Workshop 8

Mathematical Modeling

Overview

Description

This workshop presents two capstone lessons that demonstrate mathematical modeling activities in Algebra 1. In each case, students build a physical model and use it to collect data; they then generate a mathematical model of the situation they've explored. The teachers use a variety of techniques to help students apply concepts learned earlier in the year to develop the mathematical model.

- Part I: Sarah Wallick uses a pulley system to help students understand the effects of one rotating object on another. Students explore the concept in concrete, representational, and abstract forms in order to develop an algebraic understanding of the transmission factor, which is used in engineering applications.
- Part II: Orlando Pajon's students engage in hands-on simulations in order to study the process by which populations grow over time and follow certain patterns. They then model the data from the simulations using linear and exponential functions.

Featured Textbooks

- *Contemporary Mathematics in Context: A Unified Approach*. Glencoe McGraw-Hill, 2003.
- "Skeeters Are Overrunning the World," in *SIMMS Integrated Mathematics: A Modeling Approach Using Technology*; *Level 1, Volume 2.* Simon & Schuster Custom Publishing, 1996: pp. 135-154.

Featured Educators

- Sarah Wallick, The International School; Bellevue, Washington
- Orlando Pajon, Bel Air High School; El Paso, Texas

Featured Commentators

- Carol Malloy, University of North Carolina; Raleigh, North Carolina
- David C. Webb, Freudenthal Institute USA; Madison, Wisconsin

Learning Objectives

In these activities, you will learn how to help students:

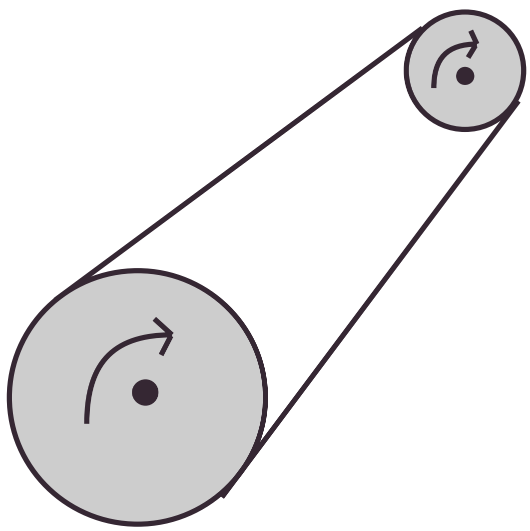
- Collect data, graph a scatterplot, and write explicit and recursive functions to model the data.
- Understand the meaning of the transmission factor.
- Apply the concept of transmission factor to a variety of problem situations.
- Conduct a simulation, analyze number patterns in the data, and write linear and exponential functions to model the data.

Workshop Session (On-Site)

Part I: Mathematical Modeling

Getting Ready (15 minutes)

Discuss the ways in which the rotation of one of these circular objects might affect the rotation of the other. Write down all of the relationships that you notice.



Watching the Video (30 minutes)

Watch Part I: Mathematical Modeling.

Going Further (15 minutes)

Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- Discuss Sarah's questioning strategies. What was accomplished? What could have been done differently, and in what ways?
- Discuss the purpose of giving students time to explore the pulleys before focusing on the specific driver-follower relationships. In what ways did it help move the lesson forward?
- Talk about how Sarah handled the variability in the data between student groups.
- Discuss the way Sarah gave instructions to the students. What does it tell you about the expectations she has for her students? How do you prepare your students to understand and complete complex problems and activities?
- Identify and discuss the different mathematical ideas and concepts that were addressed in this lesson. Discuss a mathematical modeling activity you would like to try, and identify the ideas and concepts it would involve.
- Sarah said, "Start with the concrete, then [move to] the representational, and then [move to] the abstract. The lesson should be tied together by going back to the concrete so that the students get the whole picture." Discuss the statement and decide whether and in what ways it was demonstrated in the lesson.

Workshop Session (On-Site), cont'd.

Part II: Mathematical Modeling

Getting Ready (15 minutes)

Give a specific example of something that grows in a linear fashion and something that grows exponentially. How are the two growths similar and how are they different?

Watching the Video (30 minutes)

Watch Part II: Mathematical Modeling. You may find it helpful to have the lesson plan at hand while watching this program. The lesson plan is provided in the Appendix.

Going Further (15 minutes)

Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- Discuss what you learned about the process of lesson study. What are the goals of lesson study? In what ways are the teachers in the video moving toward these goals?
- Discuss the purpose of the simulation and the ways in which it supported Orlando's objectives for the lesson.
- This lesson required a large amount of reading and following instructions. What strategies did Orlando use to prepare his class to successfully complete the lesson?
- Discuss the strategies Orlando used to prepare his students for writing the equations to model the data.
- Discuss whether or not you think the students understood how to find the equation for the purple Skeeter population, which depended on probability. If your class were at the same point, what would you do next to ensure understanding?
- Discuss the use of the assessment at the end of the lesson. Was this an effective assessment tool?
- Orlando continually reminds his class about the purpose and goals of the lesson. In what ways does this strategy promote reflective thinking and focusing on important ideas?

