ANALYSIS

In order to interpret and create patterns in today's image-packed world, it is not enough just to recognize similarities and differences; we also need to analyze them. This leads us to investigate the way that large shapes are built of smaller ones and to recognize patterns and their properties.

When children make shapes out of blocks or Legos, they often imitate the diverse compositions that they see around them (Figure 9). Nature too creates patterns. Like man-made patterns, natural patterns appear at many levels: atoms are organized into molecules, while molecules are organized into crystals and cells, which in turn are often the subunits of still higher-level organization.

When we examine patterns carefully, we find that the same forms and arrangements appear over and over again, even when the objects

FIGURE 9. Many shapes are built from smaller ones. The reinforcing beams in a bridge illustrate how repeated patterns are used in engineering and architecture, as in nature, to form a whole out of parts.

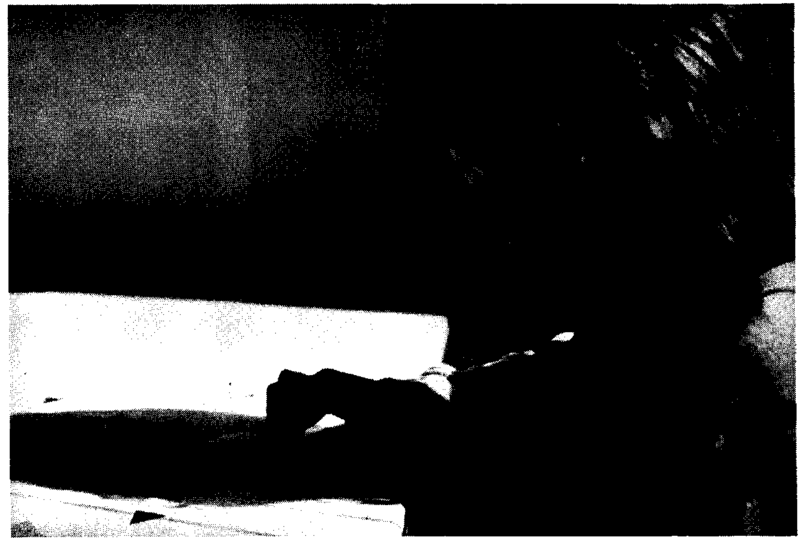

FIGURE 10. Young children can investigate the ways in which polygons can be fitted together to tile a plane surface.

involved are very different.[16] This is not just a coincidence. The geometry of most patterns is governed by a very few basic principles of formation, growth, and development. For example, in his fascinating book *Patterns in Nature*,[20] Peter Stevens discusses several ways in which natural patterns are generated, such as stress, branching, meandering, partitioning, close packing, and cracking. The results of these modes of formation are remarkably similar, despite the variety of materials on which they operate (see Figure 8).

Important aspects of pattern formation can be grasped by exploring the ways in which copies of objects can be packed together. Students quickly discover that there are only a very few ways to do this. This fundamental property of shape can be studied at many levels. For example, it can be studied intuitively and "hands-on" when the objects being packed are circles or easy-to-construct polygons such as triangles, quadrilaterals, and hexagons (Figure 10). Older children can experiment with less regular forms and discover some surprising things, such as the fact that any quadrilateral, even one that is not convex, will tile the plane (Figure 11). (This is a surprising but very simple consequence of the fact that the sum of the measures of the angles of a quadrilateral is 360°.) High school students can study deeper properties of sphere packing and tilings, such as their symmetry and how they can be generated. (Grünbaum and Shephard's *Tilings and Patterns*[10] is the definitive resource for material on tilings.)

