Workshop 5
Properties

Overview

Description

This workshop showcases ways that teachers can help students explore mathematical properties studied in Algebra 1. The activities use a variety of techniques to help students understand important mathematical concepts.

- Part I: Tom Reardon uses algebra tiles to help students understand the process of factoring quadratic expressions. He also connects the physical model of the algebra tiles to the graph of a quadratic function and to the algebraic manipulation necessary to factor quadratic expressions.
- Part II: Sarah Wallick uses probability as a vehicle to help her students understand how to write basic recursive equations and compare them to explicit equations. She helps her students understand the difference between power models and exponential models by looking at the number patterns in a table of values.

Featured Textbooks

- Textual materials written by Tom Reardon
- *Contemporary Mathematics in Context: A Unified Approach*. Glencoe McGraw-Hill, 2003.

Featured Educators

- Tom Reardon, Austintown Fitch High School; Austintown, Ohio
- Sarah Wallick, The International School; Bellevue, Washington

Featured Commentators

- Diane Briars, Pittsburgh Public Schools; Pittsburgh, Pennsylvania
- Carol Malloy, University of North Carolina; Raleigh, North Carolina

Learning Objectives

In these activities, you will learn ways that students can:

- Use algebra tiles to multiply binomials and to factor trinomials.
- Use graphing calculators to explore the relationship between the factors of a quadratic expression and the *x*-intercepts of the corresponding quadratic function.
- Use algebraic properties and techniques to multiply binomials and to factor trinomials.
- Use probability as a vehicle to study recursive equations written as Now/Next equations with a given start value.
- Look at patterns of differences in a table of values to determine whether the values fit a power model or a recursive model.
- Write recursive equations to model a variety of different situations.

Workshop Session (On-Site)

Part I: Factoring Quadratic Expressions

Getting Ready (15 minutes)

Discuss the appropriate role of factoring quadratic expressions in the Algebra 1 (or its equivalent) curriculum. Your discussion should include the amount of time that should be spent on the topic, the types of quadratic expressions to be factored, the method(s) of factoring that should be taught, and the role of technology in teaching this topic.

Watching the Video (30 minutes)

Watch Part I: Factoring Quadratic Expressions.

Going Further (15 minutes)

Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- Discuss the use of algebra tiles, and how they can help students understand the process of multiplying binomials and factoring basic trinomials.
- Discuss how Tom connected the graphs of the quadratic function to the factors of quadratic expressions. Would the use of this strategy help your students understand the concept of factoring?
- Analyze all of the patterns that the students discovered during the lesson. Were there any discoveries that surprised you? Were there any patterns that they missed?
- Analyze the discussion about a quadratic expression that doesn't factor. In what ways is this an important concept to discuss in an Algebra 1 class?
- Discuss some different ways that Tom emphasized the importance of clearly communicating ideas to students, and the importance of using correct vocabulary. How do you accomplish this in your class?

Part II: Understanding Basic Recursion

Getting Ready (15 minutes)

A. Find the next two terms for the sequence of numbers: 1, 1, 2, 3, 5, 8, 13,…. Then write instructions so that somebody else could generate the same sequence.

B. Write the first five terms for a sequence of numbers if you are given the following instructions for generating the sequence: START at 3, NEXT = NOW + 4.

Watching the Video (30 minutes)

Watch Part II: Understanding Basic Recursion.

Going Further (15 minutes)

Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- Discuss other contexts that you could use to teach the concept of recursion.
- What else could Sarah do to help her students understand and clarify the starting value when writing recursive equations?
- Discuss how Sarah helped her students understand the difference between a power model and an exponential model. Suggest some other ways this might have been done.
- What types of number patterns do you need to analyze so that you can write a recursive equation? How are these patterns different from the patterns you investigate to write an explicit equation? Explain which type of pattern you think is easier for students to understand and why.
- Discuss the advantages and disadvantages of using recursive equations along with explicit equations.
- What is the value of having students summarize what they have learned at the end of a lesson? How do you structure this activity so students benefit from it?

