature T, only a fraction, kT/E_f, of the electrons inside the Fermi surface at which electrons possess the energy, E_f, can be excited. So, with each excited electron having an energy of about kT, the free energy of the system is roughly proportional to T^2 and its temperature derivative, the specific heat, varies linearly with T. Similar qualitative arguments explain why the electron spin susceptibility (its response to a uniform external magnetic field) first calculated by Wolfgang Pauli is independent of temperature.

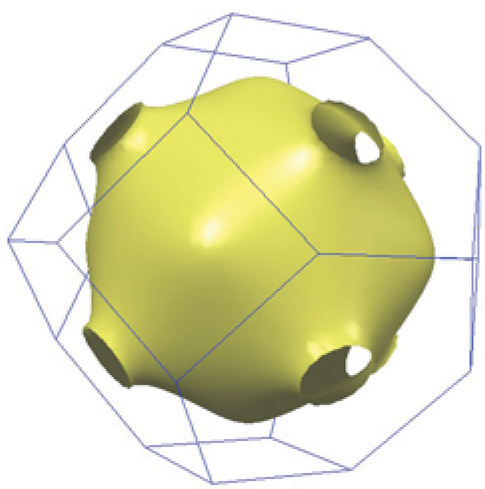

Figure 6: The Fermi surface reveals how the energy varies with momentum for the highest-energy electrons—those that have the Fermi energy.
Source: © Los Alamos National Laboratory.

Since the electrons can carry momentum in each of three independent spatial directions, it's useful to imagine a 3D coordinate system (which we call "momentum space") that characterizes the x, y, and z components of momentum.

The ground state of the electrons moving in a uniform background of positive charge would then be a simple sphere in momentum space bounded by its Fermi surface, a concept derived from the statistical work of Enrico Fermi and P.A.M. Dirac that defines the energies of electrons in a metal. When the uniform positive charge is replaced by the actual periodic array of the ions in a metallic lattice, the simple sphere becomes a more complex geometric structure that reflects the nature of the underlying ionic periodic structure, an example of which is shown in Figure 6.

To calculate the ground state wavefunction of the electrons, physicists first applied a simple approach called the Hartree-Fock approximation. This neglects the influence of the electrostatic interaction between electrons on the electronic wavefunction, but takes into account the Pauli principle. The energy per electron consists of two terms: the electrons' average kinetic energy and an attractive exchange energy arising from the Pauli principle which keeps electrons of parallel spin apart.

Emergent concepts for a quantum plasma

Conscience and Contradictions

David Bohm.
Source: © Wikimedia Commons, Public Domain. Author: Karol Langner, 30 July 2005.

David Bohm's life involved a series of contradictions. Refused security clearance for work on the atom bomb during World War II, he made critical contributions to the development of the bomb. Ever suspicious of quantum mechanics, he wrote a classic textbook on the topic before attempting to develop a theory that explained all of quantum mechanics in terms of hidden variables whose statistical behavior produced the familiar quantum results. Widely regarded as one of the greatest physicists never to win the Nobel Prize, he spent the latter part of his career working primarly on philosophy and brain science.

In the early 1940s, Bohm performed theoretical calculations of collisions of deuterons and protons for his Ph.D. under Robert Oppenheimer at the University of California, Berkeley. But when Oppenheimer recruited him for the Manhattan Project, project head General Leslie Groves refused him security clearance because of his left-wing political associations. So, when his Ph.D. research was classified, he could not even finish his thesis; Oppenheimer had to certify that he had earned his Ph.D. Bohm then spent the years from 1942–45 working at Berkeley's Radiation laboratory on the classical plasmas found in the gas discharges associated with one of the methods used to *separate uranium isotopes and became famous there for his insight into the instabilities leading to plasma turbulence.*

Politics reemerged in 1950, when the House Un-American Activities Committee cited Bohm for using the fifth amendment to refuse to answer its questions about his past political affiliations. By the time he was acquitted, the President of his university, Princeton, which had first suspended him, then refused to reappoint or promote him. Unable to obtain another scientific position in the United States because potential employers in both universities and the private sector feared being accused of appointing a communist, he became perhaps the most prominent scientific exile from the United States at a time when his scientific expertise was badly needed in the plasma-based efforts to develop a thermonuclear reactor. His exile first took him to professorships in Sao Paolo, Tel Aviv, and Bristol; he then spent the last 31 years of his life at the University of London's Birkbeck College and died in a London taxi in 1992.

The Russian physicist Lev Landau famously said, "You cannot repeal Coulomb's law." But until 1950, it appeared that the best way to deal with it was to ignore it, because microscopic attempts to include it had led to inconsistencies, or worse yet, divergent results. The breakthrough came with work carried out between 1949 and 1953 by quantum theorist David Bohm and myself, his Ph.D. student. Our research focused on the quantum plasma—electrons moving in a uniform background of positive charge, an idealized state of matter that solid-state physicist Conyers Herring called "jellium." Bohm and I discovered that when we viewed particle interactions as a coupling between density fluctuations, we could show, within an approximation we called "the random phase approximation (RPA)," that the major consequence of the long range electrostatic interaction between electrons was to produce an emergent collective mode: a plasma oscillation at a frequency, $\omega_p = (4\pi NE^2 / m)^{1/2}$, where N is the electron density and m its mass, whose quantized modes are known as plasmons. Once these had been introduced explicitly, we argued what was left was an effective short-range interaction between electrons that could be treated using perturbation-theoretic methods.

The plasma oscillation is an example of a "collisionless" collective mode, in which the restoring force is an effective field brought about by particle interaction; in this case, the fluctuations in density produce a fluctuating internal electric field. This is the first of many examples we will consider in which effective fields produced by particle interaction are responsible for emergent behavior. As we shall see later in this unit, the zero sound mode of ^{3}He furnishes another example, as does the existence of phonons in the normal state of liquid ^{4}He. All such modes are distinct from the "emergent," but familiar, sound modes in ordinary liquids, in which the restoring forces originate in the frequent collisions between particles that make possible a "protected" long wavelength description using the familiar laws of hydrodynamics.

The importance of plasmons

Following their predicted existence, plasmons were identified as the causes of peaks that experimentalists had already seen in the inelastic scattering of fast electrons passing through or reflected from thin solid films. We now know that they are present in nearly all solids. Thus, plasmons have joined electrons, phonons, and magnons (collective waves of magnetization in ferromagnets) in the family of basic elementary excitations in solids. They are as well defined an elementary excitation for an insulator like silicon as for a metal like aluminum.

By the mid 1950s, it was possible to show that the explicit introduction of plasmons in a collective description of electron interactions resolved the difficulties that had arisen in previous efforts to deal in a consistent fashion with electron interaction in metals. After taking the zero point energy of plasmons into account in a calculation of the ground state energy, what remained was a screened electrostatic interaction between electrons of comparatively short range, which could be dealt with using perturbation theory. This work provided a microscopic justification of the independent electron model for metal, in which the effects of electron interaction on "single" electron properties had been neglected to first approximation. It also proved possible to include their influence on the cohesive energy of jellium with results that agreed well with earlier efforts by Eugene Wigner to estimate this quantity, and on the exchange and correlation corrections to the Pauli spin susceptibility, with results that agreed with its subsequent direct measurement by my Illinois colleague, C.P. Slichter.

It subsequently proved possible to establish that both plasma oscillations and screening in both quantum and classical plasmas are not simply mathematical artifacts of using the random phase approximation, but represent protected emergent behavior. Thus, in the limit of long wavelengths, plasma oscillations at ω_p are found at any density or temperature in a plasma, while the effective interaction at any temperature or density is always screened, with a screening length given by s / ω_p, where s is the isothermal sound velocity. Put another way, electrons in metals are never seen in isolation but always as "quasielectrons," each consisting of a bare electron and its accompanying screening cloud (a region in space in which there is an absence of other electrons). It is these quasielectrons that interact via a short range screened electrostatic interaction. For many metals, the behavior of these quasielectrons is likewise protected, in this case by the adiabatic continuity that enables them to behave like the Landau Fermi liquids we will consider in the next section.

From plasmons to plasmonics

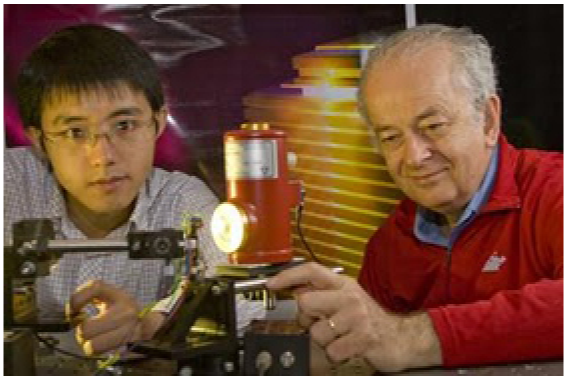

Figure 8: Harvard School of Engineering and Applied Sciences' researchers Federico Capasso (red shirt) and Nanfang Yu working on their nanoscale quantum clusters for plasmonic applications.
Source: © Eliza Grinnell.

When plasmons were proposed and subsequently identified, there seemed scant possibility that these would become a subject of practical interest. Unexpectedly, plasmons found at the surface of a metal, or at an interface between two solids, turn out to be sufficiently important in electronic applications at the nanoscale, that there now exists a distinct sub-field that marks an important intersection of physics and nanotechnology called "plasmonics." Indeed, beginning in 2006, there have been bi-annual Gordon research conferences devoted to the topic. To quote from the description of the founding conference: "Since 2001, there has been an explosive growth of scientific interest in the role of plasmons in optical phenomena, including guided-wave propagation and imaging at the subwavelength scale, nonlinear spectroscopy, and 'negative index' metamaterials. The unusual dispersion properties of metals near the plasmon resonance enables excitation of surface modes and resonant modes in nanostructures that access a very large range of wave vectors over a narrow frequency range, and, accordingly, resonant plasmon excitation allows for light localization in ultra-small volumes. This feature constitutes a critical design principle for light localization below the free space wavelength and opens the path to truly nanoscale plasmonic optical devices. This principle, combined with quantitative electromagnetic simulation methods and a broad portfolio of established and emerging nanofabrication methods, creates the conditions for dramatic scientific progress and a new class of subwavelength optical components." A description of the third such conference began with a description by Federico Capasso (Figure 8) of his bottom-up work on using self-assembled nanoclusters for plasmonic applications.

Section 3: Emergent Behavior in the Helium Liquids

The property that physicists call spin plays an essential role in the nature and emergent behavior of particles, atoms, and other units of matter. As we have noted previously, fermions have an intrinsic half-integer spin; no two fermions can occupy the same quantum state. And as we learned in Unit 6, because bosons have integral spin, any number of bosons can occupy the same quantum state. Those differences play out in the behavior at very low temperatures of the two isotopes of He—the fermion ^{3}He with spin of 1/2 owing to its single unpaired neutron and the boson ^{4}He with no net spin because it has two neutrons whose antiparallel spins sum to zero, as do the spins of the two protons in the He nucleus.

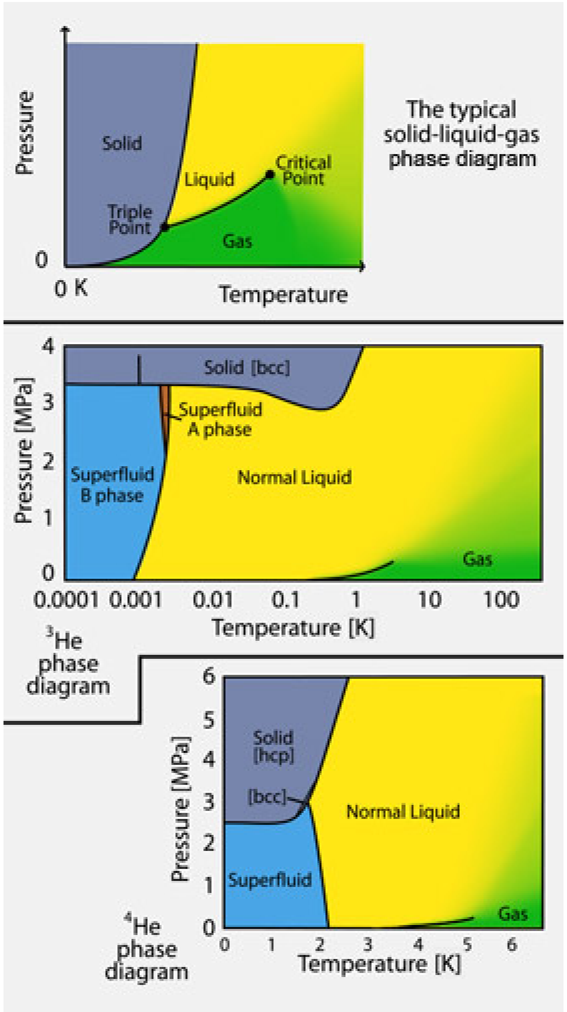

Figure 9: Temperature-pressure phase diagrams of the two quantum materials, ^{3}He and ^{4}He, that remain liquid down to the lowest temperatures in the absence of pressure compared to a typical liquid-solid phase diagram.
Source: Recreated from graphics by the Low Temperature Laboratory, Helsinki University of Technology.

The liquid forms of the two isotopes of He are the only two quantum liquids found in nature. Unlike all other atomic materials, because they are exceptionally light they do not freeze upon cooling; their zero point energy prevents them from freezing. As may be seen in Figure 9 (phase transitions) at low temperatures, the two isotopes of He exhibit remarkably different emergent behavior. Below 2.18 Kelvin (K), liquid ^{4}He becomes a superfluid that flows without appreciable resistance. Liquid ^{3}He behaves quite differently, flowing like a normal liquid down to temperatures in the millikelvin regime, some three orders of magnitude cooler, before it exhibits a transition to the superfluid state.

