

# **Workshop 1**

## **Variables and Patterns of Change**

# Overview

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## Description

This workshop presents two activities that teachers can use to help students explore how to change words into symbols and how to solve equations using manipulatives and algebra. The activities use a variety of techniques to help students understand important mathematical concepts.

- Part I: Janel Green uses a pool-tiling problem as a vehicle to help her students understand and make connections between words and symbols as used in algebraic situations.
- Part II: Jenny Novak uses manipulatives to help her students understand the equivalence transformations used to solve linear equations.

## Featured Textbooks

- Textual materials written by Janel Green
- Textual materials written by Jenny Novak

## Featured Educators

- Janel Green, Walt Whitman High School; South Huntington, New York
- Jenny Novak, River Hill High School; Clarksville, Maryland

## Featured Commentators

- Frances R. Curcio, Professor of Mathematics Education; Queens College of the City University of New York
- Miriam Leiva, Professor of Mathematics Emerita; University of North Carolina at Charlotte

## Learning Objectives

In these activities, you will learn ways that students can:

- Analyze a problem situation and write an equation to model the information.
- Use drawings and pictures to help them generalize and write a formula.
- Determine if different formulas are equivalent.
- Understand the meaning of area and perimeter, and their relation to each other.
- Solve linear equations using manipulatives.
- Solve linear equations using algebra.
- Check and verify that the solution to an equation is correct.

# Workshop Session (On-Site)

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## Part I: Words Into Symbols

### Getting Ready (15 minutes)

Discuss your goals and expectations for participating in *Insights Into Algebra 1: Teaching for Learning*. In your discussion, focus on key algebraic concepts, describe effective teaching strategies for helping students learn algebra, and address topics with which students typically have difficulty.

### Watching the Video (30 minutes)

Watch Part I: Words Into Symbols.

### Going Further (15 minutes)

Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- What aspects of cooperative learning did you see as being particularly effective in this lesson?
- Discuss the students' misconceptions concerning area and perimeter relationships. How did the students resolve these misconceptions by the end of the lesson?
- Discuss students' use of informal language in the video. In what ways might teachers help students move beyond informal language?
- Discuss the significant differences between the two problems Janel posed. In what ways did the second problem build on the first?
- Discuss Janel's teaching strategy and style. What specific examples of effective teaching stood out in your mind as you watched the lesson?

## Part II: Solving Equations Using Manipulatives and Algebra

### Getting Ready (15 minutes)

Discuss some different ways that you would help your students understand and solve the linear equation  $4x - 6 = 22$ .

### Watching the Video (30 minutes)

Watch Part II: Solving Equations Using Manipulatives and Algebra.

### Going Further (15 minutes)

Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- Discuss the use of the cups-and-chips model. What concepts did it help students understand better than they might have had they not used manipulatives?
- Discuss whether or not adding a negative chip to both sides of the equation is the same as subtracting a chip from both sides of the equation. How does the cups-and-chips model reinforce the algebraic steps the students use?
- Discuss the method of checking answers that Jenny used as compared with the method Miriam Leiva endorsed. Are the two methods significantly different?
- Discuss the way Jenny rotated students through stations as a way to practice solving equations. What were some of the benefits of this approach?
- Discuss Jenny's use of white boards as a teaching strategy in her class. What other strategies might have a similar benefit?
- Discuss the importance of having students use the cups-and-chips model and write the algebraic steps at the same time.

# Between Sessions (On Your Own)

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## Homework Assignment

If possible, try one or both of the activities from the Workshop 1 video in your classroom, and reflect in your journal about your students' understanding of the concept. As an alternative, modify an existing lesson plan on translating words into symbols or solving equations using manipulatives and algebra to incorporate teaching strategies from this workshop. Write your goals for this activity in your journal.

