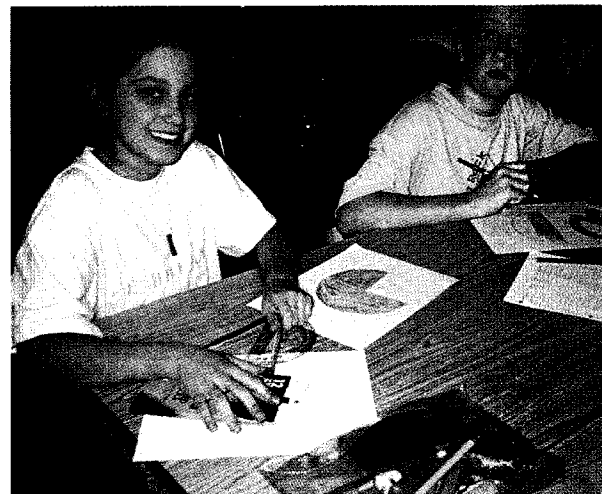




data points  $n$  is odd, or until the *two* middle values are reached, when  $n$  is even. When  $n$  is odd, the median is the middle value; when  $n$  is even, the median is the average of the two middle values. In either case, the median divides the data into halves. The lower quartile is the median of the lower half of the data, and the upper quartile is the median of the upper half.

For the large data sets, it was impractical to list all the data; however, the "Relative Cumulative Frequency" column in **table 1** provided an alternative mechanism for determining the median and the quartiles. The class considered two different cases to find the median and quartiles from the relative cumulative frequencies. First, they noted that 289 coins were minted in Philadelphia, so the median year for the P coins was the 145th value in the ordered data. This year had to be 1989, since the relative cumulative frequency first exceeded 50 percent for the year 1989. Similarly, the lower and upper quartiles for the P coins were 1985 and 1990, since the relative cumulative frequency first exceeded 25 percent in 1985 and 75 percent in 1990. Students then observed that 128 coins had been minted in Denver, so the median year was the average of the 64th and 65th values, or years, in the ordered data, which were both 1980. The lower quartile was the average of the 32d and 33d values. The students found that the 32d value was 1973 and the 33d value was 1974, so the lower quartile was 1973.5. The other medians and quartiles were found in similar ways. The five-number summaries for the four sets of data are shown in **table 2**.

Finally, each group constructed a box plot for its data. Since the box plots were to be compared, they were all created on the same scale. Each group reported its minimum and maximum year and used these values to determine an appropri-



PHOTOGRAPH BY G. KADER, ALL RIGHTS RESERVED

ate scale for the box plots. The four box plots are shown in **figure 6**.

### Interpretation

At this stage, some of the students had begun to lose sight of the overall investigation. To help them

SUMMARY STATISTIC	MINT LOCATION			
	DENVER	NONE	PHILADELPHIA	SAN FRANCISCO
Minimum	1968	1971	1980	1968
Lower quartile	1973.5	1974	1985	1968.5
Median	1980	1976	1989	1969
Upper quartile	1986	1977	1990	1969.5
Maximum	1995	1979	1995	1970

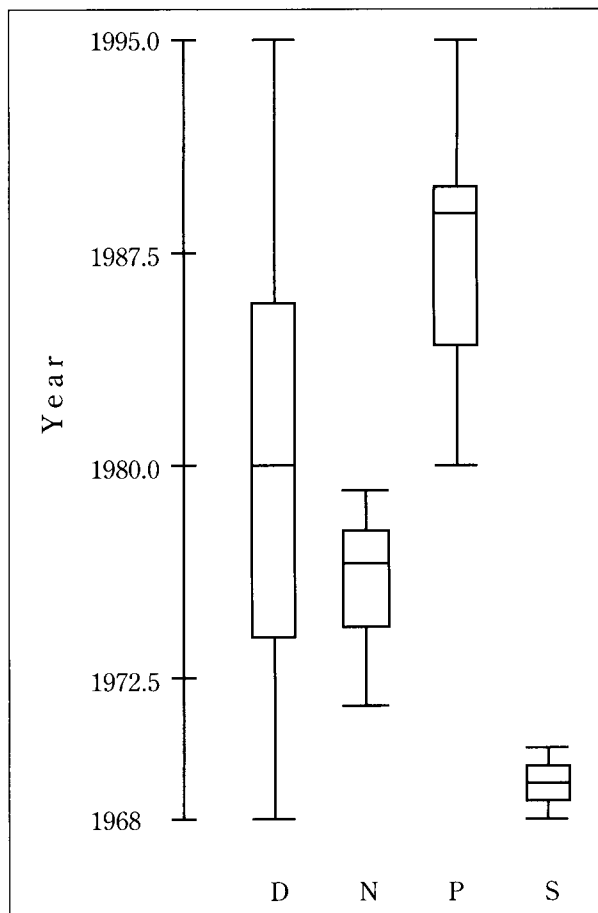


Fig. 6 Box plots of coin distribution by mint year and location



bridge the summary statistics and their original investigation, the teacher asked these questions, based on an examination of the five-number summaries and the box plots:

- Which mint locations have larger (higher) medians? What does this information show about the typical ages of the coins from these locations?
- Which mint locations have smaller (lower) medians? What does this answer show about the typical ages of the coins from these locations?