The reason is simple. Atoms of ^{4}He obey Bose-Einstein statistics. Below 2.18 K, a single quantum state, the Bose condensate, becomes macroscopically occupied; its coherent motion is responsible for its superfluid behavior. On the other hand, ^{3}He obeys Fermi-Dirac statistics, which specify that no two particles can occupy the same quantum state. While, as we shall see, its superfluidity also represents condensate motion, the condensate forms only as a result of a weak effective attraction between its quasiparticles—a bare particle plus its associated exchange and correlation cloud—rather than as an elementary consequence of its statistics.

Although physicists understood the properties of ^{3}He much later than those of ^{4}He, we shall begin by considering Landau's Fermi liquid theory that describes the emergent behavior displayed by the quasiparticles found in the normal state of liquid ^{3}He. We shall put off a consideration of their superfluid behavior until after we have discussed Bose liquid theory and its application to liquid ^{4}He, and explained, with the aid of the BCS theory that we will also meet later in this unit, how a net attractive interaction can bring about superconductivity in electronic matter and superfluidity in ^{3}He and other Fermi liquids, such as neutron matter.

Landau Fermi liquid theory

There are three gateways to the protected emergent behavior in the "Landau Fermi liquids" that include liquid ^{3}He and some simple metals: 1) adiabaticity; 2) effective fields to represent the influence of particle interactions; 3) a focus on long-wavelength, low-frequency, and low-temperature behavior. By incorporating these in his theory, Lev Landau was able to determine the compressibility, spin susceptibility, specific heat, and some transport properties of liquid ^{3}He at low temperatures.

Adiabaticity means that one can imagine turning on the interaction between particles gradually, in such a way that one can establish a one-to-one correspondence between the particle states of the noninteracting system and the quasiparticle states of the actual material. The principal effective fields introduced by Landau were scalar internal long-wavelength effective density fields, which determine the compressibility and spin susceptibility and can give rise to zero sound, and a vector effective field describing backflow that produces an increased quasiparticle mass. The focus on low-energy behavior then enabled him to determine the quasiparticle scattering amplitudes that specify its viscosity, thermal conductivity, and spin diffusion.

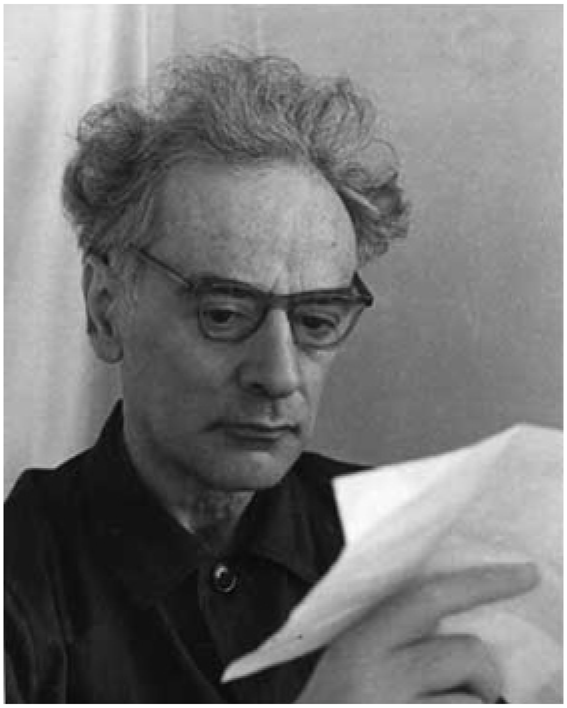

Figure 10: Landau impacted theoretical physics over much of the 20th century.
Source: © AIP Emilio Segrè Visual Archives, Physics Today Collection.

The restoring force for zero sound, a collective mode found in neutral Fermi liquids, is an internal density fluctuation field that is a generalization of that found in the random phase approximation (RPA). Its phenomenological strength is determined by the spin-symmetric spatial average of the effective interactions between parallel spin and antiparallel spin He atoms; the "Fermi liquid" correction to the Pauli spin susceptibility is determined by the corresponding spin antisymmetric average of the interactions between He atoms, i.e., the difference between the spatially averaged effective interactions between atoms of parallel and antiparallel spin. The vector field representing the strength of the effective current induced by a bare particle is just the backflow field familiar from a study of the motion of a sphere in an incompressible fluid. The collisions between quasiparticles produce an inverse quasiparticle lifetime that varies as the square of the temperature, and when modified by suitable geometric factors, give rise to the viscosity and thermal conductivity found experimentally in the normal state of liquid ^{3}He.

Fritz London

Source: © AIP Emilio Segrè Visual Archives, Physics Today Collection.

Fritz London was a seminal figure in the early days of quantum mechanics through his pioneering work on the chemical bond with Walter Heitler and his work with Edmond Bauer on the measurement problem. He was also the insufficiently celebrated heroic contributor to our understanding of superfluidity and superconductivity. As P. W. Anderson has emphasized, in his brilliant essay on London, "He was among the few pioneers who deliberately chose, once atoms and molecules were understood, not to focus his research on further subdividing the atom into its ultimate constituents, but on exploring how quantum theory could work, and be observed, on the macroscopic scale." With his younger brother Heinz, he proposed in 1934 the London equations that provided a phenomenological explanation for superconductivity, and a few years later was the first to recognize that the superfluidity of liquid ^{4}*He was an intrinsic property of the Bose condensation of its atoms. In his two books on superfluids and superconductors, he set forth the basic physical picture for their remarkable quantum behavior on a macroscopic scale, but died in 1954 before he could see his ideas brought to fruition through microscopic theory. It is a tribute to both London and John Bardeen that with his share of his 1972 Nobel Prize, Bardeen endowed the Fritz London Memorial Lectures at Duke University, where London had spent the last 15 years of his life.*

As we noted above, Landau's theory also works for electrons in comparatively simple metals, for which the adiabatic assumption is applicable. For these materials, Landau's quasiparticle interaction is the sum of the bare electrostatic interaction and a phenomenological interaction; in other words, it contains an add-on to the screening fields familiar to us from the RPA.

It is in the nonsimple metals capable of exhibiting the localized behavior predicted by Nevill Mott that is brought on by very strong electrostatic repulsion or magnetic coupling between their spins that one sees a breakdown of Landau's adiabatic assumption. This is accompanied by fluctuating fields and electronic scattering mechanisms that are much stronger than those considered by Landau. For these "non-Landau" Fermi liquids, the inverse quasiparticle lifetime may not vary as T^2, and the electron-electron interaction contribution to the resistivity will no longer vary as T^2.

We will consider Mott localization and some of the quantum states of matter that contain such non-Landau Fermi liquids in Section 7.

It is straightforward, but no longer exact, to extend Landau's picture of interacting quasiparticles to short-range behavior, and thereby obtain a physical picture of a quasiparticle in liquid ^{3}He. The theory that achieves this turns out to be equally applicable to ^{3}He and ^{4}He and provides insight into the relative importance of quantum statistics and the strong repulsive interaction between ^{3}He atoms.

The superfluid Bose liquid

While Heike Kamerlingh Onnes liquefied He in its natural ^{4}He form and then studied its properties in the 1920s, its superfluidity remained elusive until 1938. Jack Allen of Cambridge and Piotr Kapitsa in Moscow almost simultaneously found that, as the flowing liquid was cooled below 2.18 K, its viscosity suddenly dropped to an almost immeasurably low value. German-American physicist Fritz London quickly understood the gateway to the emergence of this remarkable new state of quantum matter. It was Bose condensation, the condensation of the ^{4}He atoms into a single quantum state that began at 2.18 K.

Superfluidity in liquid ^{4}He and other Bose liquids, such as those produced in the atomic condensates, is a simple consequence of statistics. Nothing prevents the particles from occupying the same momentum state. In fact, they prefer to do this, thereby creating a macroscopically occupied single quantum state, the condensate, that can move without friction at low velocities. On the other hand, the elementary excitations of the condensate—phonons and rotons in the case of ^{4}He—can and do scatter against each other and against walls or obstacles, such as paddles, inserted in the liquid. In doing so, they resemble a normal fluid.

This is the microscopic basis for the two-fluid model of ^{4}He developed by MIT's Laszlo Tisza in 1940. This posits that liquid He consists of a superfluid and a normal fluid, whose ratio changes as the temperature falls through the transition point of 2.18 K. Tisza, who died in 2009 at the age of 101, showed that the model had a striking consequence; it predicted the existence of a temperature wave, which he called second sound, and which Kapitsa's student, Vasilii Peshkov subsequently found experimentally.

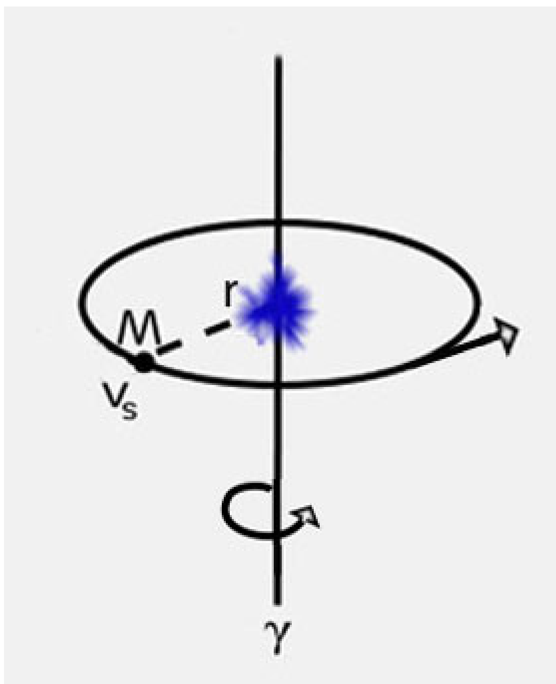

Figure 11: Geometry of a straight vortex line in a superfluid, showing how the superfluid velocity rotates about a normal core (shown in blue) whose size is the superfluid coherence length, ξ.
Source:

The superfluid flow of a condensate can also involve rotation. Norwegian-American scientist Lars Onsager and Richard Feynman independently realized that the rotational flow of the condensate of ^{4}He would be characterized by the presence of quantized vortex lines—singularities around which the liquid is rotating, whose motion describes the rotational flow. See the math

W.F. Vinen was subsequently able to detect a single vortex line, while Feynman showed that their production through friction between the superfluid flow and the pipe walls could be responsible for the existence of a critical velocity for superfluid flow in a pipe.

The introduction of rotons

It was Landau who proposed that the long wavelength elementary excitations in superfluid liquid He would be phonons; he initially expected additional phenomena at long length scales connected with vorticity, which he called rotons. He subsequently realized that to explain the experiment, his rotons must be part of the general density fluctuation spectrum, and would be located at a wave vector of the order of the inverse interatomic distance. Feynman then developed a microscopic theory of the phonon-roton spectrum, which was subsequently measured in detail by inelastic neutron scattering experiments. A key component of his work was the development with his student Michael Cohen of a ground state wavefunction that incorporated backflow, the current induced in the background liquid by the moving atom, thereby obtaining a spectrum closer to the experimental findings.

Figure 12: Lars Onsager, a physical chemist and theoretical physicist who possessed extraordinary mathematical talent and physical insight.
Source: © AIP Emilio Segrè Visual Archives, Segrè Collection.

By introducing response functions and making use of sum rules and simple physical arguments, it is possible to show that the long-wavelength behavior of a Bose liquid is protected, obtain simple quantitative expressions for the elementary excitation spectrum, and, since the superfluid cannot respond to a slowly rotating external probe, obtain an exact expression for the normal fluid density.

An elementary calculation shows that above about 1 K, the dominant excitations in liquid ^{4}He are rotons. Suggestions about their physical nature have ranged from Feynman's poetic tribute to Landau—"a roton is the ghost of a vanishing vortex ring"—to the more prosaic arguments by Allen Miller, Nozières, and myself that we can best imagine a roton as a quasiparticle—a He atom plus its polarization and backflow cloud. The interaction between rotons can be described through roton liquid theory, a generalization of Fermi liquid theory. K.S. Bedell, A. Zawadowski, and I subsequently made a strong argument in favor of their quasiparticle-like nature. We described their effective interaction in terms of an effective quasiparticle interaction potential modeled after that used to obtain the phonon-roton spectrum. By doing so, we explained a number of remarkable effects associated with two-roton bound state effects found in Raman scattering experiments.

In conclusion we note that the extension of Landau's theory to finite wave vectors enables one to explain in detail the similarities and the differences between the excitation spectra of liquid ^{3}He and liquid ^{4}He in terms of modest changes in the pseudopotentials used to obtain the effective fields responsible for the zero sound spectrum found in both liquids. Thus, like zero sound, the phonon-roton spectrum represents a collisionless sound wave and the finding of well-defined phonons in the normal state of liquid ^{4}He in neutron scattering experiments confirms this perspective.

Section 4: Gateways to a Theory of Superconductivity

Superconductivity—the ability of some metals at very low temperatures to carry electrical current without any appreciable resistance and to screen out external magnetic fields—is in many ways the poster child for the emergence of new states of quantum matter in the laboratory at very low temperatures. Gilles Holst, an assistant in the Leiden laboratory of the premier low-temperature physicist of his time, Kamerlingh Onnes, made the initial discovery of superconductivity in 1911. Although he did not share the Nobel Prize for its discovery with Kamerlingh Onnes, he went on to become the first director of the Phillips Laboratories in Eindhoven. But physicists did not understand the extraordinary properties of superconductors until 1957, when Nobel Laureate John Bardeen, his postdoctoral research associate Leon Cooper, and his graduate student Robert Schrieffer published their historic paper (known as "BCS") describing a microscopic theory of superconductivity.

