FIGURE 2

Line plot with all stick-on notes on the value 4

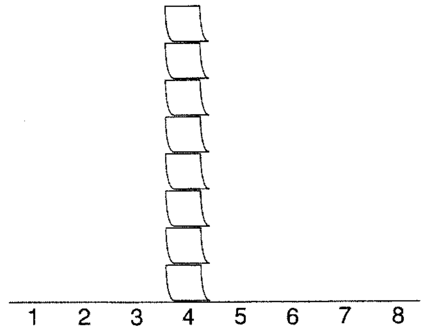
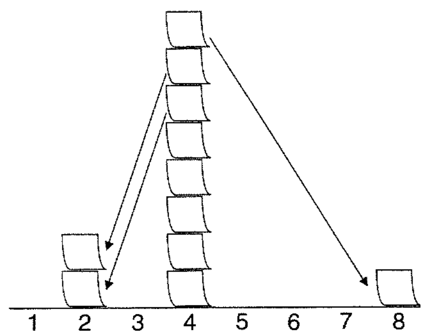


FIGURE 3

In-progress distribution with the value 8 balanced by two stick-on notes on the value 2



of thinking about the mean that involves neither fair shares nor balances. We use "unpacking" tasks. Like construction problems, unpacking tasks begin with a mean and work backward to the data. For example, if the mean family size is 4, what might the data look like? Students work with a line plot and stick-on notes, starting with all the notes stacked on the value 4. (See **fig. 2.**) Although the data distribution certainly could look like this representation, it is unlikely. Students know from their own experience that some families are the same size as the average but that many are larger and many are smaller. We ask students to make a distribution that is more realistic by asking, "If we knew that one family really had three people instead of four, what could we do to make the average come out to 4?" Students usually suggest moving one data point, represented by a stick-on note, from 4 to 5 so as to balance the 3. The teaching intervention continues by having students move more data points until they are happy that the data look "like real life" and that the average is still 4. We keep the numbers small so that students can easily calculate whether the mean is still 4.

When solving this problem, many students stick with a symmetrical distribution in which each point above the average is matched by a point the same distance below the average. Once they are comfortable with this approach, we introduce the idea of asymmetrical balancing by asking what would happen if one family has 8 people. In this situation, the move from 4 to 8 cannot be balanced by a comparable move on the left-hand side of the mean because no family can have zero people in it. In our work in classrooms, students have always come up with the idea of moving two different stick-on notes down a total of 4 units so as to balance the upward move of 4.

As students work with this complex idea using a variety of problem contexts, it is important to move slowly and let them share their ideas and strategies as they develop their ideas about balance and distance (see Friel, Mokros, and Russell [1992]).

Conclusions

Understanding averages must be grounded in many experiences with a variety of data sets. As students describe, summarize, and compare data sets, they naturally begin to talk about what is typical of a particular set of data. They develop their own descriptions of the characteristics of a number that summarizes a whole set of data. For example, during a fourth-grade class discussion of plans to compare the heights of first graders in their school with their own heights, one student described his idea saying, "We could find the one number that's sort of in the middle or that the other numbers are crowded around, and then do the same thing for the first grade (Russell and Corwin [1989]).

Once students develop some ideas about how the middle of a data set is important, they can be introduced to the definition and use of the median as a statistical measure. Used in conjunction with the range of the data, the median supplies a great deal of information about the data set. We recommend that work with the median begin in about fourth grade and continue through the upper elementary and middle school years.

Even though the procedure for finding the mean is easy to teach, we recommend delaying the teaching of this procedure until about sixth grade, and only after students have developed their own ideas about balance. Prematurely teaching the averaging algorithm does nothing to help children develop a solid understanding of the relationship between the mean and the data it represents, and may actually interfere with the development of this understanding (see Mokros and Russell [1995]). To understand what the mean represents and how it relates to the data, the concept of distance from the mean is essential.

Some questions that might be useful for teachers to ask before introducing the algorithm are the following: What do my students do if I give them an average and ask them to make up a data set that

Unpacking tasks develop ideas about balance and distribution reflects that average? Do they have some idea about an average as a kind of middle? Do they have a sense that an average is situated in the data such that high values are balanced by low values? Can they begin to think about the significance of the distances of the values from the average?

If students have not developed these ideas, which may not be understood until late in middle school, then teaching the algorithm is pointless. Students need more experiences describing and comparing sets of data. The algorithm is a useful shortcut but does not model the intricate statistical balancing act involved in finding the mean. Far more important is the relationship between the data and the average that summarizes the data. A focus on this relationship should be a top priority in statistical education.

Action Research Ideas

Action-research idea 1

Ask your students to solve the potato-chip problem as it is worded in the article.

1. See which students think of "typical or usual or average" as mode, which as median, and which as mean, by asking them for their numbers and their reason for that choice.
2. Determine which students are able to explain their choice. Do any students change their mind as a result of these explanations?
3. Some students may know an algorithm for finding the mean. Determine whether any of them can explain why their procedure is a good way to find a "typical or usual or average" value.
4. The authors indicate that many fourth graders consistently used the mode as the "typical or usual or average" value. Is this finding true in your class?

Action-research idea 2

Tell your students that the typical or usual or average family size for eight families is 4. Ask them what the data might look like.

1. If the students have trouble getting started, you might show them the distribution in **figure 2** and ask them if that would work. Then ask what other data sets might also work.
2. Ask, "If we knew that one family really had 3 people instead of 4, what could we do to make the average come out to 4?" The authors indicate that students usually suggest moving one data point, a stick-on note on the graph, from 4 to 5 so as to balance the 3. Is this suggestion made in your class? Determine whether the students want to move more data points to make the data look more realistic. After each point is moved, ask, "Is the average still 4?"
3. Introduce asymmetrical balancing by asking what would happen if one family had 8 people. The authors indicate that students always come up with the idea of moving two data points from 4 to 2. Is this suggestion made in your class? Can it be done another way?
4. Do the students have some idea about an average as a kind of middle? Which students?
5. Do the students have a sense that an average is situated in the data such that high values are balanced by low values? Which students?
6. Can the students begin to think about the significance of the distances of the values from the average? Which students?

Continue to pose this kind of "unpacking" task throughout the school year. Note how each student's understanding of the fundamental concepts of average develops over time.

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