Workshop 3
Systems of Equations and Inequalities

Overview

Description

This workshop presents two activities. The first is an example of how teachers can help students develop a conceptual understanding of solving systems of equations. The second shows ways that teachers can help students understand why the Corner Principle is useful in finding the optimal point in linear programming problems.

- Part I: Jenny Novak introduces an experiment in which students compare the speed at which they write left-handed with the speed at which they write right-handed. This activity allows them to explore the three types of possible solutions to a system of two equations with two unknowns.

- Part II: Patricia Valdez has students work through a linear programming problem to find the feasible region. They then graph a family of profit lines and try to find the point on the feasible region that will produce the greatest profit.

Featured Textbooks
- Interactive Mathematics Program, Year 2
- Howard County Schools Mathematics Curriculum

Featured Educators
- Jenny Novak, River Hill High School; Clarksville, Maryland
- Patricia Valdez, Aptos High School; Aptos, California

Featured Commentators
- Miriam Leiva, Professor of Mathematics Emerita, University of North Carolina at Charlotte
- Jane Schielack, Professor of Mathematics, Texas A&M University: College Station, Texas

Learning Objectives

In this workshop, you will learn to help students:

- Create scatterplots for two sets of data and find the equation of a line of best fit for both sets of data.
- Determine the three different types of possible solutions to a system of two equations with two unknowns.
- Interpret the meaning of the intersection of two lines as a solution to a system of equations.
- Graph the feasible region in a linear programming problem.
- Graph a family of profit lines for a given linear programming problem.
- Determine the optimum point or solution to a linear programming problem.

Workshop Session (On-Site)

Part I: Systems of Linear Equations

Getting Ready (15 minutes)
Complete the activity comparing the speed at which you write with your left hand to the speed at which you write with your right hand. (See the lesson plan, Right Hand/Left Hand Experiment, in the Appendix for instructions.) Discuss how you would graph the data on a scatterplot and find the lines of best fit, and how you would use this activity to help students understand that there are three different types of solutions to a system of equations.

Watching the Video (30 minutes)
Watch Part I: Systems of Linear Equations.

Going Further (15 minutes)
Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- What are the strengths and weaknesses of the left-hand/right-hand experiment as a vehicle to help students understand the meaning of a solution to a system of equations?

- What did you consider to be the strengths of Jenny's approach in this lesson?

- Analyze Jenny's questioning technique. What different types of questions did she ask?

- Give some examples of how Jenny demonstrated patience with her students.

- Discuss Jenny's use of graphs without scales in the introductory part of the lesson.

- Discuss how the guided discovery achieved the objectives of the lesson.

Part II: Systems of Linear Inequalities

Getting Ready (15 minutes)
Look at the homework assignment students in the video will be discussing. (See the lesson plan, Picturing Pictures, in the Appendix for instructions.) Decide what mathematics students will need to know to complete the assignment, and pinpoint those areas in which you think they may struggle.

Watching the Video (30 minutes)
Watch Part II: Systems of Linear Inequalities.

Going Further (15 minutes)
Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- Discuss the different strategies Patricia used to help her students improve their English proficiency.

- What strategies do you use to help English language learners succeed in mathematics?

- Cite some examples of how Patricia demonstrated persistence in helping her students work through the mathematics.

- What techniques do you use to help students understand how to find the optimum point in a feasible region?

- What do you think about how Patricia handled the situation when students incorrectly graphed the feasible region?

- What are some things that you do to help your students grasp the concept of the feasible region?

Between Sessions (On Your Own)

Homework Assignment

Investigate at least one of the resources listed in the Resources section of the Web site for Workshop 3, and find at least one idea that you can incorporate into your classroom. Write your thoughts on how you would use this strategy.

Ongoing Activities

You may want to carry on these activities throughout the course of the workshop:

Keep a Journal

Read the Teaching Strategies for Workshop 3 and answer the journal prompts. Include thoughts, questions, and discoveries from the workshop itself and learning experiences that take place in your own classroom. You are encouraged to use the online journaling tool at www.learner.org/channel/workshops/algebra.

Web Site: www.learner.org/channel/workshops/algebra

Investigate the Resources section to deepen your understanding of the Teaching Strategies for this workshop.

Share Ideas on Channel-Talkinsights@learner.org

Share your ideas about how you can improve your skills at building understanding and improving student communication.

Video Teacher Reflections

Jenny Novak

Below are Jenny Novak's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

When watching Workshop 3, Part I, what did you notice about your teaching strategies and student thinking?