Between Sessions (On Your Own)

Homework Assignment

If possible, try one or both of the activities from the Workshop 8 video in your classroom, and reflect in your journal about your students' understanding of the concept. As an alternative, modify an existing lesson plan on mathematical modeling so that it incorporates the use of manipulatives, graphing, or patterning. Write your goals for this activity in your journal.

Ongoing Activities

You may want to carry on these activities throughout the course of the workshop:

Keep a Journal

Read the Teaching Strategies for Workshop 8 and answer the journal prompts. Include thoughts, questions, and discoveries from the workshop itself and learning experiences that take place in your own classroom. You are encouraged to use the online journaling tool at www.learner.org/channel/workshops/algebra.

Web Site: www.learner.org/channel/workshops/algebra

Investigate the Resources section and look at resources that will deepen your understanding of listening to student discourse and improving instruction through the process of lesson study to help students understand mathematical concepts.

Share Ideas on Channel-Talkinsights@learner.org

Share your thoughts and ideas about how listening to student discourse and participating in lesson study can improve your instruction and help students better understand mathematical concepts.

Video Teacher Reflections

Sarah Wallick

Below are Sarah Wallick's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

As you watched Workshop 8, Part I, what did you notice about your teaching strategies and student thinking?

My class is built around student exploration and discussion. I try not to answer questions on topics the class needs to explore. If I answer the question, then the learning ends there. In fact, it is questionable that any learning occurs if I provide all the answers. If I redirect the question to the class, the learning continues in an active way. I want my students to be active participants in the learning process, rather than passive vessels waiting to be filled.

At the start of the lesson, the question of the day is: How far does the belt travel over the fan pulley? Is it the same or different? If I were working in the "passive vessel" model of teaching, I could wrap this lesson up in about five minutes by simply telling the students the relationship and dedicating the remainder of the period to working on practice problems. As I look back across the years I have been teaching, this is how I used to teach a lesson such as this. The problem with doing this is it limits student learning to what *I* choose to present. By using an exploration to develop student understanding, I allow the students to define how much they will learn. Under this model, the students will take the lesson as far as *they* are able to go.

During the introduction to the lesson, I ask students to predict the behavior of the pulleys. Although the first student gives a correct explanation, I continue to solicit ideas without giving an evaluative response. I do this for two reasons:

1. The minute I give an evaluative response, the students are done; I have turned them into "passive vessels."
2. Everyone in the room has some sort of idea about the behavior of the pulleys, whether they articulate the idea or not. As the students are learning, they are comparing the new information they are developing to their existing ideas. It is important for students to see that their existing ideas are not the only possibility. This prepares them for revising any misconceptions they have in light of new information. It's hard for people to let go of things they believe are correct unless they have reason to question that understanding. Sharing multiple possibilities provides a reason for them to question their prior knowledge.

Another strategy that I consistently employ is the use of words before going to symbolic representations. For example, "$TURNS_{Driver}$" is used repeatedly before I transition to the symbolic form of T_D.

Students often get lost in the "alphabet soup" of mathematics. Using the entire word helps them make the transition from the concrete to the abstract. I can't count the number of students I have had who can recite the Pythagorean theorem, $a^2 + b^2 = c^2$, but can't tell me what it means in relation to a triangle. These students went from the verbal concept to the symbolic form too fast; they were never firmly grounded in the concrete model before they went to the abstract form. Each time this occurs, the concept must be retaught.

One of the habits I try to instill in my students is interrogating their work. You will notice in the video that I have the students look at the graphs individually and then together. Questions like, "Why does this graph look different than that graph?" are important to developing student fluency and an understanding of the relationship between the visual image (graph) and the symbolic model (equation). I also spent some time asking students for equivalent forms of the equation. This gives students a chance to explore the relationship between what they wrote and the work of other students. Again, this is another way to increase student fluency.

Where/how did you get the manipulatives that you used for your demonstration and students used for their models?

The manipulatives were purchased from Glencoe McGraw-Hill [publisher of the Core Plus curriculum used in this lesson].

You may want to note that the pulleys pop out of the boards easily. This makes them hard to work with.

Video Teacher Reflections, cont'd.

If you taught this lesson again, what would you do differently and why?

Overall, I think the lesson went well. There are only two things that I would change. The first is in the exploration phase. I might restructure this to ensure that there is a variety of data gathered. This would enrich the end of the lesson, when we compare our models to the exploratory data. The second thing I would change would be to spend a little more time on the starting value for the recursive model. This is something we have been developing throughout the year; it would have been nice to continue to explore the concept here too.