Comparing the medians revealed that the P coins had the largest median (1989) and, consequently, these coins were typically *newer*. The S and the N coins had smaller medians, 1969 and 1976, respectively, and, consequently, coins from these locations were typically *older*. The D median (1980) was in the middle when compared with the other medians. When asked to write their conclusions based on these comparisons, these responses were typical:

Phylidelphia [*sic*] has the larger median. This means that P coins were made more recently than the others. S coins have the smaller medians. This means that they are the oldest coins.

The class next compared the ranges of years for each mint location. Using the five-number summaries and the box plots, they answered the following questions:

- What are the ranges of years for each mint location? Which mint locations have narrower ranges? Wider ranges?
- Which mint location typically has relatively newer coins? Older coins? Both newer and older coins?
- Do the ranges of years from all four locations always overlap? If not, which locations do not overlap? What are the beginning and ending years for each of these locations? Given these ranges, where do you think the coins without a mint mark are minted? Explain.

The S coins had the narrowest range, 1968 to 1970, or two years, and were the oldest coins in the collection. After some discussion, many students thought that the narrow range might have resulted because the collection contained only four S coins. The N coins were minted from 1970 to 1979, nine years, and were all relatively old. The P coins were minted from 1980 to 1995, fifteen years, and were relatively new. The D coins had the widest range, 1968 to 1995, or twenty-seven years, and contained both newer and older coins.



PHOTOGRAPH BY G. KADER. ALL RIGHTS RESERVED

The students were becoming coin sleuths, finding clues to answer the question “Where were the N coins minted?” Studying the box plots, they discovered that the D and S groups both had coins minted in 1968, but neither the N nor the P group had any coins this old. Further, the last mint year for the S coins was 1970, and the first mint year for the N coins was 1971; the last mint year for the N coins was 1979, and the first mint year for the P coins was 1980.

From these clues, two candidates for the mint locations of the N coins seemed likely. One possible conclusion was that the coins with no mint mark were from San Francisco. If correct, then from 1968 to 1970 coins in San Francisco had a mint mark; however, from 1971 to 1979 they did not. Another possible conclusion was that the coins with no mint mark were from Philadelphia, meaning that from 1970 to 1979 coins minted in Philadelphia did not have a mint mark but that those minted after 1980 did have one. Most students correctly identified Philadelphia as the city where the N coins were minted. Anna summarized the results:

All the locations overlap Denver, but P, S, and N do not overlap each other. D goes from 1968 to 1995, S from 1968 to 1970, N from 1971 to 1979, and P from 1980 to 1995. Since N coins end right near the year P coins begin, I think the N coins are from Philadelphia.

Adam explained why the coins could not have been minted in either Denver or San Francisco:

It couldn't be Denver because why would Denver mint two sets at one time. Also why would San Fransico [*sic*] just start not putting a mint on thier [*sic*] coins.

A few students incorrectly selected Denver. Their reasons included “because it spans more time than the others” or “because Denver overlaps it.”



## Communication

In this study, the data suggested that coins tend to stay in the region of the country where they were minted. More than 75 percent of the coins in this collection, the N and P coins combined, were minted in Philadelphia. As is often the case, this investigation raised more questions than it answered. Some students wanted to know, "Why did coins minted in Philadelphia before 1980 not have a mint mark?" Others asked, "And why were Philadelphia coins marked after 1980?"

The teacher noted that the interpretation of the results was a typical statistical solution, and the conclusion was qualified and not definitive. However, a library search verified that before 1980, coins minted in Philadelphia were not marked with a P. A detailed historical description of the Jefferson nickel and other coins is given by Davis (1976, 72–76).

## Summary

THIS ARTICLE, WHICH DESCRIBES A STATISTICAL investigation to study the circulation of coins, modeled a five-component process. The data representations used in the analysis component are not particularly sophisticated and so are appropriate for

middle-grades students. It demonstrates that statistical concepts and problem-solving strategies can be developed without employing complicated techniques or formulas. This activity was developed by the Statistics—Materials and Activities for Problem Solving (STAT-MAPS) project. STAT-MAPS (1991–95) was funded by the National Science Foundation's Materials and Development Program (grant number MDR-9150117). A similar activity, "The Age of Cents," is presented in AIMS (1987). For information on the history of mint marks, see <http://www.ustreas.gov/treasury/bureaus/mint/sub4.html>.

## References

- AIMS Educational Foundations. *Math + Science—a Solution, Grades 5–9*. Fresno, Calif.: AIMS, 1987.
- Davis, Norman. *The Complete Book of United States Coin Collecting*. New York: Macmillan Publishing Co., 1976.
- Kader, Gary, and Mike Perry. "Learning Statistics with Technology." *Mathematics Teaching in the Middle School* 1 (September–October 1994): 130–36.
- National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: NCTM, 1989. ▲

Kader, Gary and Perry, Mike (November–December, 1997). Pennies from Heaven—Nickels from Where? *Mathematics Teaching in the Middle School*, 3 (3), 240–248. Reproduced with permission from *Mathematics Teaching in the Middle School*.  
Copyright © 1997 by the National Council of Teachers of Mathematics. All rights reserved.