Workshop 6
Exponential Functions

Overview

Description

This workshop presents two activities that provide teachers the opportunity to help students develop a conceptual understanding of exponential functions as models and of the basic properties of exponents.

- Part I: Orlando Pajon uses a simulation to help students understand the meaning of the exponential growth of a population, the shape of an exponential function, and how the *y* values in a table of values change by a multiplicative factor. The lesson is an introduction to exponential growth and develops the conceptual understanding underlying the principles of exponential functions.
- Part II: Mike Melville uses a situation from *Alice in Wonderland* to help students understand exponential growth and decay. The students develop a definition of a negative exponent and come to understand the reasoning behind the division property of exponents with like bases.

Featured Textbooks

- "Skeeters Are Overrunning the World," in *SIMMS Integrated Mathematics: A Modeling Approach Using Technology*; *Level 1, Volume 2*. Simon & Schuster Custom Publishing, 1996; pp. 135-154.
- Interactive Mathematics Program, Year 2

Featured Educators

- Orlando Pajon, Bel Air High School; El Paso, Texas
- Mike Melville, Aptos High School; Aptos, California

Featured Commentators

- David C. Webb, Professor of Mathematics, Freudenthal Institute USA; Madison, Wisconsin
- Jane F. Schielack, Professor of Mathematics, Texas A&M University; College Station, Texas

Learning Objectives

In these activities, you will learn to help students:

- Use a graph and a table of values for a given data set to predict future values.
- Conduct a simulation of exponential growth of a population.
- Make a scatterplot and a table of values for the data from the simulation, and analyze patterns in the data.
- Solve problems involving multiplying powers of 2 and 1/2.
- Generalize the rules discovered while solving problems.
- Develop and verify a definition of a negative exponent, and define and verify the property of dividing exponents with like bases.

Workshop Session (On-Site)

Part I: Modeling With Exponential Functions

Getting Ready (15 minutes)

Describe all of the patterns you notice in the following set of data. Use the data to predict the world population in 1950.

Year	World Population (in millions)
1650	470
1750	629
1850	1128

Look at the data below and discuss the accuracy of your first prediction. Use the additional data given to predict the world population in 2000.

Year	World Population (in millions)
1650	470
1750	629
1850	1128
1900	1550
1950	2555

Watching the Video (30 minutes)

Watch Part I: Modeling With Exponential Functions.

Going Further (15 minutes)

Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- Discuss what you think Orlando was trying to accomplish by having students predict the world population in 1950 and 2000 based on such a limited set of data.
- Discuss the effectiveness of the techniques Orlando used to get his students to work collaboratively. How do you encourage collaboration in your classroom?
- Discuss the use of the simulation to help students understand the concept of exponential growth. What other contexts might you use?
- What mathematical ideas were learned in this lesson? How might you use a lesson like this to introduce another topic?
- List some ways in which Orlando created an atmosphere where students felt safe expressing their ideas. What evidence did you see that he was successful?

Part II: Understanding Basic Properties of Exponents

Getting Ready (15 minutes)

Solve the Rallods in Rednow Land Problem (see Appendix for solution):

> The king gives you a choice for receiving payment. You can have one billion rallods or a chessboard with 64 squares. If you choose the chessboard, the king will place one rallod on the first square, two on the second square, four on the third, and so on, so that each square has twice as many rallods as the previous square. You will receive all of the rallods on the checkerboard. Which choice do you make and why?

Watching the Video (30 minutes)

Watch Part II: Understanding Basic Properties of Exponents.

Going Further (15 minutes)

Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- Discuss the strategies that Mike used to get the level of student participation that was shown in the video.
- Discuss the instructional decisions Mike made during the lesson. Which concepts did he choose to pursue immediately, and which did he choose to put off for another class period? Discuss the reasoning behind his decisions.
- Discuss the ways that Mike handled errors that occurred during the lesson.
- What evidence did you see that Mike was steering the lesson toward a desired conclusion, even as students were presenting the problem solutions?
- In what ways did the *Alice in Wonderland* story help students understand some key ideas of exponents? What particular concepts did the story illuminate?

Between Workshops (On Your Own)

Homework Assignment

Reflect on ways that you can increase student participation and engagement in your classroom. In your online journal, explain some strategies that you can incorporate in your classroom that will enhance student collaboration. If you are currently teaching an algebra class, try out at least one strategy and record and reflect on the results in your journal.

Ongoing Activities

You may want to carry on these activities throughout the course of the workshop.

