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Teaching;
Visual displays.

Summary

This article reports some results of an informal study of very young children's reactions to some visual displays of data.

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◆ INTRODUCTION ◆

WORLDWIDE reform documents, such as those of the National Council of Teachers of Mathematics (1989) and the Department of Education and Science and the Welsh Office (1991), reflect the importance of data handling at all levels of the school curriculum. While research has generated a rich knowledge base of students' informal thinking to guide instruction in areas such as number (Carpenter and Moser, 1984), there is limited research on young students' statistical thinking.

With respect to data handling, Curcio (1987) and Wainer (1992) have characterised the kinds of responses that might be expected from upper elementary and middle school students when they are asked to read and interpret graphs. In particular, Curcio has identified three components of graph comprehension: "reading the data, reading between the data, and reading beyond the data" (p. 384).

This article examines students' statistical thinking in terms of four components of data handling: "organising, describing, representing and analysing data" (Shaughnessy *et al.*, 1996, p. 205) which, while incorporating Curcio's (1987) three levels of graph comprehension, enable us to take a broader perspective of students' statistical thinking.

◆ INVESTIGATING YOUNG STUDENTS' STATISTICAL THINKING ◆

Our investigation of young students' statistical thinking involved eight students from two classes at an elementary school in midwestern USA. They were selected by their teachers to reflect a broad range of mathematical achievement, and all of them had minimal

experiences with bar and tally graphs. Four were first grade students (Boris, Keith, Carlos, Belinda - age 6) and four were second grade students (Martina, Jane, Katrina, Jonas - age 7).

The students were interviewed using the protocol shown in figure 1. The probes were labeled D - describing, O - organising, R - representing and A - analysing, according to which of Shaughnessy *et al.*'s (1996) components it assessed.

An examination of these students' responses revealed two general patterns of thinking: (1) idiosyncratic responses that were superficial, incomplete, or inconsistent; and (2) higher-level responses that showed some attempt to interact with and make statistical sense of the data. Each of these response patterns will be discussed in relation to the four data handling components.

◆ DESCRIBING DATA DISPLAYS ◆

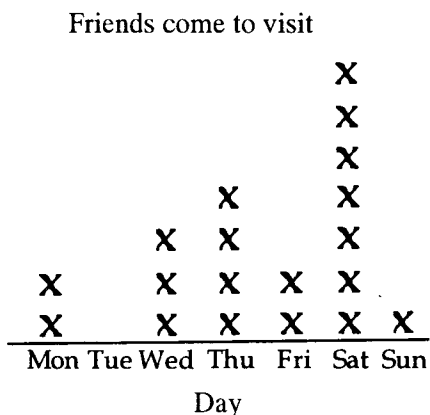
Describing data displays refers to reading information explicitly stated in the displays and recognising connections between different displays of the same data. Specific probes in the protocol (see figure 1) that related to this component were:

- ◆ What does this picture tell you? [D1]
- ◆ How are the two pictures alike? Do you think the pictures represent the same data? [D2]

Category 1 - Idiosyncratic Responses

In response to **D1** on the line plot (see figure 1a), six of the eight students gave Category 1 responses which focused on cosmetic features of the graph such as the days of the week, the graph title and the 'crosses' that formed the line plot. When asked **D1** in relation to the

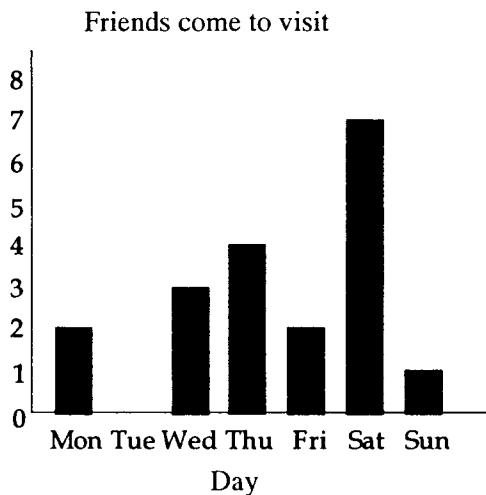
Sam had some friends come to visit each day during one week in the summer. The number of friends and the days they visited were displayed like this:



n = 19

- D1 What does this picture tell you?
- A1 How many friends came to visit each day?
- A2 What can't you tell from this picture?
- A3 Which day had the highest number of visitors?
Which day had the lowest number of visitors?
- A4 How many friends came to visit Sam during this week?
- O1 About how many friends came to visit Sam each day?
- O2 What is the average number of friends that came to visit Sam each day?
- A5 About how many friends would you expect to come to Sam's place every week during the summer vacation?

Fig 1a. Line plot of Sam's friends



n = 19

- D1 What does this picture tell you?
- D2 How are the two pictures alike? Do you think the pictures represent the same data?
- R1 Can you draw either of these graphs in another way?