In this lesson, I primarily used questioning strategies and cooperative learning activities as a means to achieve my objectives. Because this was the beginning of a unit, class discussion seemed to me a nice way for students to explore the concept of a system of equations. I also knew that I wanted to have an activity that was motivating so that the students would develop an interest in the unit. I tried to scaffold my questions so that students could build from basic observations to predictions about general trends and outcomes.

In the video, Miriam Leiva said that your demeanor gives the students confidence. What kinds of things do you do to create an atmosphere where students feel comfortable taking the time they need to build their understanding of a concept?

I try to make the atmosphere in my classroom one in which students feel comfortable taking risks and offering ideas. It is very common for me to use questioning strategies that require the students to draw conclusions and provide support for their answers. It is a common tendency for teachers to want to hear a "right" answer from the class and so they choose the top few students to participate. However, many students get overlooked in this process. Students in my class know that each student may be asked a question and when they are asked to share with the class, they are given enough time to clarify their thoughts and comments.

Write about how you structure your lessons so that you and the students can focus on building their understanding of the concept. Make reference to any evidence of this you see in the videotape.

When I began the Systems unit, I wanted students to gain a rationale for why we explore systems of equations and applications this unit may have to the real world. I wanted the students to have a strong foundation of what the systems models were before we begin to explore techniques for solving systems. By starting the unit with a data-gathering activity, students are interested and motivated. I wanted to start the day with a short connection to the linear functions unit, by having students review and discuss linear applications. Then we move into the right-hand/left-hand activity and students have an opportunity to examine sets of data that are relevant and meaningful to them. Because every student has unique data, the students are able to make comparisons between the graphs. The analysis questions for the lab provide opportunities for students to explain what the slope and y-intercept represent in the context of the experiment, as well as review how to find line of best fit. The experiment also gives the students the opportunity to consider different types of linear systems and what parallel lines or two lines sharing the same points could mean in the context of this experiment. After students have exhausted the possible system models, I wanted them to examine the car rental graph and make conclusions based on the graph. In this lesson the students moved from their specific data to broad generalizations and then back to a specific example. As a closure, I had students reflect and write about what they learned as a means for me to assess their understanding of the lesson.

When these videos were made, several students spoke little or no English, and you didn't speak their language. Write about the challenges and the rewards this presented.

In this particular class, I have three students that speak little to no English. It is often a struggle for me, and for these students, because our curriculum requires students to be able to explain and justify. I also do have quite a bit of class discussion which seemed to intimidate these students at first. I have found that during the course of this semester, these students and I have found a way to deal with the language barrier. In the video, Nancy found a way to communicate to her classmates with her hands and drawing to emphasize her ideas. I have also worked to make sure my lessons have visual aids and concrete examples when necessary to help these students that may not understand verbal instructions or explanations.

Video Teacher Reflections, cont'd.

Miriam Leiva says "we are all learning the language of mathematics." Write your thoughts on the relationship between building understanding of concepts and building proficiency in the language of mathematics.

I think it was a common notion when I studied algebra that the focus of a mathematics classroom was to find a correct answer. Nowadays, the focus of a mathematics classroom is not only find a solution, but to be able to make predictions, compare situations, identify trends, support a conclusion, and justify an answer. It seems natural that as students are developing an understanding of concepts, that they are being asked "How do you know that?", "Why is this true?", "What support do you have?" so that they can gain a strong foundation.

Patricia Valdez

Below are Patricia Valdez's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

When you watched Workshop 3, Part II, what did you notice about your teaching strategies and student thinking?

It has been very difficult to answer this question because I was actually very impressed with my students and with the way they expressed themselves. Even with the language challenges, they were very detailed in the way they explained their mathematical processes. I felt that maybe all of the strategies I have been using really **have** helped. I have been working with all of these students for two to three years, so that has also helped me teach them; consistency is so important. I don't want to sound arrogant, but I was actually very pleased and even impressed with the video. I saw myself teach like I aspire to teach every day.

How does this lesson fit into the unit you were teaching? Where were the students in terms of building their understanding of linear programming problems when this lesson was taped? How did their understanding change over the remainder of the unit?