Discuss the kinds of questions you ask students and how you proceed based on their responses.

I see my role in the classroom as that of a facilitator of learning, rather than as a teacher. As a teacher one provides the learning, and as a facilitator one provides the opportunity to learn and mediates the discussion to keep the learning on track. This philosophy drives everything I do in the classroom. In particular, it drives my questioning. Since I am not there to provide information, but to facilitate learning, I try to stay in questioning mode. When a student asks a question, I either redirect it to the group or ask a question in return.

Students generally ask three types of questions.

- Procedural clarification questions are answered by clarifying the purpose of the activity so students can structure the experience themselves.
- Factual information questions are usually redirected to the class. This keeps the students alert to the discussion and gives them an opportunity to showcase what they know. Rarely do I have to answer these questions.
- The last type of question comes under the heading of "I wonder …?" This is where the gold is. I never answer these questions directly. I will usually ask a question back to the class that delves into an essential area of focus for the original question. I keep these questions going back and forth between the students and me until they have developed as much of an understanding as they are going to for that day. These questions inform my instruction. I use them to direct the class discussion and plan future lessons.

Math education professor Carol Malloy, who provided additional on-camera commentary for this tape, was impressed by how comfortable your students were challenging each other, accepting challenges from peers, and justifying their positions. Share some thoughts on how they were at the start of the school year and how the classroom community evolved.

I put a lot of effort into developing student questioning strategies. At the start of the year, I'm an absolute shrew. If I think a student isn't listening to what the others in class are saying, I'll ask him or her a question about the discussion. If they weren't listening, I ask the student who spoke to repeat what they said. I try not to parrot student responses. Parroting student responses teaches students that they don't need to listen to one another; the teacher will be sure to repeat anything important. Students listening to one another and discussing ideas make my class productive. When students are working in groups, if they are not communicating with one another, then you don't have group work; you have individuals who happen to be sitting in furniture arranged in clusters. When students are paying attention to one another, the conversation is richer in whole-class discussion. The students can become quite passionate about their ideas. When they do, they will dig deep into their understanding of the topic to defend their ideas.

Share some thoughts on how you group students and work with student groups.

My students work in groups throughout the year. I start with assigned group seats when students arrive on the first day of school; I rearrange the seating at the end of every unit. When students come to class on the day of the unit test, they are given new seats. This discourages prearranged cheating in the groups. I randomly assign students to groups using the "seat shuffle utility" in my grade book program. I rarely make changes to this arrangement. The exceptions are:

- I try not to have a group with three boys and only one girl. Under this arrangement, girls often shut down and don't participate.
- During the course of the year, some students may develop a symbiotic learning relationship. When this occurs, I let those students remain together in a group until they request a change.
- If I have a student who needs to have special seating, I move the entire group rather than the individual student.

Video Teacher Reflections, cont'd.

It is important for students to spend a substantial amount of time together in one group. It takes time for the working relationships to be established

Describe any evidence that you see in this tape, or that you can recall from last semester, that the students are making connections between the representations.

As I reviewed the tape and reflected on the student learning last year, I could see evidence that the students were making connections between representations. I saw this when the students were relating the graphs to the data they collected. I was also pleased when they compared the transmission factor to slope, comparing and contrasting the two concepts. Throughout the year, I ask my students to look at mathematical relationships using multiple representations. It isn't enough to be able to work with an equation. They must also understand how that equation will look when graphed and how to express the relationships with words. I try to make them understand that mathematics is a way of communicating the relationships between things, and that these relationships can and should be expressed in multiple ways.

Orlando Pajon

Below are Orlando Pajon's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

As you watched Workshop 8, Part II, what did you notice about your teaching strategies and student thinking?

I noticed in reference to my teaching strategy that the students were not afraid to discuss different viewpoints. They also were very good at doing the team exploration and getting the process column filled in with the least amount of help possible. The way the exploration was structured did help on the smoothness of the lesson itself. The use of the accountable talk interactions did help foster student thinking and interaction between them and the teacher. I was very pleased with the evidence of student thinking while doing the activity, and this made me reflect that as a teacher I was experiencing some evidence of teaching for understanding. [Editor's Note: According to the Learning Research and Development Center at the University of Pittsburgh, "accountable talk seriously responds to and further develops what others in the group have said. It puts forth and demands knowledge that is accurate and relevant to the issue under discussion. Accountable talk uses evidence (such as data from investigations) and follows established norms of good reasoning." *www.instituteforlearning.org/pol.html*)]

At the start of class, you told the students that the lesson had been developed by a group of teachers who meet regularly to try to improve instruction. Why was it important to you that the students know this?