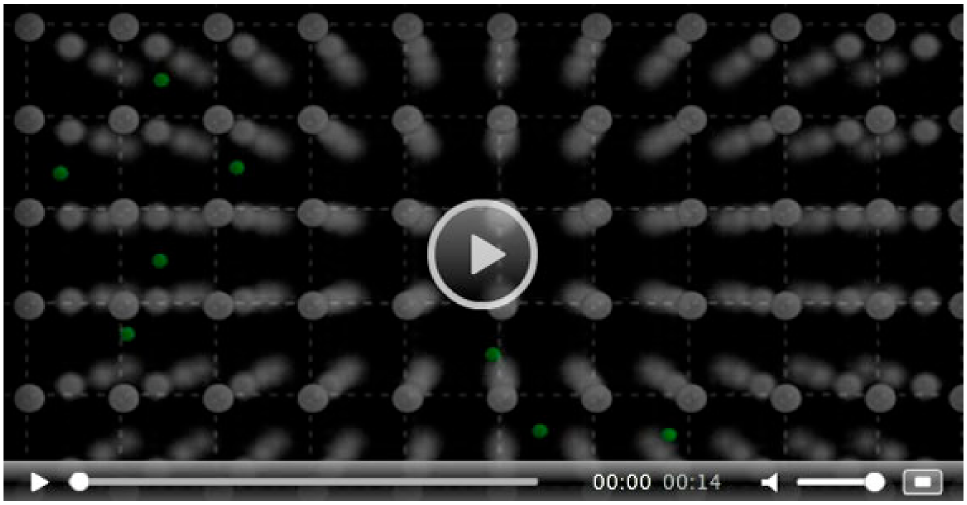

Figure 13: Superconductors carry electrical current without resistance and are almost perfect diamagnets (a more fundamental aspect of their behavior), in that they can screen out external magnetic fields within a short distance known as the "penetration depth."
Source:

We now recognize the two gateways to the emergence of the superconducting state: an effective attractive interaction between electrons (the quasiparticles of Landau's Fermi liquid theory), whose energies put them close to their Fermi surface; and the condensation of pairs of these quasiparticles of opposite spin and momentum into a macroscopically occupied single quantum state, the superfluid condensate.

BCS theory explains the superfluidity of quantum fermionic matter. It applies to conventional superconductors in which phonons, the quantized vibrations of the lattice, serve as the pairing glue that makes possible an attractive quasiparticle interaction and those discovered subsequently, such as superfluid pairing phenomena in atomic nuclei, superfluid ^{3}He, the cosmic superfluids of nuclear matter in the solid outer crust, and liquid interiors of rotating neutron stars. It also applies to the unconventional superconductors such as the cuprate, heavy electron, organic, and iron-based materials that take center stage for current work on superconductivity.

As we shall see, a remarkable feature of BCS theory is that, although it was based on an idealized model for quasiparticle behavior, it could explain all existing experiments and predict the results of many new ones. This occurs because the superconducting state is protected; its emergent behavior is independent of the details. As a result, a quite simple model that incorporates the "right stuff"—the gateways to superconducting behavior we noted above—can lead to a remarkably accurate description of its emergent behavior. In this section, we will trace the steps from 1950 to 1956 that led to the theory. The next section will outline the theory itself. And later in this unit, we will show how a simple extension of the BCS framework from the Standard Model considered in their original paper offers the prospect of explaining the properties of the unconventional superconductors at the center of current research on correlated electron matter.

Four decades of failed theories

In 1950, nearly 40 years after its discovery, the prospects for developing a microscopic theory of superconductivity still looked grim. Failed attempts to solve this outstanding physics challenge by the giants in the field, from Einstein, Bohr, Heisenberg, Bloch, and Landau to the young John Bardeen, led most theorists to look elsewhere for promising problems on which to work.

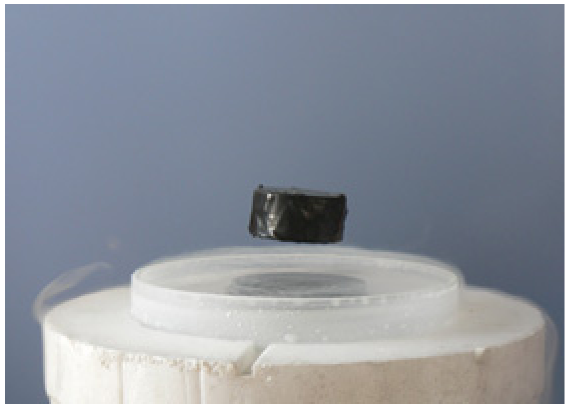

Figure 14: A photograph such as this of a levitating magnet is arguably the iconic image for superconductivity. It provides a vivid demonstration the way in which the near perfect diamagnetism of a superconducting material (the Meissner effect) makes it possible to levitate magnets above it.
Source: © Wikimedia Commons, GNU Free Documentation License, Version 1.2. Author: Mai-Linh Doan, 13 October 2007.

Despite that, experimentalists had made considerable progress on the properties of the superconducting state. They realized that a strong enough applied external magnetic field could destroy superconductivity and that a superconductor's almost perfect diamagnetism—its ability to shield out an external magnetic field within a short distance, known as the "penetration depth"—was key to an explanation. Theorists found that a two-fluid model analogous to that considered for superfluid ^{4}He in the previous section could connect many experimental results. Moreover, London had argued eloquently that the perfect diamagnetism could be explained by the rigidity of the superfluid wavefunction in the presence of an external magnetic field, while progress occurred on the "phase transition" front as Vitaly Ginzburg and Landau showed how to extend Landau's general theory of second-order phase transitions to superconductors to achieve an improved phenomenological understanding of emergent superconducting behavior.

Superconductivity was obviously an amazing emergent electronic phenomenon, in which the transition to the superconducting state must involve a fundamental change in the ground and excited states of electron matter. But efforts to understand how an electron interaction could bring this about had come to a standstill. A key reason was that the otherwise successful nearly free electron model offered no clues to how an electron interaction that seemed barely able to affect normal state properties could turn some metals into superconductors.

A promising new path

The Double Laureate and His Colleagues

Photo of John Bardeen.
Source: © Department of Physics, University of Illinois at Urbana-Champaign, courtesy AIP Emilio Segrè Visual Archives.

In 1951, after professional differences had undermined the relationship of John Bardeen and Walter Brattain with William Shockley, their team leader in the invention of the transistor at Bell Telephone Laboratories, Bardeen took a new job, as professor of electrical engineering and of physics in the University of Illinois at Urbana-Champaign. There, he was able to pursue freely his interest in superconductivity, setting out various lines of research to take on the immense challenge of understanding its nature. By 1957, when Bardeen, his postdoctoral research associate Leon Cooper, and his graduate assistant Robert Schrieffer developed what came to be known as the BCS theory, Bardeen had received the 1956 Nobel Prize in Physics for discovering the transistor. Bardeen knew that BCS was also worthy of the prize. But since no one had received a second Nobel Prize in the same subject, he reportedly worried that because of his earlier Nobel Prize, and his role in BCS, his two colleagues would not be eligible. Fortunately, the Nobel Committee eventually saw no reason to deny the prize to BCS: It awarded the three men the 1972 physics prize, in the process making Bardeen the first individual to become a double laureate in the same field.

Matters began to change in 1950 with the discovery of the isotope effect on the superconducting transition temperature, T_c, by Bernard Serin at Rutgers University, Emanuel Maxwell at the National Bureau of Standards, and their colleagues. They found that T_c for lead varied inversely as the square root of its isotopic mass. That indicated that quantized lattice vibrations must be playing a role in bringing about that transition, for their average energy was the only physical quantity displaying such a variation.

That discovery gave theorists a new route to follow. Herbert Fröhlich and Bardeen independently proposed theories in which superconductivity would arise through a change in the self-energy of individual electrons produced by the co-moving cloud of phonons that modify their mass. But, it soon became clear that efforts along these lines would not yield a satisfactory theory.

Frohlich then suggested in 1952 that perhaps the phonons played a role through their influence on the effective electron interaction. The problem with his proposal was that it was difficult to see how such an apparently weak phonon-induced interaction could play a more important role than the much stronger repulsive electrostatic interaction he had neglected. Two years later, Bardeen and his first postdoctoral research associate at the University of Illinois at Urbana-Champaign—myself—resolved that problem.

We did so by generalizing the collective coordinate formalism that David Bohm and I had developed for electron-electron interactions alone to derive their effective interaction between electrons when both the effects of the electrostatic interaction and the electron-phonon coupling are taken into account (Figure 15). Surprisingly, we found that, within the random phase approximation, the phonon-induced interaction could turn the net interaction between electrons lying within a characteristic phonon frequency of the Fermi surface from a screened repulsive interaction to an attractive one. We can imagine the phonon-induced interaction as the electronic equivalent of two children playing on a waterbed. One (the polarizer) makes a dent (a density wave) in the bed; this attracts the second child (the analyzer), so that the two wind up closer together.

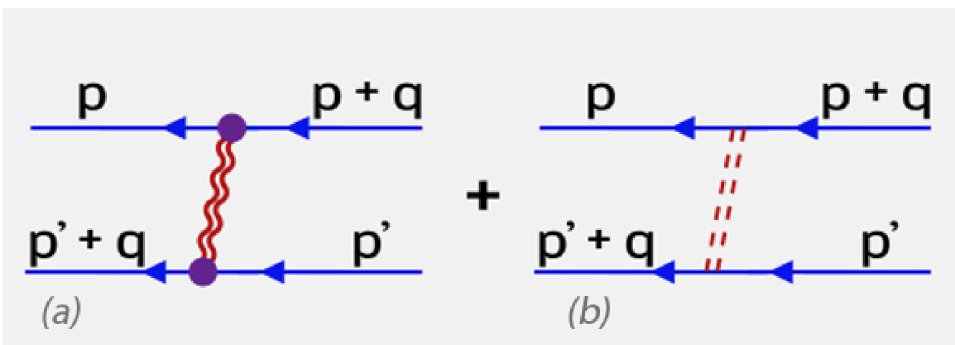

Figure 15: The net effective interaction between electrons in a metal, with (a) depicting the net phonon-induced interaction and (b) their intrinsic effective screened electrostatic interaction.
Source: © David Pines.

Leon Cooper, who replaced me in Bardeen's research group in 1955, then studied the behavior of two electrons of opposite spin and momentum near the Fermi surface using a simplified version of the Bardeen-Pines attractive interaction. In a 1956 calculation that allowed for the multiple scattering of the pair above the Fermi surface, he showed that net attraction produced an energy gap in the form of a bound state for the electron pair.

During this period, in work completed just before he went to Stockholm to accept his 1956 Nobel Prize, Bardeen had showed that many of the key experiments on superconductivity could be explained if the "normal" elementary excitations of the two-fluid model were separated from the ground state by an energy gap. So the gap that Cooper found was intriguing. But there was no obvious way to go from a single bound pair to London's coherent ground state wavefunction that would be rigid against magnetic fields. The field awaited a breakthrough.

Section 5: The BCS Theory

The breakthrough came in January 1957, when Bardeen's graduate student, Robert Schrieffer, while riding a New York City subway train following a conference in Hoboken, NJ on The Many-Body Problem, wrote down a candidate wavefunction for the ground state and began to calculate its low-lying excited states. He based his wavefunction on the idea that the superconducting condensate consists of pairs of quasiparticles of opposite spin and momenta. This gateway to emergent superconducting behavior is a quite remarkable coherent state of matter; because the pairs in the condensate are not physically located close to one another, their condensation is not the Bose condensation of pairs that preform above the superconducting transition temperature in the normal state. Instead, the pairs condense only below the superconducting transition temperature. The typical distance between them, called the "coherence length," is some hundreds of times larger than the typical spacing between particles.

To visualize this condensate and its motion, imagine a dance floor in which one part is filled with couples (the pairs of opposite spin and momentum) who, before the music starts (that is, in the absence of an external electric field), are physically far apart (Figure 16). Instead of being distributed at random, each member of the couple is connected, as if by an invisible string, to his or her partner faraway. When the music begins (an electric field is applied), each couple responds by gliding effortlessly across the dance floor, moving coherently with the same velocity and never colliding with one another: the superfluid motion without resistance.

Figure 16: Like the condensate, these coupled dancers came together when the music started and continued in a fluid motion next to each other without bumping into each other or stepping on each other's toes.
Source: © Bruce Douglas.

Sorting out the details of the theory

Back in Urbana, Schrieffer, Bardeen, and Cooper quickly worked out the details of the microscopic theory that became known as BCS. A key feature was the character of the elementary excitations that comprise the normal fluid. We can describe these as quasiparticles. But in creating them, we have to break the pair bonds in the condensate. This requires a finite amount of energy—the energy gap. Moreover, each BCS quasiparticle carries with it a memory of its origin in the pair condensate; it is a mixture of a Landau quasiparticle and a Landau quasihole (an absence of a quasiparticle) of opposite spin.

Inspiration for the Critical Breakthrough

Schrieffer found the inspiration for his wavefunction in earlier work done by T. D. Lee, Francis Low, and myself on a quite different problem: that of understanding the behavior of a polaron—an electron moving in a polar crystal that is strongly coupled to its lattice vibrations. When I arrived in Urbana in 1952, John Bardeen suggested that insight into the next step on a microscopic, phonon-based theory of superconductivity might come from a solution of the polaron problem that went beyond perturbation theory. Lee and I found such a solution by adapting an approach developed by Sin-Itro Tomonaga for mesons coupled to nuclei. With the help of our Urbana colleague Francis Low, we then wrote the wavefunction for the solution in an especially simple way. The LLP wavefunction describes a coherent state in which the phonons in the cloud of strongly coupled phonons around the electron are emitted successively into the same momentum state; it took the form:

$$\Psi_{LLP} \sim \Pi_k \exp \left[\Sigma_k \left[f(k) \left[a_k{}^* + a_{-k} \right] \Psi_0 \right.\right.$$

where Ψ_0 *was the ground state wavefunction, the operators* $a_k{}^*$ *and* a_k *act to create or destroy phonons, and f(k) describes the phonon state. Schrieffer's brilliant insight was to try a ground state wavefunction for the superconductor in which the LLP phonon field was replaced by the pair field of the condensate,* $b_k{}^* = c_{k\uparrow}{}^* c_{-k\downarrow}{}^*$*, where the c* are creation operators for a single electron (quasiparticle); his wavefunction thus took the form:*

$$\Psi \sim \Pi_k \exp \left[\Sigma_k b_k{}^* f(k) \right] \Psi_0$$

which reduces to the BCS wavefunction,

$$\Psi_{bcs} \sim \Pi_k \left[1 + \Sigma_k b_k{}^* f(k) \right] \Psi_0$$

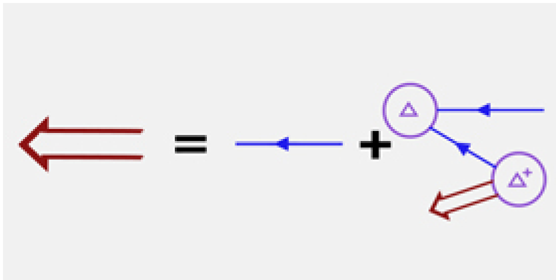

Figure 17: An illustration of the physical process by which a BCS quasiparticle becomes a mixture of a normal state quasiparticle and a quasihole and in so doing acquires an energy gap.
Source: © David Pines.