Keep a Journal

Read the Teaching Strategies for Workshop 6 and answer the journal prompts. Include thoughts, questions, and discoveries from the workshop itself and learning experiences that take place in your own classroom. You are encouraged to use the online journaling tool at www.learner.org/channel/workshops/algebra.

Web Site: www.learner.org/channel/workshops/algebra

Go online for materials and resources—including the Skeeters Are Overrunning the World lesson plan—to help you deepen your understanding and implement the practices illustrated in this workshop. The lesson plan is also provided in the Appendix for your convenience.

Share Ideas on Channel-Talkinsights@learner.org

Share your ideas on ways teachers can increase students' engagement in mathematics.

Video Teacher Reflections

Orlando Pajon

Below are Orlando Pajon's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

As you watched Workshop 6, Part I, what did you notice about your teaching strategies and student thinking?

I was very pleased with the outlined strategy and the outcome. I think that [the lesson] kept the students' attention the entire time and the students were actively engaged in discussions among themselves and they were concentrating on the activity.

How would you say these students felt about mathematics and about themselves as mathematics learners, at the time this lesson was presented? Did it change over the course of the school year? How?

I think that the students felt very comfortable about the mathematics they were learning and they were not afraid to discover new things and to link it to their prior knowledge, nor were they afraid to build their mathematical knowledge on the explorations to which they were exposed. They were in a very friendly environment where they could expose and discuss their findings. This, of course, was the result of hard work during the school year implementing team-building activities and a standards-based curriculum where they learn mathematics by doing explorations, discussing their findings and linking this into the mathematical tools that they need to solve real-life problems and applications. As a complement to this I was very pleased with the results of the accountable talk strategy.

[Editor's Note: According to the Learning Research and Development Center at the University of Pittsburgh, "accountable talk seriously responds to and further develops what others in the group have said. It puts forth and demands knowledge that is accurate and relevant to the issue under discussion. Accountable talk uses evidence (such as data from investigations) and follows established norms of good reasoning." *www.instituteforlearning.org/pol.html*)]

What specific things do you do to help students improve their confidence and attitudes about mathematics?

Every idea is based on what they see or what they find when they experiment, so there are no right or wrong answers. That makes the discussion very open and the students are not afraid to share their mathematical thinking. I also implement on a regular basis the accountable talk strategy whenever I interact with the students. I also provide them at the beginning of the year with a template with sample statements that they might use whenever they interact among themselves in different situations as well when they interact with me.

Discuss the kinds of instructional decisions that you and your lesson study group had made before you taught this lesson.

We had decided that the lesson was going to be based on a real-life problem or application. We also decided that the students would work together in teams of a maximum of four students. We knew it would be very important to use the accountable talk format to drive the discussion of the student's findings to help them build their mathematical knowledge.

How closely would you say you followed the framework or script that was developed by the lesson study group? Describe any examples you noticed in the video where you made a decision" on the fly." Reflect on what happened as a result of your decision(s).

I followed the script that was developed by the lesson study group very closely. The teachers involved in the design of the lesson, including myself, were very accurate in our predictions of what the students' responses would look like. In the video I noticed that I almost did not make any decisions "on the fly," except when trying to have more students provide examples of simulations in real life. As a result of that decision, I think that most of the students got a very strong idea of what the simulation process is and what some real life examples are.

If you were to teach this lesson again, what would you do differently? Why?

I would probably have the kids transpose their team's work onto big paper to put a display of the students' work around the classroom. This would be a good incentive for them to produce good quality work and it also accounts for their level of understanding and mastery of a particular mathematical topic. It also would be a very good opportunity to grab other students' attention about important and relevant issues, and to raise the students' self-esteem by evidencing what they are capable of producing.

Video Teacher Reflections, cont'd.

Mike Melville

Below are Mike Melville's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

When watching Workshop 6, Part II, what did you notice about your teaching strategies and student thinking?

I try to get students to hypothesize and test their ideas. It is important for them to put their understanding into their own words. These words and the understanding they have need to be made correct and precise, which is only possible when the students are willing to share what they think. When they formulate their ideas, if I am listening carefully, I can hear the germ of truth and try to lead them to a clearer recognition of that germ by questioning them. I can't do the same thing if I'm doing all the talking. Often I feel more like the ringleader at the circus, but a successful show depends on the willingness and work of those involved. Sometimes I feel like I intervene too much, usually in the interest of time, which is not the best pedagogical strategy. Sometimes I let the students go too far on their own, which can make them feel even more muddled. All in all, I liked what I saw on the video. What doesn't really come out is how I use the student initials on the board to monitor who has spoken, who has been actively involved, and who has yet to speak. In the video, it appears that I arbitrarily call on people, which is far from true. For the most precise explanations, I always choose those students I *hope* are most capable of bringing clarity to the class.

