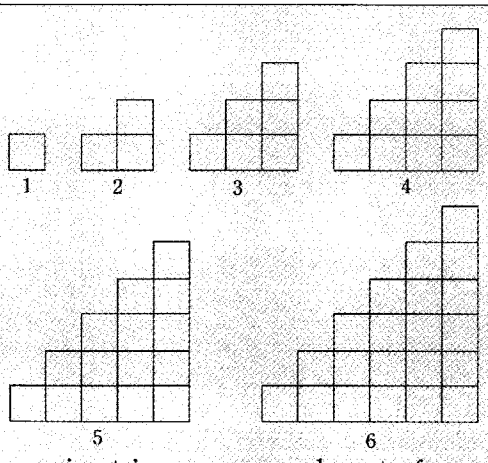
account of our experiences in solving the problem together. These accounts focused on both the mathematical content of the problem and the thought processes through which each teacher proceeded. In discussing and reflecting on these writings, we identified several aspects of our collective work that were particularly significant. The following descriptions of each of these aspects draw on contributions by several members of the group.

Teachers discovered their own learning styles. Learning in groups can be difficult. As one teacher eloquently put it, "If you embrace the notion of individuals constructing their own understanding in pursuit of 'knowledge,' then establishing a learning situation in which individuals must attend to the various paths taken by other individuals in the group is problematic." Asking a learner in the midst of thinking through a problem to stop and attend to someone else's approach risks derailing the learner's own thought process. The teacher who described this dilemma came to the realization that he did want to hear the solutions of others, but only when he was ready and had already had a chance to attack the problem on his own. He wanted to be given the opportunity to develop his own thoughts before being forced to take in others'. This example contains obvious lessons for us as teachers. For example, the next time this teacher uses groups, he will likely build in time for students to work by themselves before they share their ideas in their groups.



These six staircases are made out of square blocks. Devise as many general methods as you can for predicting the number of blocks in any staircase. If you come up with a formula for predicting the number of blocks, explain *why* the formula works.

- Suppose that the 273d staircase has 37 401 blocks. How many blocks are in the 275th?
- How many blocks are in the 100th staircase?
- Which methods of solution were most useful in answering these two questions

Fig. 1 The staircase problem

Teachers learned to value and connect with teachers of other grade levels. Initially, teachers worked on the problem in grade-level groups. The high school teachers arrived at several solutions, ranging from iterative and analytical methods to simultaneous equations and recursion. After the elementary, middle, and high school teachers had an opportunity to explore several methods, they formed new groups across grades. One high school teacher came to her new group thinking that she had discovered all possible approaches. However, one of the elementary teachers in her group offered an approach that was elegant but did not rely on algebra. She simply formed a rectangle from a staircase built with cubes by replicating the initial staircase, flipping it over, and fitting the two staircases together. The area was easily computed. The high school teacher was struck by both the simplicity and the beauty of her colleague's approach: "I never would have thought of doing it that way—that is so nice!"

As a teacher in another group who had a similar experience remarked, "It was an eye-opening experience for me to see two very different approaches to the same problem. From the high school teachers we could see how they derived the formula using the number patterns, but from the elementary teachers, we could easily see why the formula worked." Again, the lessons are clear. Many critics complain that groups saddle the high-performing students with the burden of explaining everything to the students who are not doing as well. In direct contrast, we have learned that in well-run groups, every member has the opportunity to learn from every other member.

Teachers gained appreciation for other learning styles. Another high school teacher commented on the challenge she felt when presented with the idea of going from a visual to an algebraic solution—an approach with which she had not previously felt comfortable. She wrote, "It made me wonder if, left to my own devices, I would have discovered [the problem's] richness or passed it over. It underscored the powerfulness of providing concrete models for learning along with the presentation of the abstract. This only leads to a richer understanding for all students."

Teachers' ideas of what constitutes classroom success changed. One common thread among the teach-

ers' comments was that we began to rethink what it meant to work successfully on a problem. For example, the members of one group reported that they felt tremendously successful and yet failed to come up with a solution. Their success was derived from appreciating "the variety of approaches suggested to the problem," not from reaching the "correct" answer. We do not mean to suggest that reaching a correct conclusion is unimportant but that much can be learned by trying different paths and by understanding the paths that others suggest.

Teachers reflected on what makes a problem rich. One teacher had given her students the staircase problem earlier in the year as part of a larger assignment. When a student had asked a question about it, she simply looked at the problem, thought of a way to solve it, and explained her method. A solution was given and the discussion was terminated. Of course, all future potential learning from this problem was short-circuited. Later, when this teacher worked on the problem with her fellow teachers, she thought back to her student's question and how she could have responded differently. The staircase problem is not unique. Most nontrivial mathematics problems allow different approaches and are adaptable to different levels.

Because we were participants, a particularly powerful aspect of our experience was that we could identify with our students and gain insight into their feelings. As a result, we all spent significant time reflecting on our own teaching. If our insights about different learning styles, working cooperatively, and the value of multiple solutions were valid for us, would they not be valid for our students? We were able to modify our classroom practices in accordance with our newly gained knowledge.

Our participation in these workshops has afforded us the opportunity to wrestle with these essential questions as a group, with teachers of different levels. These factors are the keys. Formerly, teachers were expected to explore mathematics by ourselves—at our desks in our studies or classrooms—the weekend before the material was to be taught. Instead, in these workshops we have been able to work on problems and topics in mathematics in groups of two through forty teachers. We have had a few uninterrupted hours with our fellow group members—time enough to explore issues in depth. The results, we have found, have been quite dramatic. Given the time and the space to work with other teachers, we have begun to develop new and firmer answers to our questions. We have begun to approach mathematics in the way that we desire our students to approach the subject. Two components are essential to the success of this endeavor: (1) that we model for our students the learning process and (2) that mathematics teachers do mathematics together.

The Challenge

IN DISCUSSING THE STAIRCASE PROBLEM IN THIS article, we have intentionally avoided specifics. Our reasoning is that we would like to challenge readers to try something similar.

- Gather a group of teachers; the number is up to you.
- Try to involve teachers of various levels. This aspect is important, but it is not essential.
- Try to solve the staircase problem. Work initially in small groups. Then come together in a large group.
- Write to us. Tell us about your experience. Describe the approaches to solutions that you came up with that differ from the ones that appear at the end of this article. Describe what it was like to work together in groups. Please mail your descriptions to the address given in the appendix.
- Good luck. Have fun. Thanks.

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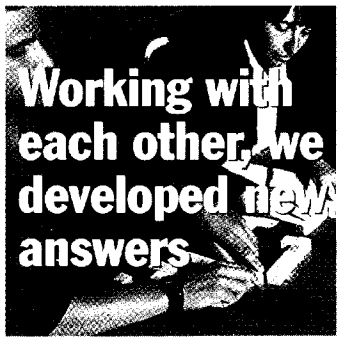

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Readers interested in the professional-development topic are urged to see the "Call for Manuscripts on Professional Development" that is on page 180 of this issue.—Ed.