Figure 17 illustrates the concept. Because the key terms in the effective interaction of the BCS quasiparticle with its neighbors are those that couple it to the condensate, a quasiparticle that scatters against the condensate emerges as a quasihole of opposite spin. The admixture of Landau quasiholes with quasiparticles in a BCS quasiparticle gives rise to interference effects that lead the superconductor to respond differently to probes that measure its density response and probes that measure its spin response. Indeed, the fact that BCS could explain the major difference between the results of acoustic attenuation measurements that probe the density and nuclear spin relaxation measurements of the spin response of the superconducting state in this way provided definitive proof of the correctness of the theory.

The basic algebra that leads to the BCS results is easily worked out. It shows that the energy spectrum of the quasiparticles in the superconducting state takes an especially simple form:

$$E_p = [\varepsilon_p^2 + \Delta^2]^{1/2}$$

Here, ε_p is the normal state quasiparticle energy and Δ the temperature-dependent superconducting energy gap, which also serves as the order parameter that characterizes the presence of a superconducting condensate. When it vanishes at T_c, the quasiparticle and the metal revert to their normal state behavior.

The impact of BCS theory

The rapid acceptance by the experimental low-temperature community of the correctness of the BCS theory is perhaps best epitomized by a remark by David Shoenberg at the opening of a 1959 international conference on superconductivity in Cambridge: "Let us now see to what extent the experiments fit the theoretical facts." Acceptance by the theoretical community came less rapidly. Some of those who had failed to devise a theory were particularly reluctant to recognize that BCS had solved the problem. (Their number did not include Feynman, who famously recognized at once that BCS had solved the problem to which he had just devoted two years of sustained effort, and reacted by throwing into the nearest wastebasket the journal containing their epochal result.) The objections of the BCS deniers initially centered on the somewhat arcane issue of gauge invariance. With the rapid resolution of that issue, the objections became more diffuse. Some critics persisted until they died, a situation not unlike the reaction of the physics community to Planck's discovery of the quantum.

Searching for Superfluid 3He

Not long after BCS published their epochal paper, both the theoretical and experimental low-temperature community recognized that ^{3}He would likely become a superfluid at some low temperature. But how low, and what form would that superfluidity take? Early on the community realized that because the spatial average of the effective interaction between the ^{3}He atoms measured by the dimensionless spin-symmetric Landau parameter, f_0^s, is positive (the strong short-range repulsion wins out over the weak long-range attraction), it was likely that the superfluid pairing would not be in the simple $^1S_0$ state found for conventional metallic superconductors. However, all attempts to predict the pairing state, much less the temperature at which superfluidity would be found, failed; while experimentalists searched for evidence for superfluid behavior at increasingly lower temperatures that were in the millikelvin range. Along the way, there were false sightings, notably a report by Peshkov at a low-temperature meeting in 1964 that was sharply and correctly criticized by John Wheatley at that same meeting. The discovery came in 1972 during an experimental study by Doug Osheroff, then a graduate student, David Lee, and Bob Richardson at Cornell University of the changes of the pressure as the volume of a sample of liquid ^{3}He that had been cooled to 2 x 10^{-3} K was slowly increased and then reduced. Tiny glitches that appeared in their results were at first attributed to solidification, but subsequent work when combined with a key interpretation of their results by Tony Leggett of Sussex University, who was visiting Cornell at the time, showed they had observed the onset of superfluid behavior, and that three different anisotropic superfluid phases could be identified. The coupling of the ^{3}He quasiparticles to the nearly antiferromagnetic spin fluctuations of the background liquid plays a key role in determining the anisotropic pairing states, which possess the common feature of being in a state in which the pairs have parallel spin and a p-wave relative orbital angular momentum, $l = 1$. Because the Nobel committee was reluctant to break its rule of awarding the prize to no more than three individuals, their work was recognized by separate Nobel Prizes, the first to Lee, Osheroff, and Richardson in 1996, and the second to Leggett in 2003.

For most physicists, however, the impact of BCS was rapid and immense. It led to the 1957 proposal of nuclear superfluidity, a 15-year search for superfluid ^{3}He, and to the exploration of the role played by pair condensation in particle physics, including the concept of the Higgs boson as a collective mode of a quark-gluon condensate by Philip W. Anderson and Peter Higgs. It led as well to the suggestion of cosmic hadron superfluidity, subsequently observed in the behavior of pulsars following a sudden jump in their rotational frequency, as we will discuss in Section 8.

In addition, BCS gave rise to the discovery of emergent behavior associated with condensate motion. That began with the proposal by a young Cambridge graduate student, Brian Josephson, of the existence of currents associated with the quantum mechanical tunneling of the condensate wavefunction through thin films, called "tunnel junctions," that separate two superconductors. In retrospect, Josephson's 1962 idea was a natural one to explore. If particles could tunnel through a thin insulating barrier separating two normal metals, why couldn't the condensate do the same thing when one had a tunnel junction made up of two superconductors separated by a thin insulating barrier? The answer soon came that it could, and such superconducting-insulating-superconductor junctions are now known as "Josephson junctions." The jump from a fundamental discovery to application also came rapidly. Superconducting quantum interference devices (SQUIDS) (Figure 18), now use such tunneling to detect minute electromagnetic fields, including an application in magnetoencephalography—using SQUIDS to detect the minute magnetic fields produced by neurocurrents in the human brain.

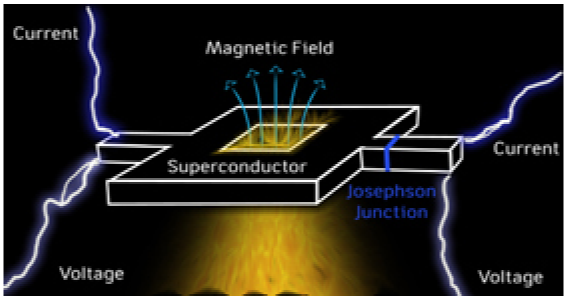

Figure 18: A Superconducting Quantum Interference Device (SQUID) is the most sensitive type of detector of magnetic fields known to science.
Source:

Still another fascinating property of a superconductor was discovered by a young Soviet theorist, Alexander Andreev, who realized that when an electron is reflected from the surface of a superconductor, because its wavefunction samples the condensate, it can break one of the pairs in the superconducting condensate, and so emerge as a hole. Measurements by point contact spectroscopy show this dramatic effect, known now as "Andreev reflection," which is the condensed matter equivalent of changing a particle into an antiparticle through a simple physical process.

Looking back at the steps that led to BCS as the Standard Model for what we now describe as conventional superconductors, a pattern emerges. Bardeen, who was key to the development of the theory at every stage from 1950 to 1957, consistently followed what we would now describe as the appropriate emergent strategy for dealing with any major unsolved problem in science:

- Focus first on the experimental results via reading and personal contact.
- Explore alternative physical pictures and mathematical descriptions without becoming wedded to any particular one.
- Thermodynamic and other macroscopic arguments have precedence over microscopic calculations.
- Aim for physical understanding, not mathematical elegance, and use the simplest possible mathematical description of system behavior.
- Keep up with new developments in theoretical techniques—for one of these may prove useful.
- Decide at a qualitative level on candidate organizing concepts that might be responsible for the most important aspect of the measured emergent behavior.
- Only then put on a "reductionist" hat, proposing and solving models that embody the candidate organizing principles.

Section 6: New Superconductors

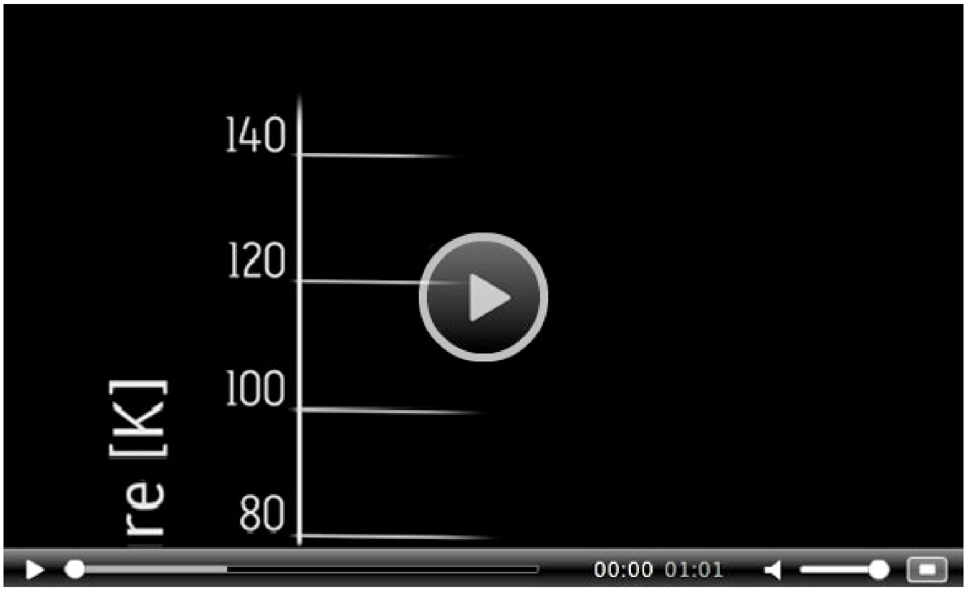

Figure 19: The first superconducting material was discovered in 1911 when mercury was cooled to 4 Kelvin (K). Seventy-five years later, thanks to the discovery of superconductivity in the family of cuprate materials by Bednorz and Mueller, scientists made a giant leap forward as they discovered many related materials that superconduct at temperatures well above 90 K.
Source:

Physics with the Whole World Watching

The discovery of materials exhibiting superconducting behavior above 23 K, the highest known transition temperature for traditional superconductors, created a huge, if delayed, response around the world. The reaction exploded at an event that became known as the "Woodstock of physics."

J. Georg Bednorz and K. Alex Müller of the IBM Zurich Research Laboratory had reported their discovery of a ceramic material with a superconducting transition temperature of 30 K in a German physics journal in April 1986. The news drew little immediate response. But excitement rose as other groups confirmed the find and discovered new high-T_c *superconductors (including one with a* T_c *of 93 K reported by Paul Chu's team at the University of Houston). By 18 March 1987 the topic had gained so much traction that the American Physical Society (APS) added a last-minute session on it at its annual meeting in New York City. When the session started at 7:30 p.m., about 2,000 people filled the hall. Others watched the event on video monitors. And although organizers limited the 51 speakers' time on the podium, the meeting continued until 3:15 the next morning.*

Characteristically, New York City embraced the event. That week, an APS badge guaranteed its wearer entry to several nightclubs without the need to pay a cover charge.

The past three decades have seen an outpouring of serendipitous discoveries of new quantum phases of matter. The most spectacular was the 1986 discovery by IBM scientists J. Georg Bednorz and K. Alex Müller of superconductivity at high temperatures in an obscure corner of the periodic table: a family of ceramic materials of which $La_xSr_{1-x}CuO_4$ (containing the elements lanthanum, strontium, copper, and oxygen) was a first example. By the American Physical Society meeting in March 1987 (often referred to as the "Woodstock of physics"), it was becoming clear that this was just the first of a large new family of cuprate superconductors that possess two factors in common. They have planes of cupric oxide (CuO_2) that can be doped with mobile electrons or holes. And the quasiparticles in the planes exhibit truly unusual behavior in their normal states while their superconducting behavior differs dramatically from that of the conventional superconductors in which phonons supply the pairing glue.

Over the past two decades, thanks to over 100,000 papers devoted to their study, we have begun to understand why the cuprate superconductors are so different. Moreover, it is now clear that they represent but one of an extended family of unconventional superconductors with three siblings: the heavy electron superconductors discovered in 1979; the organic superconducting materials discovered in 1981; and the iron-based superconductors discovered in 2006. Although there is a considerable range in their maximum values of T_c—about 160 K for a member of the cuprate family, $HgBa_2Ca_2Cu_3O_x$, under pressure; roughly 56 K in the iron pnictides ($LnFeAsO_{1-x}$); and 18.5 K for $PuGaIn_5$, a member of the 115 ($RMIn_5$) family of heavy electron materials—they show remarkable similarities in both their transport and magnetic properties in the normal and superconducting states.

In particular, for all four siblings:

- Superconductivity usually occurs on the border of antiferromagnetic order at which the magnetic moments of atoms align.
- The behavior of the quasiparticles, density fluctuations, and spin fluctuations in their normal state is anomalous, in that it is quite different from that of the Landau Fermi liquids found in the normal state of liquid ^{3}He and conventional superconductors.
- The preferred superconducting pairing state is a singlet state formed by the condensation of pairs of quasiparticles of opposite spin in an orbital angular momentum, l, state, with l = 2; as a result, the superconducting order parameter and energy gap vary in configuration and momentum space.

