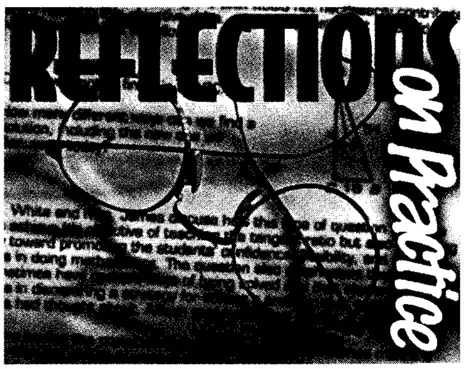

Understanding Students' Underastanding of Graphs

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STATISTICS HAS EMERGED AS A MAJOR COMPOnent of the school mathematics curriculum during the 1990s (NCTM 1989). We know that understanding the statistical-investigation process (Graham 1987) is central to working with statistics. A statistical investigation typically involves four components: (1) posing the question; (2) collecting data; (3) analyzing data; and (4) interpreting the results, in some order (Graham 1987). Kader and Perry (1994) suggest a fifth stage of a statistical investigation: communicating results.

Although a central goal is to understand how students use the process of statistical investigation within the broader context of problem solving, it also is important to look at students' understanding as related to concepts linked to this process. It has led us to consider what it means to understand and use graphs as a key part of what is involved in knowing and being able to do statistics. Typically, students are asked only to read information from graphs. However, we may need to rethink not only the nature of graphs but also questions about using and reading graphs to help students better understand the use of graphs and read graphs better.

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"Reflections on Practice" is a look back at the end result of putting theory into practice. For those interested in submitting manuscripts that concern this theme, please send them to editor Susan Friel at the School of Education, Peabody Hall, Campus Box 3500, University of North Carolina, Chapel Hill, NC 27599-3500.

Graph Sense

EXACTLY WHAT CONSTITUTES A GRAPH HAS BEEN ADdressed by a number of people (e.g., Bertin [1980]; Doblin [1980]; Fry [1983]; Twyman [1980]). Definitions have included not only graphs but also maps, diagrams, tables, and other visual representations. More recently (Wainer 1992), attention has focused on statistical graphs that are used to convey information in various fields. In the middle grades, these statistical graphs usually involve plots of univariate data, that is, line plots; bar graphs; pie or circle graphs; stem-and-leaf plots, or stem plots; histograms; and box-and-whisker plots, or box plots.

Reflection:
Do you notice any problems in the way that your students read graphs?

One way to think about what we mean by graph knowledge is to use the broader heading of *graph sense.* Similar to what has been done with regard to number sense (Sowder 1992), it is possible to characterize what may be meant by graph sense (Friel, Curcio, and Bright 1997):

> Graph sense develops gradually as a result of designing graphical displays of data, exploring their use in a variety of contexts, and relating them in ways that are not limited solely to a focus on graph construction or on simple data extraction as the purpose for reading graphs.

Reflection:
What do you look for as evidence that your students understand how to read and use information presented in graphs?

We also can describe a number of different behaviors (Friel, Curcio, and Bright 1997) that may demonstrate some presence of graph sense. These behaviors include the ability to speak the language of graphs when reasoning about information displayed in graphical form and to respond to different levels of questions associated with interpreting information displayed in graphs.

Speaking the language of graphs

A task that we have used to explore students' reasoning about graphs is shown in **figure 1.** In a line plot, the Xs indicate frequencies of occurrence; four Xs above the number thirty-five mean that four boxes contain thirty-five raisins each. If the same number of raisins are in each box, all the Xs would appear above one number. In addition, the numbers listed along the horizontal axis indicate the numbers of raisins in each box; the tallies of Xs above the numbers indicate the number of boxes having that number of raisins. Several numbers listed along the axis, many with one or more Xs indicated, mean that boxes with different numbers of raisins are included in the data set.

The first question in the task asks, "Are there the same number of raisins in each box? How can you tell?" Students' written responses to these questions show a number of different types of responses. Some students reasoned by using the language of line plots, which includes a discussion of such properties of the graph as the range of the data or the frequency of occurrence of different data values.

- "No, because the Xs are not all on one number."
- "No. If there were the same number in each box, there would be Xs all above the same number."
- "No, because the Xs show how many boxes had that many of raisins. Like twenty-eight [raisins] had five [boxes] and twenty-nine [raisins] had four [boxes]."
- "No, there are not. All you have to do is look at the numbers on the bottom, and it tells you how many raisins were in each box."

Some students reasoned by talking about the properties related to the context or to the data themselves. They did not consider the graph of the data in their response.

- "No, because they weigh the boxes until they equal one-half ounce. They don't count the raisins."
- "No, because some raisins can be smaller and that means you can have more."

Reflection:
*What do your students' written responses to the questions in **figure 1** reveal about their graph sense?*

Still other students demonstrated confusion in reading the graph. Their reasoning focused on the count of Xs or the "height" of the "X towers." It is as though each X refers to some number of raisins rather than to the values listed on the horizontal axis. For these students, the notion that "each box has the same number of raisins" means that the number of Xs above each number must be the same.

- "No, the Xs have different numbers, so there are different numbers of Xs in each box."
- "No. Because there isn't the same number of Xs above each number."
- "No, because there is more Xs in some and less in others."

Students brought several different foods to school for snacks. One snack that lots of them like is raisins. They decided they wanted to find out just how many raisins are in ounce boxes of raisins. They wondered if there was the same numbers of raisins in every box. The next day for snacks they each brought a small box of raisins. They opened their boxes and counted the number of raisins in each of their boxes.

Students are presented with a line plot showing the information the class found:

Number of Raisins in a Box

		X												
		X	X					X	X					
		X	X	X				X	X					
	X	X	X	X	X			X	X			X		
	X	X	X	X	X	X		X	X	X		X		X
26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

The following questions for assessing components of graph sense have been used with students in grades 5 through 8 to check their understanding of line plots and the linking of this representation to bar graphs.

1. Are there the same number of raisins in each box? How can you tell?
2. How many boxes of raisins had more than 34 raisins in them? How can you tell?
3. If the students opened one more box of raisins, how many raisins might they expect to find? Why do you think this?

Fig. 1 Raisins in a box (Friel and Bright 1995b)