Between Sessions (On Your Own)

Homework Assignment

If possible, try one or both of the activities from the Workshop 5 video in your classroom, and reflect in your journal about your students' understanding of the concept. Or, modify an existing lesson plan on a mathematical property so that it incorporates the use of manipulatives, graphing, or patterning. Write your goals for this activity in your journal.

Ongoing Activities

You may want to carry on these activities throughout the course of the workshop.

Keep a Journal

Read the Teaching Strategies for Workshop 5 and answer the journal prompts. Include thoughts, questions, and discoveries from the workshop itself and learning experiences that take place in your own classroom. You are encouraged to use the online journaling tool at www.learner.org/channel/workshops/algebra.

Web Site: www.learner.org/channel/workshops/algebra

Investigate the Resources section and look at resources that will deepen your understanding of using patterns and the "rule of four" to help students understand mathematical concepts.

Share Ideas on Channel-Talkinsights@learner.org

Share your thoughts and ideas about how you can use the idea of patterns and the "rule of four" in helping students better understand mathematical concepts.

Video Teacher Reflections

Tom Reardon

Below are Tom Reardon's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

As you watched Workshop 5, Part I, what did you notice about your teaching strategies and student thinking?

I noticed I really emphasized that students understand what the word "factoring" means to do...sometimes to the point of "alright, already!" But I do that with key words and I think it works.

I am trying to get the students to make connections: algebra tiles, graphs and intercepts, tables, and the expressions. I like to use the investigation or exploration approach—student self-discovery or in this case group discovery. Mathematics can be described as the study of patterns and I have the students summarize their results in a table so that the patterns are more evident. I noticed I prefer that my students write the intercepts as ordered pairs, emphasizing that the x-intercept has a y-coordinate of zero and the y-intercept has an x-coordinate of zero.

After watching the video, I now realize how much I had to readjust my lesson plan throughout the class. I [was] flexible, as we teachers have to be. We have to adapt to their perceptions and non-perceptions. We need to take advantage of observations that the students make that are good, even though they might not be what we wanted to emphasize in this particular lesson. Many unexpected things occurred in today's lesson, but I was happy with most of the ways I handled them. I was very pleased that the students were developing their own algorithms for factoring without assistance and prodding from me.

I also realized that I had to reinterpret some of the generalizations that the students made, especially with Dave. Dave has good mathematical sense, but he does not communicate well. He likes to use the word "equal" for add, multiply. So I had to help him come up with the correct wording and terminology and sometimes it was rough.

In your interview, you said you were surprised that the class was able to develop an algorithm in only one lesson. When you watched the tape, what specific things did you see that enabled them to get that far?

They picked up on the patterns quicker than I expected and they were able to verbalize the patterns much better than I thought they would be able to after just one class period. Part of that was due to good communication among the students and the other was that the table that we made helped them to organize their thoughts easier. Use of color on the SMART Board was also a contributing factor because it allowed students to focus in on a few numbers at a time while seeing the pattern.

In this lesson, students represent data numerically, graphically and geometrically, algebraically, and verbally. You make the point that each student will find one representation easiest to understand. Do you see any evidence in this tape that the students are making connections between the representations?

At one time I asked the students to factor without the algebra tiles or the graphing calculators and Andrea said something like, "How are we going to do this?" She liked either the graphing or geometric representations best. Ray and Dave jumped on the algebraic representations quickly. In fact, while we were filming, I remember going by Ray's desk and, not only did he understand the algorithm, but he also had already done the assignment for the next day and I hadn't even assigned it yet!

You could see the students pointing to the graph and picking out the intercepts and then using those to assist in the factoring.

What are the strengths and weaknesses of the structured approach you use with this class? What do you think would happen if they worked on their own or in their groups for longer periods of time before coming back to a whole-class discussion?