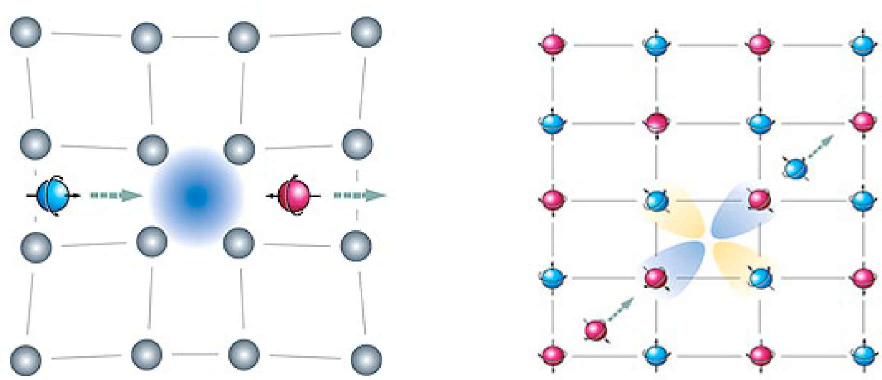

Figure 20: Left: Conventional superconductors and right: heavy-electron superconductors.
Source: © Los Alamos National Laboratory.

In this section, we can explore only a small corner of this marvelous variety of materials whose unexpected emergent properties continue to surprise and offer considerable promise for commercial application. To understand these, we must go beyond the standard model in which phonons provide the glue that leads to attraction. We explore the very real possibility that the net effective attraction between quasiparticles responsible for their superconductivity occurs without phonons and is of purely magnetic origin. In so doing, we enter territory that is still being explored, and in which consensus does not always exist on the gateways to the emergent behavior we find there.

Heavy electron materials

We begin with the heavy electron materials for three reasons. First, and importantly, they can easily be made in remarkably pure form, so that in assessing an experiment on them, one is not bedeviled by “dirt” that can make it difficult to obtain reliable results from sample to sample. Second, the candidate organizing concepts introduced to explain their behavior provide valuable insight into the unexpected emergent behavior seen in the cuprates and other families of unconventional superconductors. Third, these materials display fascinating behavior in their own right.

Heavy electron materials contain a lattice, called the "Kondo lattice," of localized outer f-electrons of cerium or uranium atoms that act like local magnetic moments magnetically coupled through their spins to a background sea of conduction electrons. It is called a Kondo lattice because in isolation each individual magnetic moment would give rise to a dramatic effect, first identified by Jun Kondo, in which the magnetic coupling of the conduction electrons to the moment acts to screen out the magnetic field produced by it, while changing the character of their resistivity, specific heat, and spin susceptibility. When one has a lattice of such magnetic moments, the results are even more dramatic; at low temperatures, their coupling to the conduction electrons produces an exotic new form of quantum matter in which some of the background conduction electrons form a heavy electron Fermi liquid for which specific heat measurements show that the average effective mass can be as large as that of a muon, some 200 or more bare electron masses. The gateway to the emergence of this new state of matter, the heavy electron non-Landau Fermi liquid, is the *collective* entanglement (hybridization) of the local moments with the conduction electrons.

Remarkably, the growth of the heavy electron Fermi liquid, which we may call a "Kondo liquid (KL)" to reflect its origin in the collective Kondo lattice, displays scaling behavior, in that its emergent coherent behavior can be characterized by the temperature, T*, at which collective hybridization begins.

Below T*, the specific heat and spin susceptibility of the emergent Kondo liquid display a logarithmic dependence on temperature that reflects their coherent behavior and collective origin. Its emergent behavior can be described by a two-fluid model that has itself emerged only recently as a candidate standard phenomenological model for understanding Kondo lattice materials. In it, the strength of the emergent KL is measured by an order parameter, f (T/T*), while the loss in strength of the second component, the local moments (LMs) that are collectively creating the KL, is measured by 1-f(T/T*). KL scaling behavior, in which the scale for the temperature dependence of all physical quantities is set by T*, persists down to a temperature, T_0 close to those at which the KL or LM components begin to become ordered.

Phase diagram of a work in progress

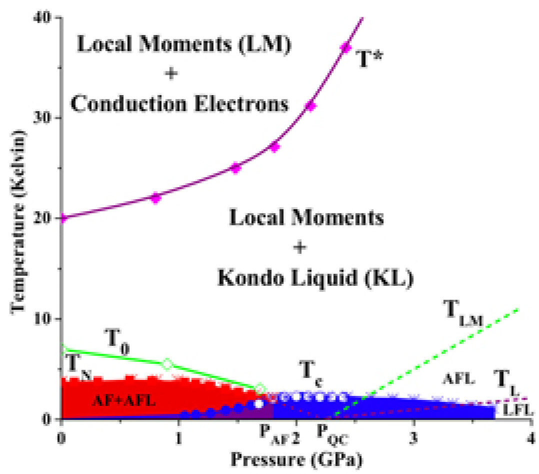

Figure 21: A candidate phase diagram for $CeRhIn_5$ depicting the changes in its emergent behavior and ordering temperatures as a function of pressure.
Source: © David Pines.

The accompanying candidate phase diagram (Figure 21) for the heavy electron material $CeRhIn_5$ (consisting of cerium, rhodium, and indium) gives a sense of the richness and complexity of the emergent phenomena encountered as a result of the magnetic couplings within and between the coexisting KL and LM components as the temperature and pressure are varied. It provides a snapshot of work in progress on these fascinating materials—work that will hopefully soon include developing a microscopic theory of emergent KL behavior.

Above T*, the local moments are found to be very weakly coupled to the conduction electrons. Below T*, as a result of an increasing collective entanglement of the local moments with the conduction electrons, a KL emerges from the latter that exhibits scaling behavior between T* and T_0; as it grows, the LM component loses strength. T_0 is the temperature below which the approach of antiferromagnetic or superconducting order influences the collective hybridization process and ends its scaling behavior.

Electronic order in metals

Electrons in a metal can become ordered through the magnetic coupling of their spins or the electrostatic interaction of their charges. The magnetic order can be ferromagnetic, as in iron, corresponding to a lattice of localized electron spins all of which point in the same direction, or antiferromagnetic, corresponding to a lattice of localized electron spins in which nearest neighbor spins point in opposite directions. The charge order can be localized, in which case electrons are no longer free to move throughout the metal, or coherent, in which case the electrons become superconducting. Interestingly, since a magnetic interaction between electron spins can bring about both superconductivity (as we discuss below) and antiferromagnetic order, one finds in some materials a competition between these two forms of order, a competition that is sometimes resolved, as is the case for $CeRhIn_5$*, by both forms of competing order coexisting in a given material.*

At T_N, the residual local moments begin to order antiferromagnetically, as do some, but not all, of the KL quasiparticles. The remaining KL quasiparticles become superconducting in a so-called d_{x2-y2} pairing state; as this state grows, the scale of antiferromagnetic order wanes, suggesting that superconductivity and antiferromagnetic order are competing to determine the low-temperature fate of the KL quasiparticles.

When the pressure, P, is greater than P_{AF}, superconductivity wins the competition, making long-range antiferromagnetic LM order impossible. The dotted line continuing TN toward zero for P greater than P_{AF} indicates that LM ordering is still possible if superconductivity is suppressed by application of a large enough external magnetic field. Experimentalists have not yet determined what the Kondo liquid is doing in this regime; one possibility is that it becomes a Landau Fermi liquid.

Starting from the high pressure side, P_{QC} denotes the point in the pressure phase diagram at which local moments reappear and a localized (AF) state of the quasiparticles first becomes possible. It is called a "quantum critical point" because in the absence of superconductivity, one would have a T = 0 quantum phase transition in the Kondo liquid from itinerant quasiparticle behavior to localized AF behavior.

Since spatial order is the enemy of superconductivity, it should not be surprising to find that in the vicinity of P_{QC}, the superconducting transition temperature reaches a maximum—a situation we will see replicated in the cuprates and one likely at work in all the unconventional superconducting materials. One explanation is that the disappearance of local moments at P_{QC} is accompanied by a jump in the size of the conduction electron Fermi surface; conversely, as the pressure is reduced below P_{QC}, a smaller Fermi surface means fewer electrons are capable of becoming superconducting, and both the superconducting transition temperature and the condensation energy, the overall gain in energy from becoming superconducting, are reduced.

In the vicinity of a quantum critical point, one expects to find fluctuations that can influence the behavior of quasiparticles for a considerable range of temperatures and pressures. Such quantum critical (QC) behavior provides yet another gateway for emergent behavior in this and other heavy electron materials. It reveals itself in transport measurements. For example, in $CeRhIn_5$ at high pressures, one gets characteristic Landau Fermi liquid behavior (a resistivity varying as T^2) at very low temperatures; but as the temperature increases, one finds a new state of matter, quantum critical matter, in which the resistivity in the normal state displays anomalous behavior brought about by the scattering of KL quasiparticles against the QC fluctuations.

What else happens when the pressure is less than P_{QC}? We do not yet know whether, once superconductivity is suppressed, those KL quasiparticles that do not order antiferromagnetically exhibit Landau Fermi liquid behavior at low temperatures. But given their behavior for P less than P_{cr}, that seems a promising possibility. And their anomalous transport properties above T_N and T_c suggest that as the temperature is lowered below T_0, the heavy electron quasiparticles exhibit the anomalous transport behavior expected for quantum critical matter.

Superconductivity without phonons

We turn now to a promising candidate gateway for the unconventional superconducting behavior seen in this and other heavy electron materials—an enhanced magnetic interaction between quasiparticles brought about by their proximity to an antiferromagnetically ordered state. In so doing, we will continue to use BCS theory to describe the onset of superconductivity and the properties of the superconducting state. However, we will consider its generalization to superconducting states in which pairs of quasiparticles condense into states of higher relative angular momentum described by order parameters that vary in both configuration and momentum space.

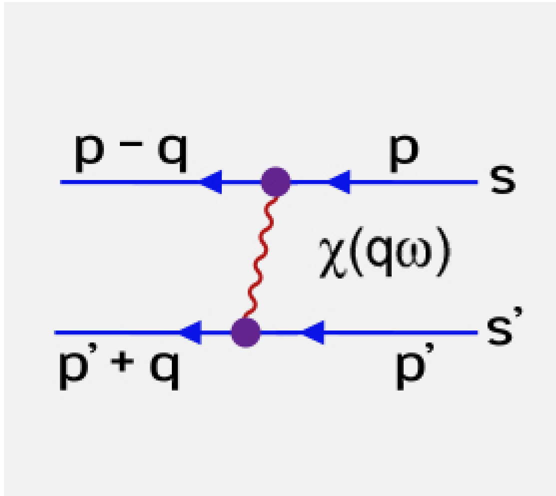

Figure 22: The magnetic quasiparticle interaction between spins s and s' induced by their coupling to the spin fluctuations, $\chi q\omega$, of the magnetic background material.
Source: © David Pines.

To see how the magnetic gateway operates, we turn to Figure 22 which illustrates how the magnetic interaction between two quasiparticles, whose spins are s and s', can be modified by their coupling to the spin fluctuations characteristic of the background magnetic behavior of the material in which they are located. Quite generally, spin, s, located, say, at the origin, acts to polarize the material by inducing a spin fluctuation; this induced magnetization, in turn, couples to the second spin, s', located a distance r away, producing an induced effective magnetic interaction which is analogous to the phonon-induced interaction responsible for superconductivity in ordinary BCS superconductors.

This induced effective magnetic interaction is highly sensitive to the magnetic properties of the background material. For an ordinary paramagnet exhibiting weakly magnetic behavior, the resulting magnetic interaction is quite weak and unremarkable. If, however, the background material is close to being antiferromagnetic, the spectrum of the spin fluctuations that provide the glue connecting the two spins becomes highly momentum dependent, exhibiting a significant peak for wave vectors that are close to those for which one finds a peak in the wave vector dependent magnetic susceptibility of the almost magnetically ordered material. As a result, the induced magnetic quasiparticle interaction will be strong and spatially varying.

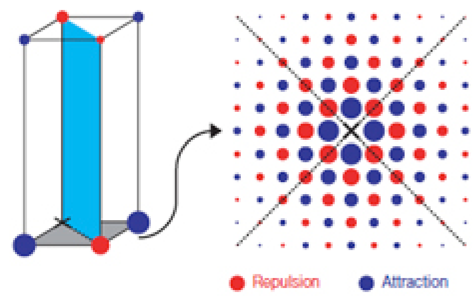

Figure 23: Magnetic interaction potential in a lattice.
Source: © Reprinted by permission from Macmillan Publishers Ltd: Nature 450, 1177–1183 (18 December 2007).

Consider, for example, a magnetic material that is at a pressure near the point at which the material exhibits simple two-dimensional planar commensurate AF order (in which the nearest neighbor spins point in opposite directions). Its momentum dependent susceptibility will then have a peak at the commensurate wave vector Q =[π/a, π/a] where a is the lattice spacing, as will its spin fluctuation spectrum. The corresponding induced magnetic quasiparticle interaction in configuration space will then be repulsive at the origin, attractive at its nearest neighbor sites, repulsive at next nearest neighbor sites, etc., as shown in Figure 23. See the math

Such an interaction, with its mixture of repulsion and attraction, does not give rise to the net attraction required for superconductivity in the conventional BCS singlet s-wave pairing state with an order parameter and energy gap that do not vary in space. The interaction can, however, be remarkably effective in bringing about superconductivity in a pairing state that varies in momentum and configuration space in such a way as to take maximum advantage of the attraction while possessing nodes (zeros) that minimize the repulsion. A dx2-y2 pairing state, the singlet d-wave pairing state characterized by an order parameter and energy gap Δ_{x2-y2} (k)= Δ [cos (k_xa)-cos (k_ya)], does just that, since it has nodes (zeroes) where the interaction is repulsive (at the origin or along the diagonals, for example) and is maximal where the interaction is maximally attractive (e.g., at the four nearest neighbor sites) as may also be seen in Figure 23.

