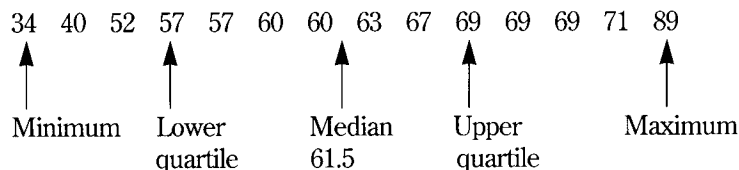
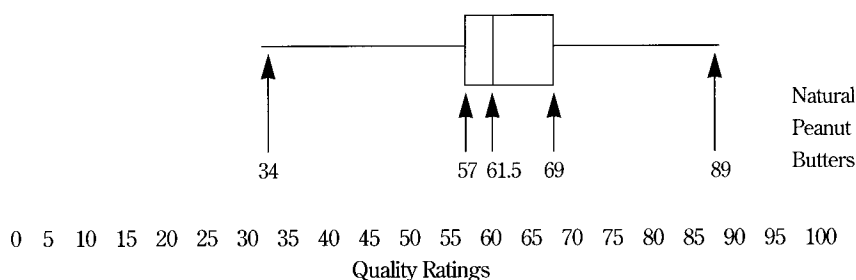


Fig. 5
Making box
plots

To make a box plot, students need to identify the five-number summary for each data set. This summary includes the minimum value, maximum value, median, lower quartile, and upper quartile of the data. The lower quartile is the median of the data values below the median. The upper quartile is the median of the data values above the median. So for natural peanut butters, the quality ratings can be listed in order from smallest to largest and the five values identified:



The five-number summary is used to make a box plot. The “box” extends from the lower quartile, includes the median, and ends with the upper quartile. The “whiskers” are the lines that extend from the two quartiles to the minimum and the maximum values.



Several box plots can be made using the same scale to compare different distributions of data.

the other one [regular peanut-butter quality ratings].” A statistician looking at the distributions also might note that the quality ratings for natural peanut butters are shifted to the right and would compare the amount of shift between each set of corresponding points on the box plots to see if the shift is about the same for each set; for example, he or she might compare the shift of the minimum value for natural peanut butters with the minimum for regular peanut butters. The students did not make such comparisons, although during the discussion the teacher could suggest that they note the amount of the shift. Other students in the class observed that “three-fourths of the natural peanut butters’ ratings are higher than three-fourths of the regular peanut butters’.” These comments seem to reflect an awareness of the structure of the box plot, that is, that 25 percent of the data are found in each of the four quartiles.

Box plots can be tools for reflecting on the importance of the differences among multiple data sets. If we assume that the quality ratings are reasonably reliable, we note that natural brands generally have higher quality ratings than regular peanut

butters. In looking at other possible comparisons (figs. 6b and 6c), we can see that the shifts in these data may not be as obvious as the shifts found when we compare natural and regular peanut butters. Students have opportunities to consider whether such shifts do suggest differences that are worth noting.

Outliers

ALTHOUGH NOT CONSIDERED IN THE PREVIOUS discussion, one other attribute of box plots may be useful. It is possible to determine outliers, that is, values that are considered unusual in relation to the other values in the data set. To decide whether any outliers are present, we determine the length of the box plot of the data—the difference between the lower and upper quartiles; this difference is called the interquartile range (IQR). If a data value is greater than 1.5 times the IQR added to the upper quartile or less than 1.5 times the IQR subtracted from the lower quartile, it is an outlier. The whiskers are drawn to the greatest and least data values that are not outliers, and each outlier is

marked with an asterisk. For example, for the natural brands, the IQR is 12. Subtracting 1.5 times the

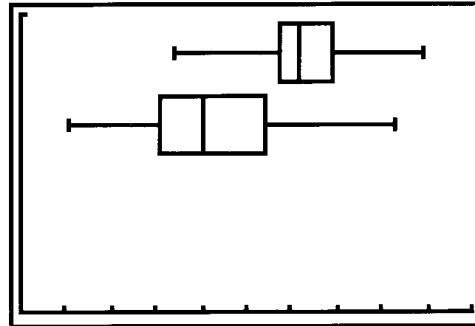
IQR, or 18, from the lower quartile, 57, yields 39, which means that any quality rating less than 39 is

The scale on the graphing screen is in units of 10, from 0 to 100.