How does this lesson fit into your curriculum? What had you done with students in preparation for this lesson?

We used the model of Alice in Wonderland eating cake to grow and drinking beverage to shrink as a context for understanding rules and meanings of exponents and some ideas around the notion of exponential growth. Prior to looking at this lesson on negative exponents, the students had developed a general idea for the exponential curve and had established various rules for manipulating exponents when multiplying. They had also used this model to understand and explain the convention of using zero as an exponent. We had used both the Alice model and patterns with numbers in understanding these rules. Soon in our work, we will apply this to scientific notation.

To what extent did this lesson follow the plan you wrote for it?

This lesson pretty much followed the plan I had written for it. I was surprised with the discovery of the possibility of fractional/decimal exponents. The explanation of why we subtract exponents when we divide and the clarity that came from the students was unexpected. Both of these discussions were positive enhancements of the lesson I had written.

Reflect on the times during this lesson that you made decisions on the spot. What was the decision, what was the reasoning behind it, and what were the results?

Typically as I move through a lesson, and in particular in this one, I need to decide when the expression of an idea or a situation is getting caught up in too many details or needs more clarity. On the spot, I decide when it's time to move on or time to review with more clarity. My judgments are not always perfect, and there are errors, but the key considerations are use of time, clarity, and whether the issue involves an individual or a group of students.

When the possibility of fractional exponents came up, I decided to let the students know that we still needed to explore more ideas about exponents, and so the question could be raised, but not yet answered. When the issue of negative results from using exponents came up, I decided to push into the idea in order to keep focus on our key idea and to avoid confusion. When one student misrepresented the idea of negative exponents, I sent him to the overhead calculator to clarify the mathematics for himself and for the class. I decided on the spot to write the rules myself, rather than have students do it, for fear that something would be written wrong and precision would be lost. Along with this, I made a decision during the lesson to have the students copy down the connections between the various aspects of the problem. I believe this helped the students follow the investigation to reach the definition we were seeking. The choice of which students to make presentations is always on the spot, and is largely based on a sense of who has ideas that will help us move to deeper understanding, even if what they have is in error. When a student said, "Oh, I get it now," I made the decision to have him explain his big "aha" to the class in order to clarify why we subtract exponents when we divide. It was at that point that I decided to summarize together all the cases for "2 raised to the c – b."

Video Teacher Reflections, cont'd.

Describe how you followed up on some of the questions that were left unanswered during this class period.

In later work in this same unit, we used Alice eating cake as a model for what happens to her height when she does not eat whole ounces of cake to develop an understanding of fractional and decimal exponents and what they might represent, and what the conventions mean.

When Jane Schielack viewed your lesson on videotape for the first time, she commented that, while you may appear to be on the sidelines, you are in fact carefully orchestrating the lesson so that the students achieve the intended goal.

I see my role in the classroom as being a facilitator of learning, and I have a plan for the logical development of concepts. I believe the learning will be more powerful and complete for the students the more it comes from their own words and presentations. So, once I've put the lesson in motion, I am on the alert for opportunities to use student discoveries and ideas to move the lesson along toward better understanding.

What are some things you do to help your students improve their appreciation of mathematics and their confidence in themselves as math learners?

In my class, students are constantly called on to share ideas within their groups, present solutions to problems, and sometimes they are asked to work through problems they're not sure about. I try to show them that they can accomplish things they thought they were not able to do. I ask them to relate their new ideas to old problems and situations and to predict for future ones. My students are asked often to put their understanding into words, orally and in writing. In these ways, I believe they build their own confidence. Seeing the many connections each idea has to other situations helps my students build an appreciation for the beauty and power of mathematics.

If you were to teach this lesson again, what would you do differently?

As I watched the video, I realized that I would have liked to force more use of actual numeric fractions. This would have meant some work with using the math button to convert decimals to fractions. It would definitely have meant doing things like showing the equality of "one-half squared" to "one divided by two squared."

Next time, I would have more students using the overhead calculator to demonstrate the process of their work and their thinking. I might spend a little more time and effort reinforcing what it means to multiply by a fraction, and definitely reiterate the known idea that to multiply by one-half is the same as to divide by two. I would also make my own language more precise by saying "one divided by…" rather than "one over…" I also think that I would deal more with the issue of "negative Alice" if it came up.