We call such superconductors "gapless" because of the presence of these nodes in the gap function. Because it costs very little energy to excite quasiparticles whose position on the Fermi surface puts them at or near a node, it is the nodal quasiparticle excitations which play the lead role in determining the normal fluid density. Their presence is easily detected in experiments that measure it, such as the low-temperature specific heat and the temperature dependence of the London penetration depth. Nodal quasiparticle excitations are also easily detected in NMR measurements of the uniform susceptibility and spin-lattice relaxation rate, and the latter measurements have verified that the pairing state found in the "high T_c" heavy electron family of $CeMIn_5$ materials, of which $CeRhIn_5$ is a member, is indeed d_{x2-y2}, the state expected from their proximity to antiferromagnetic order.

To summarize: In heavy electron materials, the coexistence of local moments and itinerant quasiparticles and their mutual interactions can lead to at least four distinct emergent states of matter: the Kondo heavy electron liquid, quantum critical matter, antiferromagnetic local moment order, and itinerant quasiparticle d_{x2-y2}, superconductivity, while the maximum superconducting transition temperature is found close to the pressure at which one finds a QCP that reflects the onset of local moment behavior. In the next section, we will consider the extent to which comparable emergent behavior is observed in the cuprate superconductors.

Physics for the 21st Century

Section 7: Emergent Behavior in the Cuprate Superconductors

Nevill Mott and his exploits

Nevill Mott at University of Bristol; International Conference on the Physics of Metals, organized by N. F. Mott and A. M. Tyndall.
Source: © Archives of HH Wills Physics Laboratory, University of Bristol, courtesy AIP Emilio Segrè Visual Archives.

Nevill Mott was a world leader in atomic and solid-state physics who combined a keen interest in experiment with a gift for insight and exposition during a career in theoretical physics that spanned over 60 years.

We can best appreciate the remarkable properties of the cuprate superconductors by considering a candidate phase diagram (Figure 24) that has emerged following almost 25 years of experimental and theoretical study described in well over 100,000 papers. In it, we see how the introduction of holes in the CuO planes through chemical substitution in materials such corresponding to $La_{1-x}Sr_xCuO_4$ or $YBa_2Cu_3O_{6+x}$, the low-temperature phase is one of antiferromagnetic order. The gateway to this emergent behavior is the very strong electrostatic repulsion between the planar quasiparticles. This causes the planar Cu d electron spins to localize (a process called "Mott localization" in honor of its inventor, Nevill Mott rather than be itinerant, while an effective antiferromagnetic coupling between these spins causes them to order antiferromagnetically. The magnetic behavior of these localized spins is remarkably well described by a simple model of their nearly two-dimensional behavior, called the "two-dimensional Heisenberg model;" it assumes that the only interaction of importance is a nearest neighbor coupling between spins of strength J.

The impact of adding holes

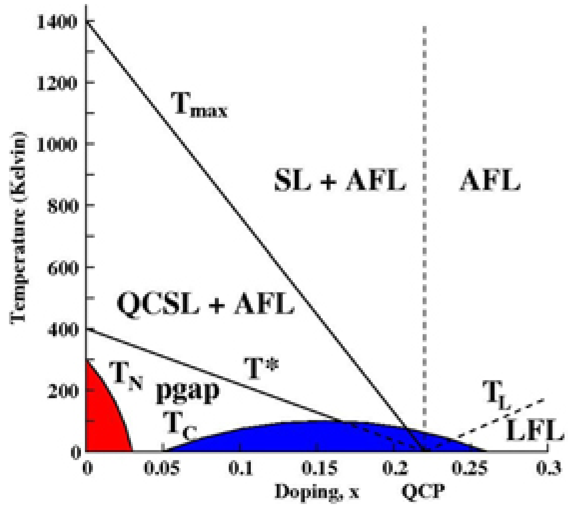

Figure 24: A candidate phase diagram based, in part, on magnetic measurements of normal state behavior, for the cuprate superconductors, in which are shown the changes in its emergent behavior and ordering temperatures as a function of the concentration of holes in the CuO planes.
Source: © David Pines.

When one adds holes to the plane, their presence has a number of interesting consequences for the localized Cu spins. Those, in turn, can markedly influence the behavior of the holes that coexist with them. The accompanying phase diagram (Figure 24) indicates some of the effects. Among them:

- Holes interfere with the long-range antiferromagnetic order of the localized spins, initially reducing its onset temperature, T_N, and then eliminating it altogether for hole doping levels x > 0.03.
- At higher hole doping levels, 0.03 < x < 0.22, the local spins no longer exhibit long-range order. Instead they form a spin liquid (SL) that exhibits short-range spin order and scaling behavior controlled by their doping-dependent interaction. The measured scaling behavior of the SL can be probed in measurements using nuclear magnetic resonance to probe the temperature-dependent uniform magnetic susceptibility and measure the relaxation time of ^{63}Cu probe nuclei. These show that for temperatures above T*(x), the SL can still be described by the 2-d Heisenberg model, with a doping-dependent interaction, $J_{eff}(x)$, between nearest neighbor spins whose magnitude is close to the temperature, $T_{max}(x)$, at which the SL magnetic susceptibility reaches a maximum. As the density of holes increases, both quantities decrease linearly with x.
- x = 0.22 is a quantum critical point (QCP) in that, absent superconductivity, one would expect a quantum phase transition there from localized to itinerant behavior for the remaining Cu spins.
- Between T_{max} and T*, the holes form an anomalous fermi liquid (AFL), whose anomalous transport properties are those expected for quantum critical matter in which the quasiparticles are scattered by the QC fluctuations emanating from the QCP at x ~ 0.22. Careful analysis of the nuclear spin-lattice relaxation rate shows that in this temperature range, the SL exhibits the dynamic quantum

critical behavior expected in the vicinity of 2d AF order, hence its designation as a quantum critical spin lLiquid, QCSL.

- Below T*, a quite unexpected new state of quantum matter emerges, pseudogap matter, so called because in it some parts of the quasihole Fermi surface become localized and develop an energy gap; the SL, which is strongly coupled to the holes, ceases to follow the two-dimensional Heisenberg scaling behavior found at higher temperatures.

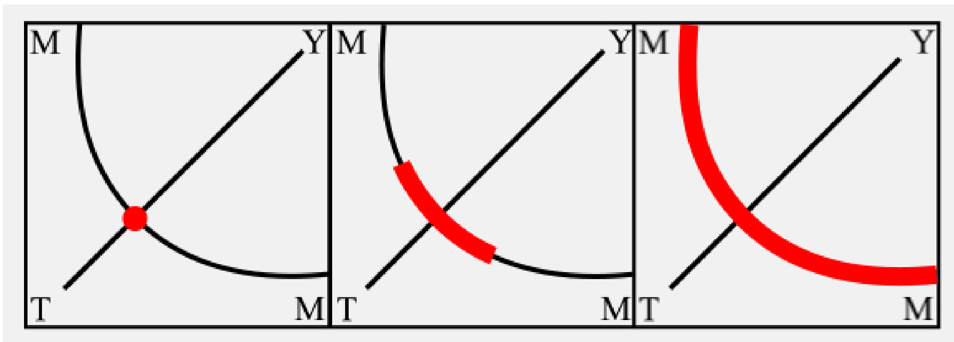

Figure 25: Schematic illustration of the temperature evolution of the Fermi surface in underdoped cuprates. The d-wave node below T_c (left panel) becomes a gapless arc in the pseudogap state above T_c (middle panel), which expands with increasing T to form the full Fermi surface at T* (right panel).
Source:

- For $0.05 < x < 0.17$, the hole concentration that marks the intersection of the T* line with T_c, the superconducting state that emerges from the pseudogap state is "weak"; some of the available quasiparticles have chosen, at a temperature higher than T_c, to become localized by condensing into the pseudogap state, and are therefore not available for condensation into the superconducting state. Their absence from itinerant behavior, illustrated in Figure 25, is seen, for example, in an ARPES (angle-resolved photoemission spectroscopy) probe of quasiparticles at the Fermi surface. Pseudogap matter and superconductivity thus compete for the low-temperature ordered state of the hole Fermi liquid in much the same way as antiferromagnetism and superconductivity compete in heavy electron materials.

- For $x > 0.17$, superconductivity wins the competition and is "strong," in that all available quasiparticles condense into the superconducting state. At these dopings, the pseudogap state does not form unless a magnetic field strong enough to destroy superconductivity is applied; when it is, the pseudogap state continues to form until one reaches the QCP at $x \sim 0.22$, behavior analogous to that found for the AF state in $CeRhIn_5$.

- Whether the superconductivity is weak or strong, the pairing state turns out to be the d_{x2-y2} state that, in the case of heavy electron materials, is the signature of a magnetic mechanism in which the magnetic quantum critical spin fluctuations provide the pairing glue. It is not unreasonable to conclude that the same physics is at work in the cuprates, with the nearly antiferromagnetic spin fluctuations playing a role for these unconventional superconductors that is analogous to that of phonons for conventional superconductors.

- The pseudogap state tends to form stripes. This tendency toward "inhomogeneous spatial ordering" reflects the competition between localization and itinerant behavior. It leads to the formation of fluctuating spatial domains that have somewhat fewer holes than the average expected for their doping level that are separated by hole-rich domain walls.

- Scanning tunneling microscope experiments (STM) (Figure 26) on the BSCCO members of the cuprate family at low temperatures show that, for doping levels less than x ~ 0.22, even the samples least contaminated by impurities exhibit a substantial degree of spatial inhomogeneity, reflected in a distribution of superconducting and pseudogap matter energy gaps.
- Just as in the case of heavy electrons, the maximum T_c is not far from the doping level at which the spatial order manifested in pseudogap behavior enters.

Ingredients of a theory

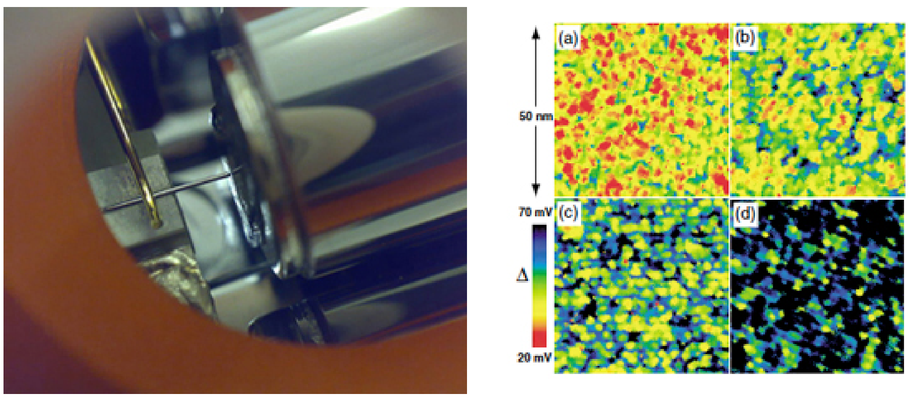

Figure 26: Left: A scanning tunneling microscope (STM) is a powerful instrument for imaging surfaces at the atomic level. Right: Inhomogeneous energy gaps measured in BSCCO; (a)-(d) correspond to doping levels that range from near optimal values of x = 0.19 seen in sample (a), for which T_c is 89 K, through levels of 0.15 (b), 0.13 (c) to the very underdoped material(d), for which x = 0.11 and T_c = 65 K; the color scales are identical.
Source: © Left: Wikimedia Commons, CC Share Alike 2.5 Generic License. Author: Royce Hunt, 5 March 2007. Right: K. McElroy, D.-H. Lee, J. E. Hoffman, K. M. Lang, J. Lee, E.W. Hudson, H. Eisaki, S. Uchida, and J. C. Davis.

We do not yet possess a full microscopic theory that explains these amazing emergent behaviors, but we see that the basic ingredients for developing such a theory are remarkably similar to those encountered in heavy electron materials. In both cuprates and heavy electron materials, local moments coexist with quasiparticles over a considerable portion of their generalized phase diagrams. Their mutual interaction and proximity to antiferromagnetism and a "delocalizing" quantum critical point lead to the emergence of quantum critical matter and d_{x2-y2} superconductivity, with the maximum T_c for the latter located not far from the QCP at which quasiparticle localization first becomes possible.

The principal differences are twofold: First, in the cuprates, the physical origin of the local moments is intrinsic, residing in the phenomenon of Mott localization brought about by strong electrostatic repulsion); second, in place of the AF order seen in heavy electron materials, one finds a novel ordered state, the pseudogap, emerging from the coupling of quasiparticles to one another and to the spin liquid formed by the Cu spins. It is the task of theory to explain this last result.

We can qualitatively understand the much higher values of T_c found in the cuprates as resulting from a mix of their much higher intrinsic magnetic energy scales as measured by the nearest neighbor LM interaction—J ~1000 K compared to the 50 K typically found in heavy electron materials—and their increased two-dimensionality.

Theories in competition

Our present understanding of emergent behaviors in the cuprates would not have been possible without the continued improvement in sample preparation that has led to materials of remarkable purity; the substantive advances in the use of probes such as nuclear magnetic resonance and inelastic neutron scattering, to study static and dynamic magnetic behavior in these materials; and the development of probes such as ARPES, STM, and the de Haas von Alphen effect that enable one to track their quasiparticle behavior in unprecedented detail. The brief summary presented here has scarcely done justice to the much more detailed information that has emerged from these and other experiments, while it is even more difficult to present at a level appropriate for this unit an overview of the continued efforts by theorists to develop a microscopic explanation of this remarkable range of observed emergent behaviors.

The theoretical effort devoted to understanding the cuprate superconductors is some orders of magnitude greater than that which went into the search for a microscopic theory of conventional superconductors. Yet, as of this writing, it has not been crowned by comparable success. Part of the reason is that the rich variety of emergent behaviors found in these materials by a variety of different experimental probes are highly sample-dependent; it has not yet proved possible to study a sample of known concentration and purity using all the different probes of its behavior. This has made it difficult to reconcile the results of different probes and arrive at candidate phenomenological pictures such as that presented above, much less to arrive at a fundamental theory.

Another aspect is the existence of a large number of competing theories, each of which can claim success in explaining some aspect of the phase diagram shown in Figure 24. The proponents of each have been reluctant to abandon their approach, much less accept the possibility that another approach has been successful. Since none of these approaches can presently explain the complete candidate phase diagram discussed above, developing a microscopic theory that can achieve this goal continues to be a major challenge in condensed matter theory.