Fig 1b. Bar graph of Sam's friends

bar graph (see figure 1b), students again responded idiosyncratically. For example, Carlos pointed to the bars from tallest to shortest and said "That is the biggest person he invited over and that is the medium kind of person, and a small kind of person...". While trying to make sense of the bar graph, and being aware that it was also about Sam's friends, he did not attempt to relate it to the earlier line plot. When asked what the numbers on the vertical axis told him, he focused on yet another characteristic of the friends, "how old they are". Even when asked the follow-up probes "How are the two pictures alike?" and "Do the pictures represent the same data?" he saw no inconsistencies in his explanations. While stating that they represented the same data and noting that the bars and the Xs looked alike, he said "They are the same information because each of these [bars] are people and this shows how tall they are and

this [vertical axis] shows how big [old] they are". For these students, links with the two data representations are tenuous.

Category 2 - Higher-Level Responses

The major feature that characterises this level is reflected in Boris' and Martina's responses to probe **D1** (figure 1b). Both recognised without even being asked probe **D2** that the two graphs describe the same story. One said "It tells the same thing as this graph [line plot]" and "it shows how many friends come to visit"; the other said "It tells the same thing as this [line plot], except there are no Xs". Moreover, when asked about the two graphs, the students agreed that they represented the same data and explained this by comparing, for each of the days, the number of Xs with the height of the bars.

Organising data incorporates mental actions such as grouping, reducing and summarising data (Moore, 1997). The following two probes on summarising data referred to the line plot (figure 1a).

- ◆ About how many friends came to visit Sam each day? [O1]
- ◆ What is the average number of friends that came to visit Sam each day? [O2]

Category 1 - Idiosyncratic Responses

All students except Boris gave idiosyncratic responses to these probes. Belinda's responses to O1 and O2 were typical. For O1 she said "One, because one person came every day". Like a number of other students in this study, Belinda ignored the fact that no friends came on Tuesday. For O2 she said "Five. I thought it up in my head." For these seven students, there was little evidence of informal knowledge of typicality or average.

Category 2 - Higher-Level Responses

Boris' response to O1 when he said "Four - because two is close to four, and three is close to four, and one is close to four, and two is close to four, and seven is close to four" indicates some understanding of typicality. He did not recognise the word 'average' in O2, but his response to O1 suggests that he might be looking for a 'balancing' number like the mean or possibly a middle number like the median.

Representing data involves constructing visual displays that exhibit different organisations of the data. In this investigation only one question on representing data was asked:

- ◆ Can you draw either of these graphs in another way? [R1] (see figure 1b)

Category 1 - Idiosyncratic Responses

Five of the eight students gave idiosyncratic responses. Jane's graph vividly illustrates such thinking. She drew what appeared to be a pictograph (see figure 2a).

Her explanation - "This is like a bunch of baby snakes on top of each other and [some are] more, [some are] less, and there is one lesser" - suggested an understanding of the need for some kind of icon or symbol to generate a graph, but the frequencies represented in her graph bore no relationship to either the line plot or the bar graph. In essence, Jane's graph showed no connection to the data on Sam's friends. Although the idiosyncratic responses of the other four students were not as vivid as Jane's, they were all characterised by the fact that their graphs did not represent the original data.

Category 2 - Higher-Level Responses

A concern for maintaining the integrity of the original data characterised the higher-level responses given by