## Ongoing Activities

You may want to carry on these activities throughout the course of the workshop:

### Keep a Journal

Read the Teaching Strategies for Workshop 1 and answer the journal prompts. Keep a journal, including thoughts, questions, and discoveries from the workshop itself and learning experiences that take place in your own classroom. You are encouraged to use the online journaling tool at [www.learner.org/channel/workshops/algebra](http://www.learner.org/channel/workshops/algebra).

### Web Site: [www.learner.org/channel/workshops/algebra](http://www.learner.org/channel/workshops/algebra)

Investigate the Resources section to deepen your understanding of cooperative learning and using manipulatives as means to help students understand mathematical concepts.

### Share Ideas on Channel-Talkinsights@learner.org

Share your thoughts and ideas about how cooperative learning and using manipulatives is important in helping students better understand mathematical concepts. You can subscribe to an e-mail discussion list and communicate with other workshop participants online. To subscribe to Channel-Talkinsights, visit:

<http://www.learner.org/mailman/listinfo/channel-talkinsights>.

# Video Teacher Reflections

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## Janel Green

Below are Janel Green's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

### ***As you watched Workshop 1, Part I, what did you notice about your teaching strategies and student thinking?***

I allowed students to work out solutions for themselves. They are capable of solving more problems by themselves than we may think. I was careful not to form a judgment about students' answers. For example, in the beginning of the lesson, I accepted all guesses and encouraged students to explain how they arrived at their solutions. As they explained their answers, their fellow classmates were able to make corrections. Students seem to accept correction better from their peers than from their teachers.

### ***What experiences had these students had with symbolic manipulation prior to this lesson? In what ways would you say the "dream pool" context was effective in extending their understanding of symbolic manipulation?***

The students were familiar with using variables to represent an unknown. They also had experience adding, subtracting, multiplying, and dividing variables. This was the first time that they connected a real-life application to symbolic manipulation. The "dream pool" problem allowed students to see a need for variables, and later they had to use mathematical operations on the variables.

### ***For what purposes do you use cooperative groups?***

I use cooperative learning for "self-discovery learning." When I want students to discover solutions, formulas, or patterns for themselves, I put them in groups. In problem solving, sometimes "two heads are better than one." For example, in this lesson, I challenged the students to find and discover patterns. Pattern recognition and symbolic manipulation was fairly new to them. This lesson might have been intimidating to students so close to the beginning of the year. However, since they were in groups, they were comfortable discovering and discussing mathematics.

***Teachers who watched this video were impressed when you refrained from correcting students during the group work, thereby allowing them the opportunity to correct each others' misconceptions in their presentations. For example, you gave Sean the opportunity to correct another team's misuse of the term "perimeter" in front of the class. Discuss why you were able to feel comfortable allowing it to happen this way.***

I was comfortable with students doing self-correction because I have trained myself to give the students the power over their learning. It is important for teachers to learn to give up their power to the students.

I ask questions that provoke critical thinking, and I try to be careful with the way I ask a question. I try not to lead students. If students aren't familiar with this style of learning, it may take time for them to adapt to it. From the beginning of the year, I've encouraged the students to explain their answers and to correct each other.

### ***As you watched the video, what were your thoughts on how the students handled the second pool problem? What were the pros and cons of introducing this problem to them during the same class period?***

The students handled the second problem better than I thought they would; I didn't expect them to finish it. However, some of the students realized that it was an extension of the first problem and applied what they had just learned. The benefit of the second problem was that the students were able to take what they learned and apply it in a similar situation. That is an important skill in itself. The drawback of the second problem was that I wasn't able to assess fully how well students could apply what they learned to a totally new situation.

### ***If you were to teach the pool problem again, what might you do differently?***

I would give a second problem that was totally different from the first problem.