The unit question is "How many dozens of plain cookies and dozens of iced cookies should Abby and Bing Woo make so that their profit is as high as possible?" Students are given the following information and constraints:

The Woos know that each dozen of their plain cookies requires 1 pound of cookie dough and no icing, and each dozen of their iced cookies requires 0.7 pounds of cookie dough and 0.4 pounds of icing. Each dozen of their plain cookies requires 0.1 hours of preparation time and each dozen of their iced cookies requires 0.15 hours of preparation time. They also know that no matter how many of each kind they make, they will be able to sell them all. The Woos' decision is limited by three factors:

1. The ingredients they have on hand: They have 110 pounds of cookie dough.

2. The amount of oven space: They have room to bake 140 dozen cookies for tomorrow.

3. The amount of preparation time available: They have 15 hours.

Video Teacher Reflections, cont'd.

The plain cookies sell for $6.00 per dozen and cost $4.50 a dozen to make. The iced cookies sell for $7.00 a dozen and cost $5.00 a dozen to make.

By the time this lesson was taped, the students had converted the constraints into inequalities and learned to graph inequalities. They had also had several experiences with graphing two to four inequalities together to form a "feasible region."

The lesson that was videotaped was the first time the students were asked to put everything together and make sense of the profit lines. After this lesson, students continued to graph feasible regions and profit lines for 10 other situations before returning to the unit question. They also had assignments in which they practiced graphing equations of lines and finding the intersection of lines algebraically (solving systems of linear equations) as well as finding the intersection of lines using a graphing calculator (estimation).

The students seemed to feel very confident by the time we returned to the unit question. They had learned and practiced all of the mathematics needed to solve the unit question.

It is clear from the video that some students are more proficient in English than others, and some students are more proficient in mathematics than others. Discuss the challenges and rewards of working with a diverse group of learners. What strategies do you recommend for addressing the needs of all students?

I love working with diverse groups of learners. They all have something to offer the class and each other. Many times they feel that they do not have anything to offer, so my first challenge is to build their self-esteem. I feel that my first job is to show them how much they **do know** and to get them to trust their instincts. My first strategy for building self-esteem is to use appropriate material that will challenge and interest them. I use "real-life" situations from the Interactive Mathematics Program. My second strategy is asking lots of questions rather than telling them things. This is also a great challenge for me. It is much easier to tell them the answer. It is much more difficult to come up with questions that will lead them to think and to figure out the answer for themselves.

For English Language Learners, I use several strategies. I am very aware of the language that I use for mathematics. I require that all students answer questions in complete sentences in writing and while presenting. This means that I give them phrases to help them express their thoughts or questions. For example, the first time I answer a question regarding a graph on the board, I will write: **"According to my graph,** *Bernie is spending about $6.00 a day and will not have enough money to pay his rent by the end of the month. Bernie will have about $360 by May first."* After I give them these sentence starters, the students start to use them in their presentations and in their written work.

For any type of writing that I want the students to learn, I do it for them in the beginning and then help them less and less until they can do it for themselves. For example, I help them with portfolios and problems of the week. I also give them rubrics and teach them how to make rubrics so that they know what is expected of them. I also make posters of important mathematical concepts and refer to the posters constantly. The students need lots of models and rubrics.

What kinds of things do you do to help the students build confidence in themselves as mathematics learners and to encourage them to support each other's learning?

First of all, I always start my classes with the understanding that I expect everyone to present in front of the class. This said, I acknowledge that this is a very scary thing and that because of this we must all be kind to one another during presentations. All students must be helpful and supportive. We discuss what it means to be supportive through writing activities and discussions. If needed, I resort to disciplinary action when a student is rude, mean, or critical. Next, we learn to be helpful. If a student makes mistakes during a presentation, I teach the students to come up with questions that will help the presenter figure out the mistake. This is a long process and I have had to practice this kind of questioning myself for many years. I very seldom just give answers; I try first to ask good questions.

It is also important to note that I try to use as many of the SDAIE (Specially Designed Academic Instruction in English, http://www.fremont.k12.ca.us/state-federal/sdaie.html) teaching strategies as possible.

Video Teacher Reflections, cont'd.

As you watched this tape, what strategies did you feel were particularly successful in building understanding? What might you do differently the next time you teach this lesson?

I noticed that the students seemed to understand the algebra better when I referred to the **meaning** of each variable, equation, or inequality in the context of the problem. I also think that it was helpful to the students for me to ask questions and **wait** until they answered the questions.

This is the first year that I developed the use of the triangle to investigate the profit lines. Profit lines are the stumbling block for many of the students. They get confused between the inequalities of the constraints and the equations of the profit lines. Next year I will try to review the concepts thoroughly before going on to the profit lines. I will use the triangle again.