Strengths: By keeping it structured I was able to keep students from straying and focusing in on ideas that were extraneous to what we wanted to accomplish today. By supplying and suggesting the table to keep track of our findings, it made the students discover the patterns quicker than if they didn't keep track of their findings.

Weaknesses: The students become too dependent upon me leading them to discover the concepts. It might be better to let them find the pattern on their own, encouraging more examples, with less assistance from me, and try to make them more independent thinkers.

I chose the structured approach because I wanted to accomplish "x" amount in today's lesson. I was partially driven by the time factor.

Video Teacher Reflections, cont'd.

If you were to teach a factoring lesson again, what would you do differently?

If I were teaching this lesson again, I don't think I would change too much because I was very pleased with the results. They exceeded my expectations. Maybe I would try to have them factor something like $x^2 - 7x + 12$ and see what happens with that.

Sarah Wallick

Below are Sarah Wallick's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

As you watched Workshop 5, Part II, what did you notice about your teaching strategies and student thinking?

It might be helpful for you to know that my students have a very weak background in probability. The last time they had any formal instruction in probability was in middle school, two years prior to this lesson. This weakness is substantiated by our dismal performance in probability on our state assessment, The Washington Assessment of Student Learning.

To compensate for their lack of experience in probability, I chose to open the lesson with a situation familiar to most of them—playing Monopoly. I posed the "question of the day": What is the probability that you will get out of jail by rolling doubles? I use a question like this so that students will focus their learning on my objective. This is very important since the lesson is structured around student questions and responses. The question of the day helps keep the lesson on track. Otherwise it is very easy for the lesson to meander through the world of mathematics (not always a bad thing), and never get to my objective. When you view the video you will see that the lesson could very easily have wandered into a deep exploration of the starting value for the recursive equation or a deeper exploration of finite differences and power models. Since these weren't my objectives, the question of the day made it easy to redirect the conversation back to my lesson objectives.

After posing the question of the day, I moved into an exploration that was simple with which the students were already familiar. I did this for two reasons. One was to acknowledge their weakness in probability. I didn't know how they would handle the analysis; I wanted every student to have an entry point to the learning. The other reason I chose to use a simple model is that I don't really care whether they can work with probability for coins or dice. I want them to have strategies they can use to explore situations where probability is involved. I want them to be able to use what they learn in a new setting or situation. Using a simple situation with which they were already familiar allowed them to focus on the process and not the particular situation.

Throughout the lesson I use questioning and exploration to guide students to construct their own understanding. The difficulty with this type of teaching is that some students may draw incorrect conclusions. Throughout the lesson it is essential that I monitor the students' work and intervene with individuals who are off track or who have made incorrect conclusions. In general, when I teach I try to customize the learning to the needs of individuals and the group as a whole. This means that if you view the same lesson for different classes, it will look different for each, since each group is made up of individuals with different experiences and needs.

Reflect on the way you have the students summarize their learning at the end of this and every lesson.

At the end of any lesson I have students summarize their learning. I do this for three reasons.

- First, since students are constructing their own understanding, it is important for me to know what they learned. This guides my instruction for subsequent lessons.
- Second, during the course of a class period a lot of ideas—both correct and incorrect—are discussed. I need to be sure students have not integrated incorrect information into their understanding of the content.
- Third, the summary gives students a chance to focus on the important points of a lesson. Throughout the course of a lesson we harvest a lot of experiences; the summary gives us a chance to separate the wheat from the chaff.