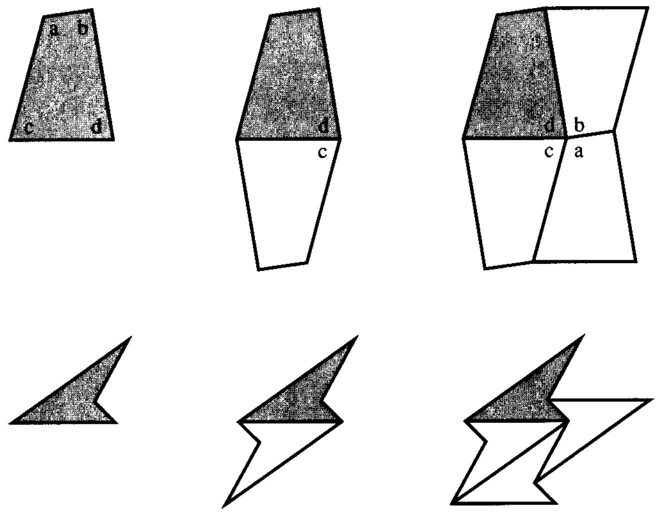

FIGURE 11. Any quadrilateral will tile the plane, because the sum of the measures of its angles is 360°, which is the same as the total number of degrees around each vertex. So four copies of a quadrilateral arranged around a point with each angle used once will fit just perfectly.

Discovering Symmetry

One of the most striking things about patterns of many kinds is their symmetry, and this symmetry is an important tool in their analysis. A pattern is something that repeats in some sense; symmetry is the concept that makes that sense precise.

The study of symmetry begins by decomposing figures into congruent parts. Although some shapes do not at first appear to be made of smaller parts, it is often helpful to think of them as if they were. For example, mirror lines divide a square into eight congruent sectors, which the symmetries of the square permute. This decomposition helps us study the way symmetries work. In particular, it reveals that symmetry is self-congruence. It is this self-congruence that we consider beautiful and that makes symmetry a meaningful organizing principle in the analysis of structure.

Young children learn quite easily to recognize symmetry, not only in squares and butterflies, but also in animals, flowers, household utensils, toys, buildings, and arrays of every kind. Symmetry can be found almost everywhere. Older children can get great pleasure, and gain great insight, by creating symmetrical patterns and discovering the rules that govern them.

One of the most interesting but underappreciated techniques for exploring patterns is paper folding. We are all familiar with the pretty

patterns that result when folded paper is cut and then unfolded. The snowflakes, chains of dolls, and other repeating patterns that appear are not created by magic but are simple consequences of the geometry of reflection. Many geometrical constructions, and even aspects of number theory (some of them decidedly nontrivial), can be represented by unfolded designs. Conversely, many interesting three-dimensional shapes can be created by folding paper: the polyhedral nets of Figure 2 are one example; origami puzzles are another. Paper-folding problems stimulate the geometrical imagination in many ways.

Mirror Geometry

Mirrors can be used to study the principles of reflection. In particular, building a kaleidoscope is an excellent way to discover how reflections interact to generate the orderly arrangements that we call kaleidoscopic patterns. The kaleidoscope is much more than a toy: it is a lesson in mirror geometry. Even one mirror has much to teach us: adults as well as children are challenged by the "mirror cards" used in elementary school classes. The kaleidoscope is more complex, but it too is based on the principles of reflection in a mirror.

To explore the operation of a simple kaleidoscope, you just need two rectangular pocket mirrors and some tiny colored objects—bits of plastic or glass will do very well. Tape the two mirrors together along one edge, with their reflecting surfaces facing each other. Place the objects on a table, between the standing mirrors (Figure 12). If you look in the mirrors you will see the objects repeated in a delightful pattern. A little experimentation will show that some angles produce lovelier configurations than others. Only certain angles produce, in the words of the kaleidoscope's inventor, Sir David Brewster, "a perfect whole"—a finite number of identical regions arranged in a circular pattern. By playing with the mirrors, it is not difficult for children to discover which angles produce this perfect kaleidoscopic image. By doing so they will have learned an important lesson in the modern study of shape.

Reflections generate patterns with a finite number of subunits, patterns that have rotational as well as mirror symmetry. The rotations and reflections can be performed one after the other, always leaving the "perfect whole" apparently unchanged. Formally, such a system of motions is known as a *symmetry group.* Many properties of shapes can be analyzed by studying their symmetry groups; indeed, for more than a century this strategy has been a guiding principle in the study of geometry. By using a kaleidoscope, students can understand this fundamental idea by direct experience without making a lengthy detour through the formal and abstract algebraic language in which it is usually expressed.