Still another challenge is finding new families of superconductors. Theory has not notably guided that quest in the past. However, the striking similarities in the families of novel unconventional superconductors thus far discovered suggest one strategy to pursue in searching for new families of unconventional (and possibly higher T_c) superconductors: Follow the antiferromagnetism, search for layered materials with high values of J, and pay attention to the role that magnetic order can play in maximizing T_c. In so doing, we may argue that we have learned enough to speculate that, just as there was a "phonon" ceiling of some 30 K for T_c in conventional superconductors, there may be a "magnetic" ceiling for T_c in unconventional superconductors. Both may be regarded as reflecting a tendency for strong quasiparticle interactions to produce localization rather than superconductivity. The question, then, is whether we have reached this ceiling with a T_c of about 160 K or whether new materials will yield higher transition temperatures using magnetic glues, and whether there are nonmagnetic electronic routes to achieving still higher values of T_c.

Section 8: Superfluidity on a Cosmic Scale

The Quick and the Dense: Pulsars and Neutron Stars

When a star with between four and eight times the mass of our Sun approaches the end of its life, it undergoes gravitational collapse, seen as a supernova, with the end-point being a comparatively tiny object—a star in which the inward pressure of gravity is balanced by the outward quantum pressure of the mostly neutrons it contains. These neutron-rich celestial objects arise from when the relentless gravitational pressure within its parent object exceeds the thermal pressure produced by its nuclear. The resulting stellar collapse drives a conversion in the supernova core of protons and electrons into neutrons, essentially compressing the atomic matter into nuclear material. This eliminates the empty space in atoms, and produces an object of extraordinarily high density. The typical neutron star packs a mass about 1.4 times that of the Sun into a diameter of order 10 kilometers so that its density is of the order of that found in atomic nuclei, equivalent to packing all the people on Earth into a single raindrop, so that a teaspoonful of the matter in a neutron star would weigh one billion tons on Earth.

Although the possibility of neutron stars was first suggested by Walter Baade and Fritz Zwicky in 1934, it was not until 1967 that astronomers took the proposal seriously. That year, a Cambridge University graduate student, Jocelyn Bell, convinced her thesis supervisor, Antony Hewish, that her observation of a radio source pulsing on and off with extraordinary regularity was not due to a system malfunction. The Cambridge radio astronomers first called the object LGM-1 (for little green men 1) because its precision seemed to indicate communication from intelligent life. But within a few months, theorists, led by Cornell astrophysicist Thomas Gold, persuaded the astronomical community that a pulsar had to be a rotating neutron star containing a giant residual magnetic field whose magnitude may be estimated by assuming that flux is conserved in the implosion which formed it, so that a progenitor core magnetic field of ~1 gauss becomes amplified into a field of $\sim 10^{12}$ *gauss, which could emit electron and electromagnetic beams of radiation as it spins.*

So, while pulsars have not revealed anything about life elsewhere in the universe in the decades since their discovery, they have provided astrophysicists with a cosmic laboratory that can be used to study the behavior of matter at extraordinarily high densities, densities that are indeed the highest observable in our universe.

We conclude this unit with an introduction to some truly high T_c superfluids—the neutron superfluids found in the crust and core of a neutron star, for which T_c can be as large as 600,000 K. As we will see, the remarkable behavior of these superfluids not only enables us to study their emergent behavior by observing pulsars located many light-years away, but also establishes that these represent the most abundant superfluids in our universe.

Figure 27: The first director of the Los Alamos National Laboratory, Robert Oppenheimer (ca. 1944) was a brilliant theoretical physicist and inspired teacher who became famous for his remarkably effective leadership of the Manhattan Project.
Source: © Los Alamos National Laboratory.

Russian physicist Arkady Migdal suggested in the 1960s that cosmic hadron superfluids might exist in neutron stars. If the neutron stars studied by Robert Oppenheimer in 1939 existed, he reasoned, then in light of the fact that nuclei in the outer shells of terrestrial nuclei exhibited superfluid behavior, the neutrons these stars contained would surely be superfluid. Not long afterwards, Vitaly Ginzburg and David Kirshnitz argued that neutron stars, if indeed they existed, would surely rotate, in which case their rotation should be described in terms of the quantized vortex lines seen in liquid He.

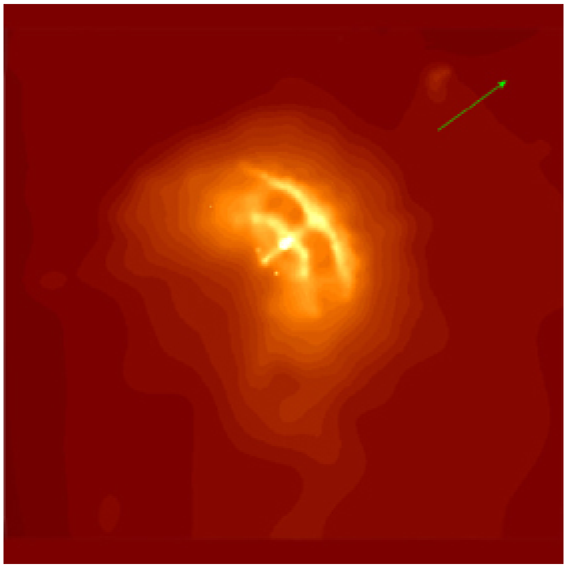

Figure 28: An image taken by the Chandra X-ray telescope of the Vela supernova remnant that shows dramatic bow-like structures produced by the interaction of radiation and electron beams coming from the rapidly rotating neutron star in its center with its immediate environment.
Source: © NASA, PSU, G. Pavlov et al., courtesy of Chandra X-ray Observatory.

The issue of such cosmic superfluidity remained a gleam in the theorist's eye until 1967. In that year, Cambridge graduate student Jocelyn Bell, who was studying atmospheric-produced scintillations of radio signals in Antony Hewish's radio astronomy laboratory, discovered pulsars. Astronomers soon identified these objects as rotating neutron stars which slowed down in remarkably regular fashion as they transferred their rotational energy into electromagnetic waves and accelerated electron beams.

Two years later, V. Radhakrishnan and Richard Manchester, using a radiotelescope in Australia, and Paul Reichley and George Downs, based at Caltech's Jet Propulsion Laboratory, independently observed that a comparatively young and fast pulsar, the Vela pulsar with an 89 ms period of rotation, "glitched." First, instead of continuing a remarkably regular spin-down produced by the transformation of its rotational energy into the beams of radio emission observed on Earth, it sped up by a few parts in a million.

Then, over some days to weeks, the sudden spin-up decayed. That a sudden spin-up of a tiny astronomical object with a radius of about 10 kilometers but a mass of the order of our Sun should occur at all is remarkable. Indeed, astronomers might have treated the glitch report as a malfunction of an observing radio telescope had not observers working independently in Australia and California both seen it. But perhaps more remarkable is the fact that astronomers could actually observe a glitch, since a response time for ordinary neutron matter would be about 10^{-4} seconds, the time it takes a sound signal to cross the star. So, under normal circumstances, a glitch and its response would be gone in less time than the twinkling of an eye.

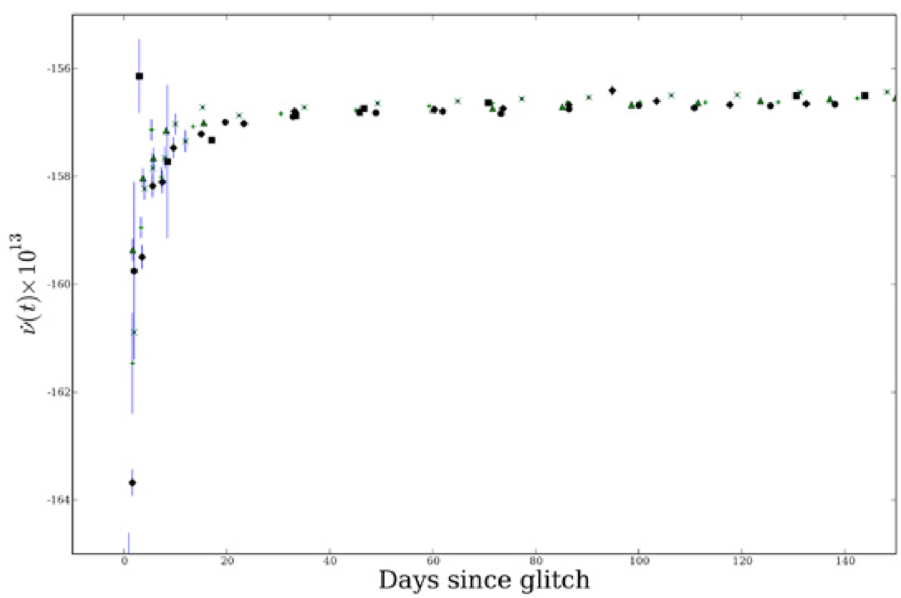

Figure 29: Radiotelescope observations of postglitch behavior in the Vela pulsar.
Source: © Sarah Buchner, Hartebeesthoek Radio Astronomy Observatory, South Africa.

The explanation was soon forthcoming: The slow decay time provided unambiguous evidence for the presence of superfluid neutrons in the pulsar. The reason was that these can change their angular momentum only by the postglitch motion of the vortex lines they carry and that process could easily be imagined to take days to months.

Why glitches occur

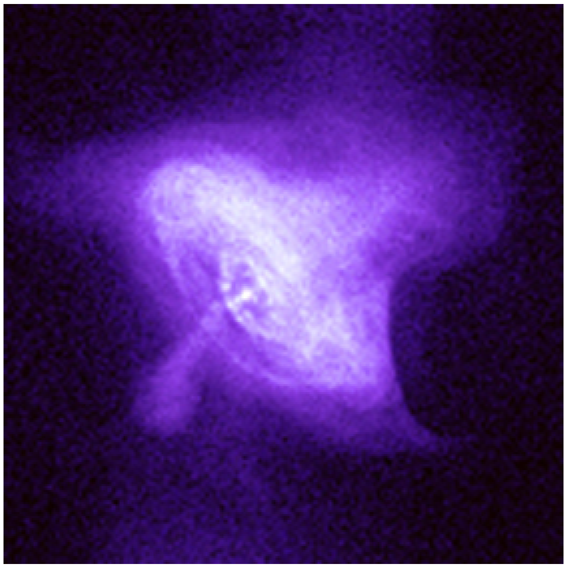

Figure 30: This recent image from the Chandra X-ray telescope shows the Crab Nebula, the remnant of a supernova explosion seen on Earth in 1054 AD that accompanied the formation of a rapidly rotating neutron star at its center.
Source: © NASA, CXC, and SAO.

Theorists initially thought that the origin of the glitch was extrinsic to the neutron superfluid. They envisioned a starquake in which part of the stellar crust crumbled suddenly in response to the forces produced by changes in the star's shape induced by pulsar spindown. But the observation of a second glitch in the Vela pulsar quickly ruled out that explanation for Vela pulsar glitches, since an elementary calculation showed that the expected time between such massive starquakes would be some thousands of years. Typical intervals between Vela pulsar glitches are some two years. It should be noted that for the much smaller glitches (a few parts in 100 million) seen in very young pulsars, such as that located in the Crab Nebula, whose age is less than 1,000 years, starquakes continue to provide a plausible explanation of their origin.

In 1975, Philip Anderson and Naoki Itoh came up with what astrophysicists now recognize as the correct explanation of the frequent pulsar glitches seen in the Vela and other older pulsars. Glitches, they argued, are an intrinsic property of the crustal neutron superfluid and come about because the vortex lines that carry the angular momentum of the crustal superfluid are pinned to the crustal nuclei with which they coexist. As a result, the superfluid's angular velocity, which can change only through the motion of its vortices, will lag that of the crust. The lag will persist until a sufficiently large number of vortices are pinned, at which point these unpin catastrophically, bringing about a sudden jump in the angular momentum of the star—a glitch—while their subsequent motion determines the postglitch behavior produced by superfluid response to the glitch.

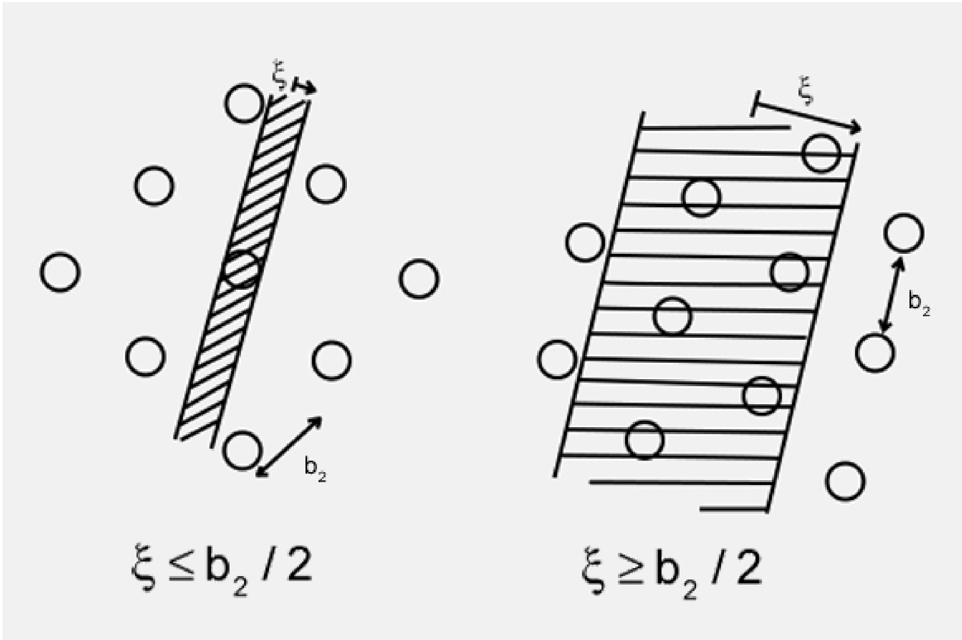

Figure 31: An illustration of two possible regimes of pinning for superfluid vortices in the crust of a neutron star. Left: the weak pinning expected when the size of the normal core, ξ of a vortex in the crustal neutron superfluid vortex is comparable to that of a crustal nucleus. Right the superweak pinning expected for a much larger vortex normal core.
Source: © Reproduced by permission of the AAS. Courtesy of M.A. Alpar, P.W. Anderson, D. Pines, and J. Shaham, 1983.