# Video Teacher Reflections, cont'd.

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## **Jenny Novak**

Below are Jenny Novak's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

### ***As you watched Workshop 1, Part II, what did you notice about your teaching strategies and student thinking?***

For this lesson, I chose to use a combination of cooperative learning, direct instruction, and questioning techniques to teach students how to solve two-step equations. I wanted to use manipulatives so students have a hands-on strategy to help develop the algebraic skills. In the warm-up, I chose to include a two-step subtraction problem that could be solved using other strategies to help transition into the lesson. Throughout the lesson, I found that students made common errors, such as adding rather than subtracting, but after the cups-and-chips activity, it appeared that the students struggling were able to work through some of their mistakes. It also seemed that the activity helped the students to make the connection between the algebraic operations and a rationale for why the operations were being performed.

### ***Give more detail than is presented in the video about how you use cooperative groups. How do you select the students to put together, and what things do you watch for as they begin to work in a new group? What specific things do you do to make sure all students get the most out of their experience working in groups?***

As I am forming cooperative groups, I try to make sure the group is as diverse as possible in terms of academic ability, gender, and ethnicity. In the beginning of the course, I do a lot of shifting of groups so students get to know each other, and I learn what groups work well and what groups don't. I try to take into account students who may be shy or introverted and do my best to place them with students with whom they feel comfortable sharing. I want to make sure that one student in a group is not emerging as the "controlling force" of the group. My students know that each member of the group is responsible for making contributions and there is individual accountability.

### ***For what purposes do you use cooperative groups? For what purposes do you find individual work or whole-class discussion more effective?***

I believe working in cooperative groups serves an important function in the mathematics classroom. I think that students learn better when they have the opportunity to share their ideas with their peers, and I generally find that cooperative groups are a motivation for students to complete activities. It provides a way for all students to be actively engaged in the lesson. I use cooperative groups for exploratory or introductory activities, data analysis and/or data gathering experiments, and during the practice stage of a lesson. I try to balance cooperative group time with individual time and class discussion time. The group discussion is also an important part of the lesson because it often serves as an introduction to an idea or a means to connect different ideas or insights discovered in the cooperative groups.

If students have been taught a skill and are asked to practice a skill, I sometimes use Think-Pair-Share as a strategy to gauge understanding. If I employ this strategy, I often have the students work on the dry-erase boards so I can check their work. This strategy is effective because it gives students individual time to work through a problem, a chance to confer with a partner or small group and reach a consensus, and then the opportunity to share with the class.

### ***As you watched the video, what did you notice about the students' use of manipulatives? What about their ability to move between the cups and chips and the symbolic manipulation? Do you think it is important to differentiate between adding a negative (as the cups-and-chips model suggests) as compared to subtracting (which the students did in the algebraic steps?)***

The use of manipulatives in this lesson served as a tool for the students to gain an understanding of the algebraic manipulation. Students typically like hands-on activities and are motivated to use the manipulatives. I find that my students with special needs seem to understand the symbolic manipulation better after using manipulatives. Because they have a concrete example to explore, the transition to symbolic manipulation is fairly smooth. I believe that a brief discussion should be held during this activity to differentiate between adding a negative and subtracting. If this discussion is not held, some students may get confused. For example, if a problem was  $-2 + x = 4$ , some students may want to subtract 2 from both sides of the equation.

# Video Teacher Reflections, cont'd.

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***Do you agree with Miriam Leiva's statement that if you introduce manipulatives, you need to introduce the related symbols and symbolic manipulation during the same lesson? Why or why not?***

I think it is very important that students are writing their algebraic steps as they are using the manipulatives so that they receive constant reinforcement of the skills that they are learning. The ultimate goal is for students to solve equations algebraically, so by having them write their steps side-by-side, the transition will be easier to make.

***If you were to teach this lesson again, what might you do differently?***

If I were to teach this lesson again, I think I would like to do more with the dry-erase boards using Think-Pair-Share so students would have more opportunities to practice two-step equations. I would probably move the equation windows activity to a second day and have it serve as a review activity at the beginning of the lesson. Even though the written work turned in showed that students appeared to understand the skill, I would like the students to have a little more practice with solving these equations.

# Notes

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