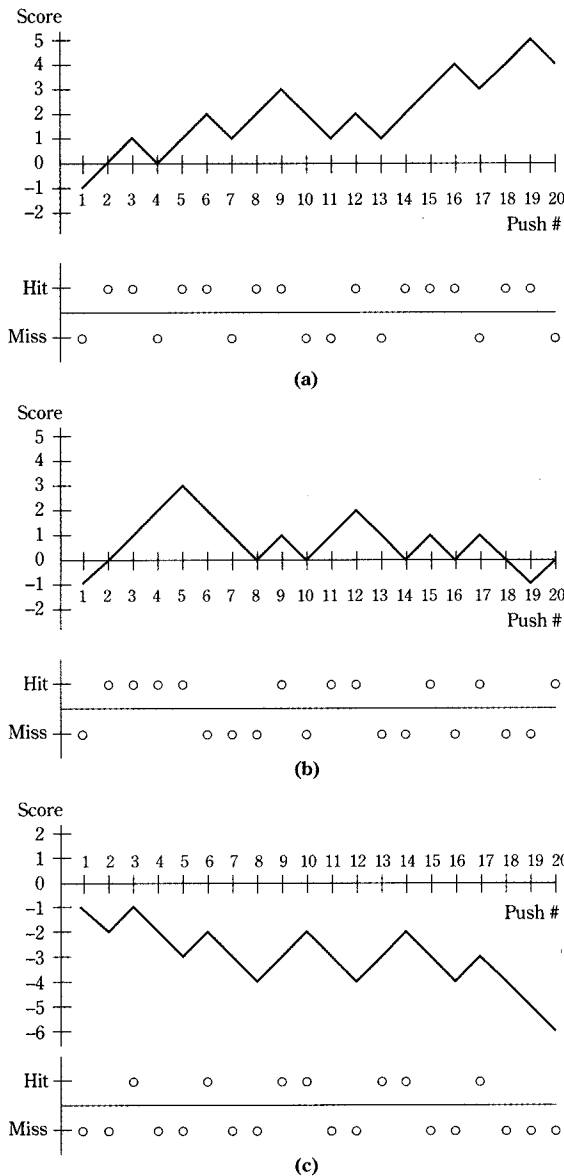


**Fig. 3**  
Graphs for  
the data in  
figure 2



**Pooled class data**

The class data are combined, summarized, and analyzed in the ways outlined subsequently. Examples and illustrations of these summaries are given in the “Interpretation” section.

- (a) Record the number of hits for all games played by the class.  
Determine the percent of hits for the class.
- (b) Record the final scores for all students in a frequency table.  
Draw a bar graph to represent this frequency table.  
Determine the percent for each final score.  
Determine the mean final score for the class.

- (c) Record all the maximum scores for the class in a frequency table.  
Draw a bar graph to represent this frequency table.  
Determine the percent for each maximum score.
- (d) Record all the maximum run lengths for the class in a frequency table.  
Draw a bar graph to represent this frequency table.  
Determine the percent for each maximum run length.

**Interpretation**

IF A PLAYER’S PUSH IS “RANDOM,” THE CHANCE that the coin will fall on a line is 50 percent. Look at **figure 4**. The shaded bands centered about the lines are one “quarter” wide. The unshaded bands have the same width. Half the board is shaded, and the other half is unshaded. If the center of the coin is in the unshaded band, the coin will not touch the line. If the center of the coin is in the shaded band, the coin will touch the line. A “random” push will put the center of the coin in the shaded band 50 percent of the time. Even with practice, most players will effectively be making random pushes, so you can expect the class as a group to get hits on about 50 percent of their pushes. The analysis of the pooled data will allow students to confirm their overall hit rate.

**Questions for Rules 1 and 2**

We first focus on the questions about rules 1 and 2.

**Individual results:** Refer to the game represented by the graphs in **figure 3a** and the corresponding data in **figure 2a**. After push 20, the final score was +4. This value is the height of the final plotted point in the graph of the score versus the push number. We also note that 60 percent of the pushes resulted in a hit. The maximum score, however, was +5, the highest point on the graph, which occurred at push 19.

Next look at the game in **figures 2b** and **3b**. After push 20, the final score was 0 and 50 percent of the pushes resulted in a hit. The maximum score was +3. The results for the game in **figure 3c** are a final score of -6 and a maximum score of -1.

