Push-Penny: Are you a Random Player?

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Summary
Several measures of "success" for a simple game are discussed. Comparisons between players, and between a player and a simulation, can be made in several ways.

◆ INTRODUCTION ◆

MUCH RESEARCH has given attention to the subjective understanding of statistical concepts (see Shaughnessy and Bergman (1993) for a review of this literature). These findings show that students need to experience random variation in practical ways if they are to develop a proper intuition of probability.

This Push-Penny activity is designed for grades 7-8 (USA ages 12-13) and is intended to develop students' awareness of and intuitive feeling for the consequences of variation and the role of chance variation in the analysis of data. Informal inferences from experimental data are used to decide whether a proposed theoretical model is realistic.

◆ THE GAME OF PUSH-PENNY ◆

The game of "Push-Penny" (Holmes (1980)) is played with a coin and a game board which is a 36 inch × 24 inch poster board lined as shown in figure 1. The object of the game is to push the coin and have it land on a line. (Note - the objective in some versions of the game is to land on a blank space.) A blank poster board is used as a buffer between the player and the lined board. A coin is needed for each board. A U.S. "quarter" is about the right size [approximately 1 inch (2.54cm.) diameter].

If a player's push is "random" there is a 50% chance that the coin will fall on a line. Look at figure 1. If the

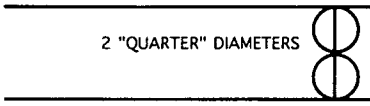

Fig 1.

centre of the coin is moved along a line perpendicular to the lines on the board, the coin will touch the line half of the time.

For this activity, a play of the game consists of four pushes. The score is the number of hits.

◆ WHO IS THE BEST PLAYER? ◆

Students want to know how they compare with the other students in the class and who the best players are.

The Question

How do I compare with the other players in the class?

Criteria For "Best" Player

We must first agree on criteria for comparing players. Several reasonable criteria are illustrated below. These are based on twenty plays of the game. Students should be invited to invent others.

Criterion 1
For each student, determine the total number of hits in twenty games. The student with the largest total is the "best".
Criterion 2
The student with the most scores which are "3" or "4" is "best".
Criterion 3
The student with the most scores which are "2" or "3" or "4" is "best".

Data Collection, Analysis, Interpretation

Each student plays the game twenty times. Part of a recording worksheet is shown in figure 2. Set up stations around the room with three students at each station - one student manages the board, another records results, while the other plays the game.

Game No.	Sequence of Hits or Misses					No. Hits
1.	H	H	H	H	-	4
2.	H	H	M	H	-	3
3.	M	M	H	H	-	2
4.	M	H	M	H	-	2
5.	H	M	H	M	-	2
etc.						

Fig 2. Data recording

Each student completes a frequency table for his/her individual data, records the value for each criterion and draws a bar chart to represent the frequencies. These are displayed together on the wall. Then each student compares his/her scores with the summary sheets of the other students. Figure 3 shows such a summary sheet for a class of 25 students.

For instance, student (a) in figure 3 has a Criterion 2 value of 6. Inspection of figure 3 reveals that this student did better than thirteen (52%) of the values produced by the entire class. To put it another way, the class produced twelve rounds out of twenty-five (48%) that did this well or better. This student's value is close to the middle of the class. Student (c) has a Criterion 2 value of 8 which is better than seventeen (68%) of the others. After these comparisons have been made for each student then the best (based on Criterion 2) players in the class can be determined. In a similar way comparisons can be made based on Criterion 1 or Criterion 3.

Further Analysis

This type of comparison of results is facilitated by summarising class results in a frequency table. Figure 4 shows the frequency table for the Criterion 2 values for the students in figure 3; similar tables can obviously be constructed for Criteria 1 and 3. However, this is a significant "data reduction" and students should first make the comparisons from the wall displays so they can appreciate this additional summarisation.