Fig. 6 Box plots of quality ratings

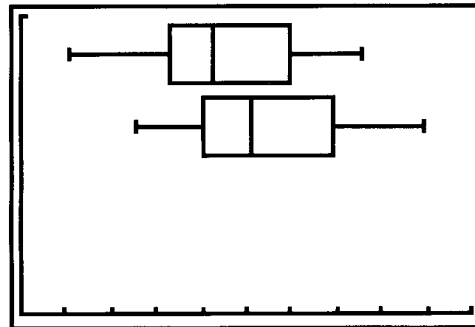
a. Natural or regular peanut butters

The lower quartile of the natural peanut butters (top box plot) is greater than the upper quartile of the regular peanut butters (bottom box plot). More than 75 percent of the natural peanut butters have higher quality ratings than 75 percent of the regular peanut butters.



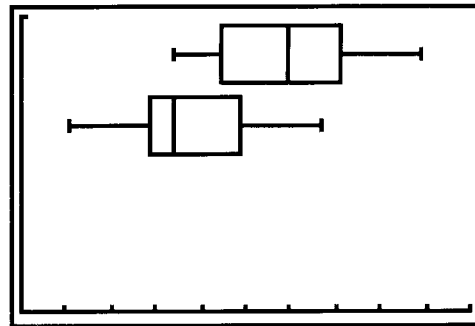
b. Chunky or creamy peanut butters

Less variation occurs between these two samples. The median rating for chunky (bottom box plot) is 54, and the median rating for creamy (top box plot) is 43, which is a shift of eleven points. The box plot for quality ratings of chunky peanut butters is shifted to the right.



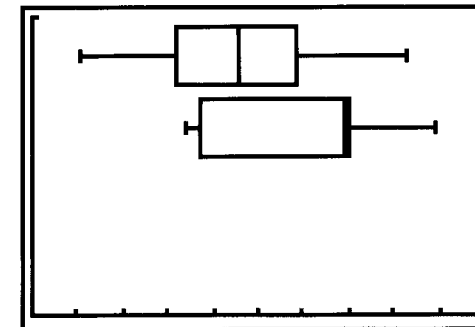
c. Name- or store-brand peanut butters

Close to 100 percent of name brand peanut butters (top box plot) have quality ratings that are greater than the median quality rating of the store brands (bottom box plot).



d. Salted or unsalted peanut butters

Twenty-eight salted and nine unsalted peanut butters were evaluated. The bottom box plot (unsalted) may appear confusing; the median (69) and the upper quartile (70) are almost identical. It is important that students discuss the size of the sample of unsalted peanut butters and the implications for using these data.



considered an outlier. The whisker is drawn from the lower quartile to the lowest value in the data set that is greater than or equal to the outlier boundary of 39, that is, a rating of 40. A similar process is used for the upper quartile. The quality ratings of 34 and 89 are outliers; the regular peanut butters have no outliers. The box plots may be drawn to show the outliers (**fig. 7**). By noting outliers, we can see that the overall variation in quality ratings appears to be much less for natural peanut butters than for regular peanut butters.