Compare the graphs in **figures 3a, 3b,** and **3c**. The sequences of hits and misses, the percents of hits, the final scores, and the maximum scores differ. This outcome is a consequence of the randomness in the pushes, what we call *random variation*. When outcomes are due to randomness, the outcomes will vary. Have your students display their graphs together so that they can compare them and see this random variation as well as determine winners.

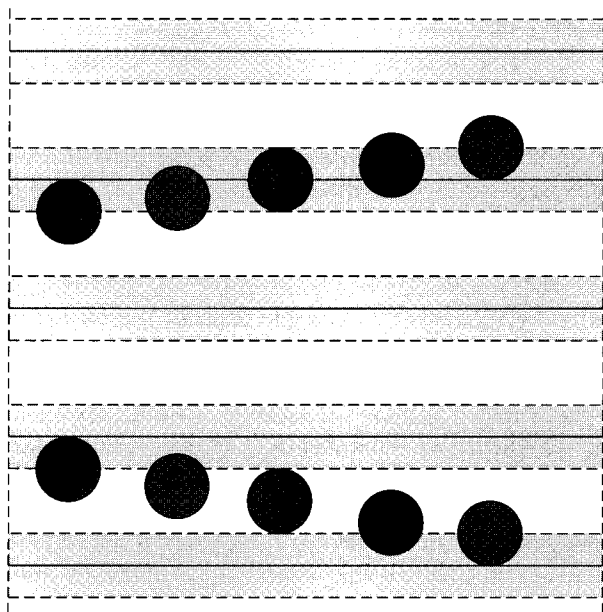


Fig. 4 A random push has a 50 percent chance of hitting a line.

Probability tells us very little about individual plays of the game. However, it tells a lot about a large number of plays of the game. With random pushes and a 50 percent chance of hitting a line, we would expect that 100 plays of the game ( $100 \times 20 = 2000$  pushes) will produce about 1000 hits (50 percent of 2000) and that the average of the 100 final scores will be about 0. Predictability is found in repetitions of the game. Students' reasoning and intuition about this concept can be developed by studying experimental data as illustrated subsequently in the pooled results.

**Pooled results:** Table 1 gives a summary for 100 games. The largest final score was 12. The summary resulted from  $100 \times 20 = 2000$  die rolls. The percent of hits was about 50. The average, or mean, of the final scores was 0.06. The frequency table enables us to answer the question about which scores occur most often. A score of 0 occurs most often (frequency 16), and scores from  $-6$  to  $+4$  are not far behind, with frequencies of 12 or more. Only one person made the highest score of  $+12$ .

**Table 2** gives a frequency table for the maximum scores. The largest maximum score was 12. A maximum score of 2 occurred most often, but scores ranging from 0 to 4 were quite common.

### Questions for Rule 3

**Individual results:** Look at the sequence of hits or misses in the graphs in figure 3a and the data in figure 2a. This game had six runs of hits. The first run (pushes 2, 3) is of length 2. The next run (pushes

5, 6) is of length 2, followed by runs of lengths 2, 1, 3, and 2. The maximum run length is 3.

Note the relationship between the plot of hits and misses and the line graph that shows the final score. A run of hits corresponds to a line segment with a positive slope; a run of misses corresponds to a line segment with a negative slope.

Next look at the game in figures 3b and 2b. The run lengths are 4, 1, 2, 1, and 1. The maximum run length is 4. The game in figure 3c has run lengths of 1, 1, 2, 2, 1. The maximum run length is 2.

Compare the graphs for a, b, and c in figure 3. The three sequences of hits or misses, the patterns and lengths of runs, and the maximum run lengths

TABLE 1  
Rule-1 Results for 100 Games

NO. OF HITS	FINAL SCORE		FREQUENCY
4	-12		0
5	-10		0
6	-8	*	1
7	-6	*****	12
8	-4	*****	14
9	-2	*****	15
10	0	*****	16
11	2	*****	16
12	4	*****	15
13	6	*****	7
14	8	**	2
15	10	*	1
16	12	*	1

# hits: 1003; mean final score: 0.06; % hits: 50.15

TABLE 2  
Rule-2 Results for 100 Games

MAXIMUM SCORE		FREQUENCY
0	*****	14
1	*****	15
2	*****	22
3	*****	13
4	*****	14
5	*****	7
6	*****	8
7	***	3
8	*	1
9		0
10	**	2
11		0
12	*	1