It is important for the students to know that the lesson is a result of collaboration between teachers because it [mirrors] what they do in the classroom in the sense that they work together as a team towards a common goal. It is important also to know that there is power in collaboration and that the result is always better in quantity and quality. [Also], they will see teachers in the classroom following up [on] the outcome of the lesson and taking notes of the students' response according to the outlined script and what is happening in the classroom. This will help us improve the lesson.

Video Teacher Reflections, cont'd.

Explain a little about the lesson study collaboration. How was it started? Who usually participates in the meetings? What do you do at the meetings, and what do you do between meetings?

The lesson study process in the math department started three years ago as a result of the participation of two teachers—James Salazar and myself—in the Lead Learner program at the University of Texas at El Paso, through the El Paso Collaborative for Academic Excellence. There we became familiar with new strategies to improve instruction and student learning, among them lesson study. We looked at the fact that in most countries where education is at a high level, the lesson-study process is part of the education system, especially in Japan. We then decided to gather together a group of teachers in the math department once a month to produce lessons on a particular topic, provided that every participant has input on what the lesson will look like and that there has to be an implementation of the lesson to evaluate its effectiveness. During our meetings, we would discuss strategies to present the lesson, potential student responses to every action taken by teacher, and what the assessment will look like, among other things. After the meeting, there is a follow-up process. We visit each other's classrooms when we implement the lesson to experience the outcomes of the lesson [as it is] taught and to collect evidence of student understanding. We tape the lesson to use it in further analysis when we meet the next time. Then, the lesson is revised and proper adjustments are made based on the evidence of student work and assessment.

How has the collaboration process evolved since it began? How would the team like it to evolve in the future?

The collaboration process has evolved very positively since it began. At the beginning, to produce a lesson took a more significant amount of time [than it does now]. We were using a different framework than the one we are using now. Also, we were on our own. As the process evolved, we invited a professor from the university, Mr. Mourat Tshoshanov to assist us in the process, as well as Mrs. Bertie Lopez from the El Paso Collaborative and other USP mentors and teachers from other schools. They played a key role in helping us reshape the template of the lesson-study process and giving us their particular expertise in different areas. It is also very important that the administration supported us by providing us with the time and location for our meetings, and trusting in what we do.

The lesson-study process continues today at Bel Air High School. It is now a school-wide initiative, even though neither James Salazar nor myself (the pioneers) remain at the school. Today we are staff developers for the school district in the area of mathematics. This makes me think that everything is about building capacity, so that even when we are not there, the lessons and the products developed in collaboration among teachers are available and other teachers can use them with a reasonable degree of success.

Discuss the kinds of questions you ask students and how you proceed based on their responses.

I think the questioning of the students' thinking played a key role in the outcome of the lesson. During the lesson-study process, we think about the types of questions that we should ask the students based on the different levels of cognitive demands that we want to make. We try to come up with what we think the student response will be based on our own experience from teaching that particular topic before—we depict a scenario of the student responses. When the lesson has not been taught yet, this scenario is smaller than when we come back to revise it based on the observations made by other teachers during the implementation of the lesson. There is always room for adaptation and accommodation based on the teacher's opinion while implementing the lesson—for example, if I see that the students are having problems understanding a key concept or a skill necessary to accomplish the lesson goal.

At a certain point in this lesson, I asked one team of students what they had in the process column. They were not sure what to write there. I wanted to make sure that all the students knew how to fill the process column, because it is a necessary step to enable them to develop the rules for the linear and exponential growth. I made the decision to stop and ask the students for some input to measure their understanding of what they needed to do, and I modeled how to fill in the process column for one type of Skeeter population. After I made sure that every student had an idea of what to write there, they then continued with the exploration activity.

Share some thoughts on how you group students and work with student groups.

I group students randomly. At the beginning of every project (we have divided the Algebra 1 curriculum into 10 main projects or modules), we change groups. I like to reinforce the idea that in real life they

Video Teacher Reflections, cont'd.

have to work most of the time in constantly changing groups, and sometimes with people with whom they are not familiar or that they might not like. If the group has a common goal, that should be the focus of their attention. I try to make adjustments in cases of Limited English Proficiency students, [students with special needs], and gifted and talented students, so that they could be distributed in the best possible way to meet their needs.

While the students are investigating or discussing in their groups, I go around [to] the different teams to ask for clarification and make sure the teams know what to do. I request that students who have a question address it first to their team, and if nobody on the team has an answer to it, then they should call me. At that time when I approach their table, everybody is willing to listen. I randomly ask somebody on the team about the question to make sure that they have made an attempt to discuss it within the team. When I clarify their questions, I try to do it by asking questions to the team that will help them come up with the desired explanation on their own.

If you taught this lesson again, what would you do differently? Why?

I would show some more examples of linear or exponential growth and would ask them to come up with the rules based on the model. [I would] ask more discussion questions about the differences between linear and exponential growth. I think that by doing this, I would have a wider variety of situations where the students can show a wider level of understanding and mastery of that particular topic.