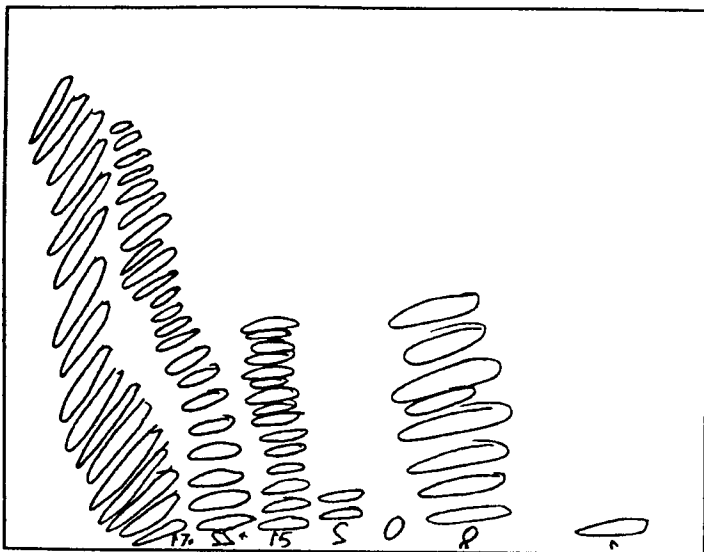


Fig 2a. Jane's graph

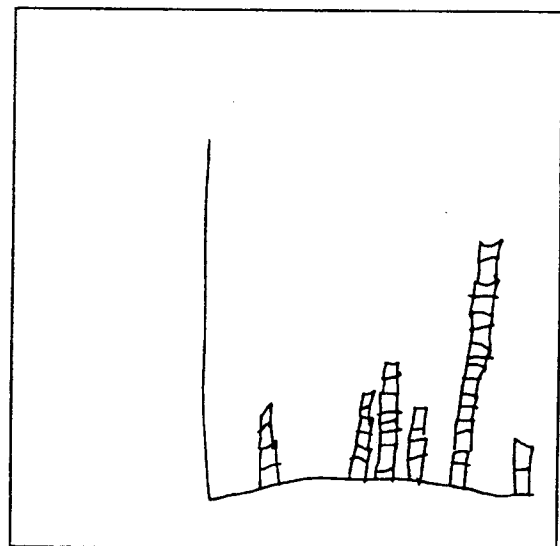


Fig 2b. Keith's graph

Boris, Martina and Keith. For example, Keith drew two unlabeled axes at right angles and redrew the line plot replacing Xs with 'doors' (see figure 2b).

When asked about the doors, he replied "That's where the people can walk in". Although Keith did not label the axes, his frequencies matched those on the line plot and the bar graph in both order and magnitude.

◆ ANALYSING DATA ◆

Analysing data incorporates recognising patterns in the data and making inferences, interpretations or predictions from the data. In this investigation all of the probes on analysing data referred to the line plot (see figure 1a):

- ◆ How many friends came to visit each day? [A1]
- ◆ What can't you tell from this picture? [A2]
- ◆ Which day had the highest number of visitors? Which day had the lowest number of visitors? [A3]
- ◆ How many friends came to visit Sam during this week? [A4]
- ◆ About how many friends would you expect to come to Sam's place every week during the summer vacation? [A5]

Category 1 - Idiosyncratic Responses

Jane's response to A1 was typical of the six students who gave idiosyncratic responses. She gave the total for the week (19) and the largest number for any day (7), but she did not list the number of friends who visited *each* day. Not surprisingly, when asked A2, she seemed to ignore the original data and responded "It doesn't tell you *how many kids came over*".

Students generally made sense of the three probes A3, A4 and A5, and showed evidence of being able to read "between the data" (Curcio, 1987, p. 384). In fact, all students gave a correct response to the probe about the highest number of visitors (7), and all but one correctly calculated how many friends came to visit Sam during the week. However, in response to the probe about the least number of friends visiting Sam, they overlooked the 0 (Tuesday) and maintained that 1 (Sunday) was the lowest number. Responses to A5 were typically idiosyncratic, four of the six focusing on personal situations. For example, Keith said "Four, because during the summer I had four people come over"; Jonas said "Maybe 2, because if I had 7 over during the summer, then it would probably be too crowded in my house".

Category 2 - Higher-Level Responses

By way of contrast, Martina and Boris understood that each X represented one friend and readily listed the number of friends who came each day of the week. Their responses to A2 also revealed an understanding that the data didn't tell "the names of the people that came [to visit]" or "what friend came, if they were a boy or a girl, or if they were a grown-up or not".

They both gave correct responses to the three probes about the lowest and highest number of friends and the number who visited during the week. However, neither student recognised the possibility that the same friend could be represented on different days, making the total less than 19. In response to A5, Martina showed evidence of thinking "beyond the data" (Curcio, 1987, p. 384) by using the 19 determined from the original data to predict for the other weeks of the summer.

◆ SUMMARY ◆

Although 70% of the responses of the eight 1st and 2nd grade students in this investigation were characterised as idiosyncratic, Boris (grade 1) and Martina (grade 2) consistently gave statistically meaningful and valid responses. Notwithstanding the prevalence of idiosyncratic thinking among these students, there was evidence that all but Keith could make meaningful interpretations (read between the data) once they understood what the data were saying. However, it should be underscored that these interpretations were only made following probes that enabled the students to build some understanding of data handling at the level of "reading the data" (Curcio, 1987, p. 384).

One implication arising out of this investigation is that student learning in data handling might be enhanced if teachers were more aware of the scope and range of students' thinking in relation to *describing data displays and representing data*. Such awareness might be achieved if teachers used specific and open-ended probes like those in this investigation to encourage students to describe or predict what they thought a visual display represented. Such probes also seem to enhance students' understanding of data and enable them to demonstrate higher levels of analysis and interpretation. Students' representations of data in this study are also very revealing. There is evidence that teachers could gain further insights into their statistical thinking by having students construct their own visual displays using data they had been given, that they had gathered themselves or that had been represented in another visual display.

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