Value	Frequency
9	4
8	4
7	2
6	2
5	5
4	4
3	2
2	2
1	0

Fig 4. Frequency table for Criterion 2 values

◆ CAN YOU BEAT THE DICE? ◆

Do we develop skill at the game of Push-Penny? If we have "skill" then we should be able to do better than a "random" player. Random pushes have a 50% chance

	Student																								
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)	(n)	(o)	(p)	(q)	(r)	(s)	(t)	(u)	(v)	(w)	(x)	(y)
no. Hits	no. games																								
0	0	0	0	1	3	1	0	3	0	0	3	2	2	1	1	0	3	0	3	3	2	2	1	0	1
1	4	6	5	3	4	6	2	3	6	6	7	4	5	6	3	4	4	3	8	3	2	8	4	2	7
2	10	5	7	11	11	6	13	5	10	9	7	6	5	9	7	10	9	10	6	5	12	8	10	10	7
3	4	8	5	5	2	6	4	6	4	4	1	6	6	3	6	6	2	3	2	8	4	2	3	6	4
4	2	1	3	0	0	1	1	3	0	1	2	2	2	1	3	0	2	4	1	1	0	0	2	2	1
Criterion																									
1	44	44	46	40	32	40	44	43	38	40	32	42	41	37	47	42	36	48	30	41	38	30	41	48	37
2	6	9	8	5	2	7	5	9	4	5	3	8	8	4	9	6	4	7	3	9	4	2	5	8	5
3	16	14	15	16	13	13	18	14	14	14	10	14	13	13	16	16	13	17	9	14	16	10	15	18	12

Fig 3. Results of actual plays (25 students, 20 games each)

of falling on a line. If you have no skill, then you might as well toss a fair die to determine if you hit the line or not.

The Question

Am I better than a player who is making random pushes?

Data Collection, Analysis, Interpretation

The summaries of actual plays from part 1 of this activity are also used for part 2. In addition, each student plays a round of twenty simulated games and the results are recorded as before. A simulated play of the game consists of four tosses of a die, with an even outcome recorded as a "hit." Students work in pairs. A minimum of twenty-five rounds of simulated plays is recommended. If there are fewer than twenty-five students, have some or all of the class complete more than one round of twenty games to get a large enough number of summaries.

Each student's actual scores are compared with simulated scores. The summary of simulated plays is of the same type as the summary used for actual plays - a frequency table for twenty simulated scores, the values for each criterion and a bar graph. The summary sheets for simulated games, one for each student, are displayed together on the wall.

The simulated Criterion 2 values for twenty-five students are summarised in figure 5. This time we compare the results for each student with the summary of simulated results. For instance, student (a) in figure 3 has a value of 6 for Criterion 2. Figure 5 reveals that randomness did as well as this student or better in 20 rounds (80%). The highest value for the students in figure 3 is 9, achieved by four students. Randomness produced five rounds out of twenty-five (20%) that scored this well or better. Similar evaluations of Criterion 1 or Criterion 3 results can obviously be made.

Value	Frequency
10	3
9	2
8	6
7	7
6	2
5	3
4	1
3	0
2	0
1	1

Fig 5. Frequency table for Criterion 2 values for simulated plays

Inferences

Our analysis provides a peculiar sort of answer to the question: "Am I better than a player who is making random pushes?"

If I (like player (a)) obtain a Criterion 2 value of 6, then the evidence of the analysis would appear to suggest a rather emphatic "No." After all, given 25 opportunities, the dice did at least as well as I did 80% of the time!

If I (like player (b)) obtain a Criterion 2 value of 9, then the evidence of the analysis is not so emphatic. However, I seem to be getting the best of the dice most of the time. Given 25 opportunities, the dice did as well as I did or better only 20% of the time.

If I obtain a value of 10, then the evidence of the analysis would appear to give fairly strong support for a "Yes." After all, given 25 opportunities, the dice did as well as I did only 12% of the time.

◆ WHAT'S NEXT? ◆

Other binomial models can be introduced by changing the number of pushes in a game or the probability of a hit can be decreased by increasing the distance between lines. The geometric random variable can be introduced by playing a game defined by "push until you hit a line." It is most effective in this version of the game to use small probabilities for successful pushes. For instance, setting lines six coin diameters apart makes the probability of a hit 1/6.

This activity might naturally be followed in the next level of the curriculum by the use of the mathematical description of the binomial distribution to describe a random player and beyond that with more formal inference procedures for testing the hypothesis of the random player.

References

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