Unexpectedly, perhaps, careful study of the range of pinning possibilities and the nonlinear response of the superfluid to unpinning events has made it possible to identify the distinct pinning regions in the stellar crust shown in Figure 30 by their different response to a glitch and to explain the characteristic response times identified in the postglitch behavior of the Vela and other pulsars. Still more surprising, a careful analysis of the glitch magnitude and the superfluid postglitch response of a given pulsar now makes it possible to predict with some accuracy (roughly tens of days, say, for glitches separated by intervals of order years) the time to its next glitch.

A summary of our present theoretical understanding of the components of a neutron star is given in Figure 32, while some recent observations of the two best-known neutron stars, those found in the Crab and Vela constellations, are illustrated in Figures 28 and 30.

Superfluids in the stars

Based on the roughly 25 glitching pulsars observed so far, we can easily establish that the amount of cosmic neutron superfluid observed thus far is several solar masses, or some 10^{34} grams. That far exceeds the quantity of terrestrial superfluid ever produced or observed. Further, the amount of cosmic neutron superfluid contained in neutron stars that have yet to be seen to glitch is likely an order of magnitude larger.

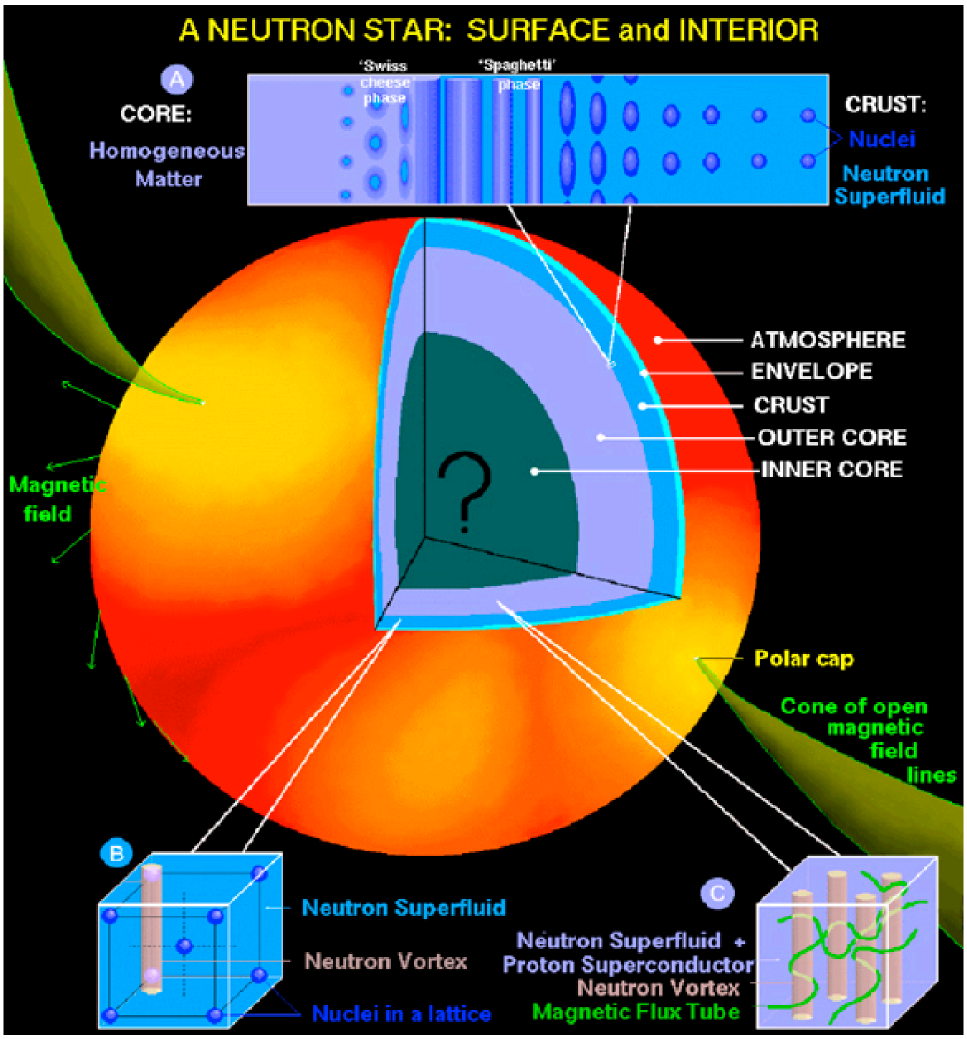

Figure 32: A cross section of a neutron star shows the rich variety of emergent quantum matter expected in its crust and core.
Source: © Dany Page.

Interestingly, glitch observations also provide us with important information on the hadron equation of state, since one that is too soft will not yield a crust sufficiently thick (~ 1 km) to support the region of pinned crustal superfluid we need to explain glitches. On combining this with information from direct measurements of pulsar masses in binary systems, theorists now conclude that the hadron equation of state that describes the behavior of matter in the inner core of the star depicted in Figure 32, is sufficiently stiff that one will not find quark or other proposed exotic forms of matter there.

Developing an emergent perspective

While the author was receiving radiation treatment for prostate cancer at UCSF in San Francisco in the spring of 1999, with time on his hands following his early morning irradiations, he arranged to visit Stanford two days a week to discuss with his colleague Bob Laughlin various issues relating to the then newly formed Institute for Complex Adaptive Matter.

What emerged from those discussions was a paper, "The Theory of Everything." In it, we pointed out the obvious—that there can be no "theory of everything" in an emergent universe in which it is impossible to calculate with precision the result of bringing more than a dozen or so particles together, to say nothing of the difficulties in dealing with the living matter (discussed in Unit 9). We then called attention to the not so obvious; that despite this, one knows many examples of the existence of higher organizing principles in nature—gateways to emergence that lead to protected behavior in the form of exact descriptions of phenomena that are insensitive to microscopic details.

In this unit, we have considered a number of well-established quantum protectorates: the low-energy excitation spectrum of a conventional crystalline insulator, which consists of transverse and longitudinal sound, regardless of microscopic details; the low energy screening of electron interactions in quantum plasmas; the low-energy behavior of a Landau Fermi liquid; and the low-energy excitation spectrum of a conventional superconductor which is characterized by a handful of parameters that may be determined experimentally but cannot be computed from first principles. We have also considered a newly discovered candidate protectorate, the emergence of the Kondo liquid in heavy electron materials.

In "The Theory of Everything," we emphasized the importance of developing an emergent perspective on science, a perspective espoused years earlier by P. W. Anderson in his seminal article, "More is Different." The importance of acquiring and applying that emergent perspective—the realization that we have to study the system as a whole and search for the organizing principles that must be at work to bring about the observed emergent behavior—is arguably the most important takeaway message of this unit.

An emergent perspective is also needed as we confront emerging major societal challenges—human-induced climate change, terrorism, our current global economic meltdown. These are all caused by humans; and in searching for an appropriate emergent response, we begin by seeking to identify their origins in societal behavior. But now there is a difference. Because these emerging challenges have no unique cause, it follows that there is no unique or even "best" solution. So we must try many different partial solutions, invent many new institutions, and, above all, experiment, experiment, experiment, as we address the various candidate causes, hoping (and expecting) that in the process some of these experiments will work. If all goes well, because everything is pretty much connected to everything else, a set of related solutions that begin to produce the desired result will emerge over time.

The selection of the examples of emergent behavior in quantum matter to be discussed in this unit has been a quite personal one. There are so many interesting examples of emergent behavior in quantum matter that the unit could easily have been 10 times its present length; in choosing which to present, the author decided to focus on examples drawn from his personal experience. He hopes the reader/viewer will be inspired to explore a number of other important examples on her/his own. Among those highly recommended are the discovery and explanation of quantum Hall states, metal-insulator transitions, dynamical mean field theory, quantum critical behavior, the recently discovered topological insulators, and the emerging fields of spintronics, nanoscience and nanotechnology, and quantum information.

Section 9: Further Reading

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- Philippe Nozieres and David Pines. "The Theory of Quantum Liquids," Vol 1. *Normal Fermi Liquids* and Vol. 2. *The Superfluid Bose Liquid*, Perseus Books, 1999.
- David Pines. *Elementary Excitations in Solids*, WA Benjamin, 1962.

Glossary

antiferromagnetic order: An antiferromagnet is a magnet in which the microscopic magnetic moments inside the material line up in a grid on which neighboring moments point in opposite directions. The interaction energy between two magnetic moments in an antiferromagnet is lower when the two moments point in opposite directions. This can lead to a frustrated system with multiple ground states.

BCS theory: BCS theory is the theory of superconductivity put forward in 1957 by John Bardeen, Leon Cooper, and John Schreiffer, who received the 1972 Nobel Prize for their effort. The basic premise of BCS theory is that under the right conditions inside a conductor, electrons can form weakly bound pairs called "Cooper pairs" that form a condensate. Pairs in the condensate experience no resistance as they travel through the conductor.

doping: In condensed matter physics, doping refers to the deliberate introduction of impurities into an extremely pure crystal. For example, a crystal of pure silicon might be doped with boron atoms that change the material's electrical properties, making it a more effective semiconductor.

emergent behavior: Emergent behavior is behavior of a complex system that is not easily predicted from a microscopic description of the system's constituent parts and the rules that govern them.

Fermi surface: According to the Pauli exclusion principle, it is not possible for identical fermions to occupy the same quantum state. In a system with many identical fermions, such as electrons in a metal, the fermions fill in the available quantum states in order of increasing energy. The energy of the highest occupied quantum state defines the energy of the Fermi surface, which is a surface of constant energy in momentum space.

ferromagnet: A ferromagnet is a magnet in which the microscopic magnetic moments inside the material all point in the same direction. Most magnetic materials we encounter in daily life are ferromagnets.

inelastic neutron scattering: Inelastic neutron scattering is an experimental technique for studying various properties of materials. A beam of neutrons of a particular energy is shot at a sample at a particular angle with respect to the crystal lattice. The energy of neutrons scattered by the sample is recorded, and the experiment is repeated at different angles and beam energies. The scattered neutrons lose some of their energy to the sample, so the scattering is inelastic. The results of inelastic neutron scattering are readily interpreted in terms of the wave nature of particles. The incident neutron beam is a wave with a frequency proportional to the neutron energy. The crystal preferentially absorbs waves with frequencies that correspond to its natural modes of vibration. Note that the vibrations can be magnetic or acoustic. Thus, the modes of the sample can be inferred by mapping out how much energy is absorbed from the incident beam as a function of the incident beam energy. Inelastic neutron scattering has also been used to study acoustic oscillations and their corresponding quasiparticles in liquids.

itinerant: In condensed matter physics, the term itinerant is used to describe particles (or quasiparticles) that travel essentially freely through a material and are not bound to particular sites on the crystal lattice.

magnons: Magnons are the quasiparticles associated with spin waves in a crystal lattice.

phase: In physics, the term phase has two distinct meanings. The first is a property of waves. If we think of a wave as having peaks and valleys with a zero-crossing between them, the phase of the wave is defined as the distance between the first zero-crossing and the point in space defined as the origin. Two waves with the same frequency are "in phase" if they have the same phase and therefore line up everywhere. Waves with the same frequency but different phases are "out of phase." The term phase also refers to states of matter. For example, water can exist in liquid, solid, and gas phases. In each phase, the water molecules interact differently, and the aggregate of many molecules has distinct physical properties. Condensed matter systems can have interesting and exotic phases, such as superfluid, superconducting, and quantum critical phases. Quantum fields such as the Higgs field can also exist in different phases.

phonon: Phonons are the quasiparticles associated with acoustic waves, or vibrations, in a crystal lattice or other material.

plasma: A plasma is a gas of ionized (i.e., electrically charged) particles. It has distinctly different properties than a gas of neutral particles because it is electrically conductive, and responds strongly to electromagnetic fields. Plasmas are typically either very hot or very diffuse because in a cool, relatively dense gas the positively and negatively charged particles will bind into electrically neutral units. The early universe is thought to have passed through a stage in which it was a plasma of quarks and gluons, and then a stage in which it was a plasma of free protons and electrons. The electron gas inside a conductor is another example of a plasma. The intergalactic medium is an example of a cold, diffuse plasma. It is possible to create an ultracold plasma using the techniques of atom cooling and trapping.

plasmons: Plasmons are the quasiparticle associated with oscillations of charge density in a plasma.

pulsar: A pulsar is a spinning neutron star with a strong magnetic field that emits electromagnetic radiation along its magnetic axis. Because the star's rotation axis is not aligned with its magnetic axis, we observe pulses of radiation as the star's magnetic axis passes through our line of sight. The time between pulses ranges from a few milliseconds to a few seconds, and tends to slow down over time.

quasiparticles: Just as particles can be described as waves through the wave-particle duality, waves can be described as particles. Quasiparticles are the quantized particles associated with various types of waves in condensed matter systems. They are similar to particles in that they have a well-defined set of quantum numbers and can be described using the same mathematical formalism as individual particles. They differ in that they are the result of the collective behavior of a physical system.

SQUID: A superconducting quantum interference device, or SQUID, is a tool used in laboratories to measure extremely small magnetic fields. It consists of two half-circles of a superconducting material separated by a small gap. The quantum mechanical properties of the superconductor make this arrangement exquisitely sensitive to tiny changes in the local magnetic field. A typical SQUID is sensitive to magnetic fields hundreds of trillions of times weaker than that of a simple refrigerator magnet.