At the beginning of every school year, I spend time teaching students how to summarize their learning. At the start of the year the summaries are developed in the classroom. Students discuss the main ideas in their groups and then we share as a whole class. I do this because summarizing and identifying the big ideas in a lesson is a learned skill. Students don't generally have experience doing this in a math class. The small group experience gives them a chance to practice the skill; working as a large group gives them a chance to compare their ideas to the ideas of others in the class

Video Teacher Reflections, cont'd.

and to my thoughts about what was important. It also gives me an opportunity to assess their skill in identifying the big ideas at the start of the year so I can direct my instruction to their needs. As time goes on and the students develop better summarizing skills, I transition them to doing their summaries in groups and finally to doing them independently. It is my goal for students to develop the habit of summarizing their learning. This helps them understand what they know, what they need to know, and what their questions are on a given topic. This is a critical skill for success beyond high school.

If you taught this lesson again, what would you do differently and why?

When I teach this lesson again, the main thing I will do differently is in preparation. I would prepare more completely for the question of the starting value in the recursive model. Frankly, I was caught flat-footed on that one. This is one of the reasons I elected to defer the discussion for another day. If I find that I am not well-prepared to take on a topic with the students, I think it is better to just accept that it is not the time to teach that topic. The starting point was a place where I felt I could do a better job if I waited until I was thoroughly prepared before pursuing the topic in class.

During the course of the lesson I would have the students present their ideas at the board or on the overhead. When I looked at the video, I noticed that they never got out of their seats. ☹

The last thing I would do is prepare my rectangles ahead of time. I'm a poor artist, and my rectangles were pathetic.

Reflect on the way you built this lesson around the idea of patterns, including the finite differences. What impact did this have on student understanding of the algebraic concepts?

When I work with middle school students, I've noticed that they look at tables of data by relating each value to the previous value in the same column. Since this seems to be the way they naturally look at the data I thought it would be a good idea to build on student comfort examining data in this way. By the time they get to my class, they have had a lot of experience identifying patterns in a sequence of numbers. I have the advantage of teaching one of the middle school courses; I make sure I build these experiences into that course so I can use them in this course.

I want the students to develop fluency in mathematics. In order to be fluent they must be able to look at data and decide what type of model might be reasonable. To accomplish this I try to contrast the current model with models they have seen in the past. In their course last year they had an introduction to the exponential function.

Earlier this year, in a unit on power models, we looked at inverse power models. When students looked at the graph of the data from the coin experiment, they saw a graph that in many ways resembled the inverse power model for $x > 0$. Looking at the finite differences of the power model and contrasting that to the finite differences in the probability model gave the students another tool to help them discern the difference between the model they were working with and the power model. Once they knew that it couldn't be a power model they explored other possibilities, specifically the exponential model.

In this lesson, students represent data numerically, graphically and geometrically, algebraically, and verbally. Reflect on how you used this so-called "Rule of Four" to help students understand the concepts of probability and recursion.

As I mentioned at the beginning, my students are weak in probability. Add to this the difficulty inherent in recursion and it was essential that I provide as many ways for students to "grab" the concept as I could. Every student has a comfort zone, so I tried to give each student something they could be comfortable with. Once a student has become comfortable with the work he or she is free to explore in areas that are not as comfortable. It is OK to step into an area that is more challenging because they have their comfort zone to fall back on. I motivate students to step out of their comfort zone by including the requirement to work with all the representations, numerical, graphic, geometric, algebraic and verbal, on classroom assessment items.

Describe any evidence that you see in this tape or that you can recall from last semester that the students are making connections between the representations.

I can see that students are making connections between the representations because they started with collecting data, then they graphed the data. Once they had a graph they started trying to fit an appropriate model to the graph. If they hadn't made a connection between models and the graphs they produce, they wouldn't have been able to begin this exercise. The fact that they started with a power model is probably due to having just finished a unit on power

Video Teacher Reflections, cont'd.

models. It was also evident that the students connected the geometric model to the table of data. They used the geometric model to produce the values in the table and they used the table to check the geometric models. (When students are doing this, it is important for the teacher to monitor the work. If the same error occurs in both representations, the students may come to an incorrect conclusion.) Finally, they used the graph and table to develop a rudimentary Now-Next Equation and an Explicit Equation. When they have settled the question of the starting value, they will have a finished model.