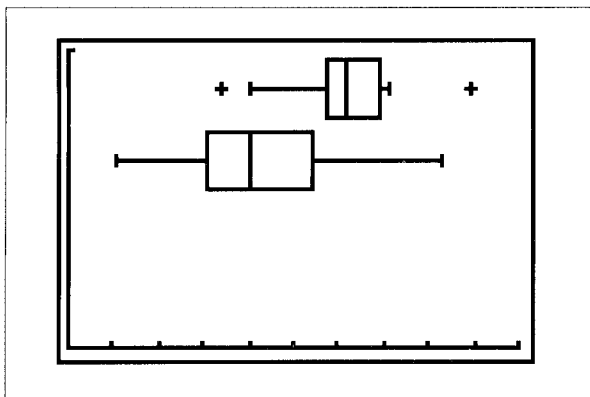


Fig. 7 Box plot drawn to show outliers

Extending the Analysis

ONCE STUDENTS WERE COMFORTABLE MAKING and reading box plots, they continued their comparisons of quality ratings by using the remaining attributes of creamy or chunky, salted or unsalted, and name or store brands. These plots are shown in **figures 6b to 6d**. In examining these data, we can determine that the higher quality ratings appear to belong to natural, chunky, and name-brand peanut butters. The sample size for the unsalted category may be too small to warrant consideration here.

Choosing a brand by applying the characteristics associated with higher quality ratings essentially gives equal weight to the relationship between each attribute and the quality rating, however, and may not account for individual variation in the products. For example, Health Valley 100% Natural is natural, chunky, and a name brand, yet it has a quality rating of 40. This kind of situation does permit students to consider the variation found in the quality ratings. It is important to remember that quality ratings are subjective ratings. With a well-trained *Consumer Reports* taster, we would expect that when asked to rate the same peanut butter at two different times, he or she would both times give similar ratings; these ratings would be similar to those of other tasters. Variations in the reliability of ratings could account for some of the differences reported in these data.

Students can also be encouraged to explore the relationships between the different attributes and price. Students did find that the higher quality ratings are associated with higher prices.

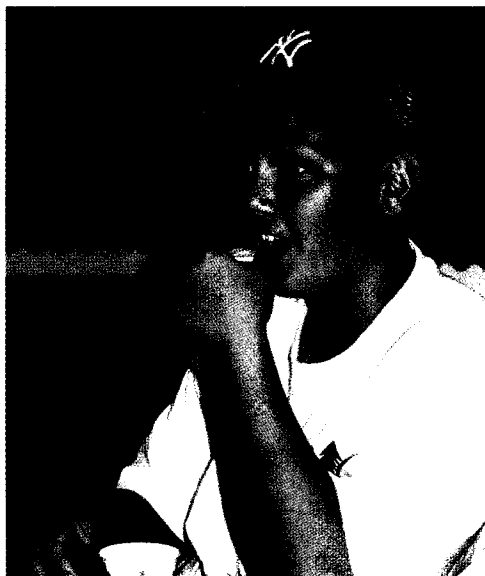
What constitutes a “best buy” may involve more than quality ratings or price for many consumers. As the students noted, sometimes a best buy depends on how much money you have. Often people choose peanut butter for reasons that have nothing to do with these ratings or price. Perhaps a particular brand is “the brand that has always been used in the family.” A low-fat brand may be the choice, and these data do not supply information on this attribute. Or some people choose peanut butters recommended by their friends or family. These data provide some, but not all, the information that consumers may need or want.

The results of the *Consumer Reports* study give students an opportunity to explore what it means to compare two or more data sets by using a kind of statistical reasoning that is more sophisticated than the reasoning involved in describing a distribution. When Gal and Wagner (1992) asked students to compare two data sets that did not have the same numbers of data items and were displayed in such representations as side-by-side bar graphs or his-

tograms, students often argued that they could not make those comparisons because the data sets had different numbers of values. In working with box plots, unlike with stem plots, students are able to ignore sizes of data sets. All box plots involve the five-number summaries. Students make comparisons on the basis of these values. It may be valuable to have students regularly note the sizes of the data sets that they are comparing and to discuss the ways in which they can use statistics to compare data sets of different sizes. Such discussions may focus students' attention on sample size as yet another component to consider in reasoning about data.

Engaging students in activities that involve comparing different sets of data lays the groundwork for developing more advanced statistical reasoning skills. Using real data sets within meaningful contexts presents opportunities for students to explore these con-

cepts through various representations and statistical measures. Further, they can consider this context and the results from their data analysis in light of consumer decision making.



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