Underwood Dudley
(dudley@depauw.edu)

Underwood Dudley taught his first calculus course in 1957 and is amazed that after all the years he has been teaching them, students are still making the same mistakes. He hopes to see the new millennium in at DePauw University, here he has taught for 19% of the institution's existence. His last book, *Numerology*, was published by the MAA in 1997. Woody is the editor-elect of the *CMJ* (the *College Mathematics Journal*).

Is Mathematics Necessary

Underwood Dudley
DePauw University

Is mathematics necessary? Necessary, that is, for citizens of the United States to function in the world of work? You would get that impression from reading various recent documents, some coming from high and official places. For example, *Moving Beyond Myths*, published by the National Academy of Sciences, says so [**5**, p. 11]:

> ***Myth:*** Most jobs require little mathematics.
> ***Reality:*** The truth is just the opposite: more and more jobs—especially those involving the use of computers—require the ability to use quantitative skills. Although a working knowledge of arithmetic may have sufficed for jobs of the past, it is clearly not enough for today, for the next decade, or the next century.

The anonymous author of that item presumably had mathematical training and thus should know that theorems are not proved by assertion. But if we look in the document for evidence for that supposed reality, we look in vain, so an assertion is all that it is.

Here is an excerpt from *Everybody Counts* [**6**, p. 4], written anonymously for the National Research Council. (Do documents issuing from important national organizations gain more weight when their authors are not identified? Some members of important national organizations evidently think so.) This report too says that mathematics is a vocational necessity:

> Just because students do not use algebra anywhere except in algebra class does not mean that they will not need mathematics in the future. Over 75 percent of all jobs require proficiency in simple algebra and geometry, either as a prerequisite to a training program or as part of a licensure examination.

A quick reading of that passage might leave the impression that algebra and geometry are used in 75% of *all* jobs, but that is silly. Just look at the next eight workers that you see and ask yourself if at least six of them require proficiency in algebra to do their jobs. (If you are a teacher of mathematics, it is not fair to look at eight colleagues.)

The anonymous author was careful to

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add the qualification that the algebra and geometry may be necessary only for training or licensing. But I find even this claim, another bald assertion with nothing to back it up, unbelievable. Over 75% of all jobs? Incredible! I cannot imagine how that wildly inflated percentage was arrived at, unless the author was including having a high school diploma under "licensure examination." Authors who want to indulge in unsubstantiated percentages should be careful to have them consonant with common sense. For example, "99% of the mathematics done by the average person relates to money" [**2**]—now *that* I can believe.

Almost all jobs, I counter-assert, require no knowledge of algebra and geometry at all. You need none to be President of the United States, none to be a clerk at Wal-Mart, none to be a professor of philosophy, ... the list extends indefinitely. Few jobs require knowing any mathematics beyond algebra. You might think that engineers, of all people, would need and use calculus, but this seems not to be so [**7**]:

> Why do 50% (probably closer to 70%) of engineers and science practitioners seldom, if ever, use mathematics above the elementary algebra/trigonometry level in their daily practice?
>
> My work has brought me into contact with thousands of engineers, but at this moment I cannot recall, on average, more than three out of ten who were well versed enough in calculus and ordinary differential equations to use either in their daily work.

If 70% of *engineers* don't need calculus to do their jobs, then how many of the 500,000 or so students that we put through calculus every year will? Minutely few, so we should not tell them how tremendously *useful* calculus is going to be to them when they go to work. If most engineers can do quite well with only algebra and trigonometry (or perhaps even less), is it not reasonable that nonengineers can survive and flourish with arithmetic, or even less? Yes, it is.

Were algebra necessary for 75 percent of all jobs, our algebra textbooks would be filled with on-the-job problems, since examples would be so plentiful. But they are not. Open any textbook at random—I will open a new one, just published—and what you find are problems like this:

> Through experience and analysis, the manager of a storage facility has determined that the function $s(t) = -3t^2 + 12t + 10$ models the approximate amount of product left in the inventory after t days from the last resupply. We want to find when the supply of the product will be exhausted and a new resupply needed.

Real inventories do not behave this way. For one thing, they do not *increase* after the resupply, from 10 at $t = 0$ to 22 at $t = 2$. For another, they usually decrease linearly, not quadratically. Besides, I doubt that warehouse managers, even 75% of them, use formulas to decide when to reorder.

Making fun of the "applications" that appear in textbooks is as easy as swatting mosquitoes in a swamp in midsummer, and as useful. What such problems actually illustrate is that the mathematics in the textbooks has no application to the world of warehouses and work. Does this mean that we should teach less mathematics? No; we should teach more. *Everyone* should learn algebra, but not because it is necessary for managing warehouses. Does it mean that we should stop assigning "applied" problems? Certainly not; we should assign more. Problems expressed in words are the best kind, but they should all start with "Suppose that...." If we can't be realistic, we can at least be honest.

Those who know not history ...

Let us look at history. Those ignorant of history too often assume, knowing no better, that the world has always been much as it is now, which is seldom so. Today, with near-universal instruction in arithmetic and algebra, it is easy to suppose the curriculum has always been like that. But it has not. Algebra was not always taught to everyone. Not only that, even arithmetic itself is a relative newcomer.

Here is a report from Massachusetts in the early 1800s. Not the 1700s, nor the 1600s, the 1800s [**3**, p. 13]:

> Until within a few years no studies have been permitted in the day school but spelling, reading and writing. Arithmetic was taught by a few instructors one or two evenings a week. But in spite of the most determined opposition, arithmetic is now being permitted in the day school.

Opposition to arithmetic! How could anyone possibly be opposed to arithmetic? It is difficult for us to imagine.

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The explanation is that arithmetic was a vulgar subject. As Patricia Cline Cohen tells us in *A Calculating People: The Spread of Numeracy in Early America*, a book that deserves to be more widely known [**1**, p. 26]:

> Those of high social rank, theoretically above the world of getting and spending, did not deign to study the subject. The most respectable English public schools, like Eton and Harrow, did not offer any instruction in arithmetic until well into the nineteenth century.

The English attitude was exported to the colonies [**1**, p. 49]:

> The founding generation arrived in Massachusetts in the 1630s with the highest number of university degrees and the highest rate of literacy of any migratory group. Within a decade they instructed towns to establish local grammar schools and had set up Harvard College to provide high-level training for homegrown ministers. But arithmetic was not among the subjects considered basic for Puritan children to learn.

Nevertheless, the colonies, and England, not only survived but thrived, economically as well as culturally. Some people believe that the eighteenth century represented a peak of civilization from which we have declined. I would not go that far, and I much prefer living in our times, with its plumbing and penicillin, computers and compact disks, anesthesia and even its automobiles, yet history clearly shows that arithmetic in the schools is not needed for a high civilization. How can that be? Easily enough: workers learn what they need *on the job*. What happens in the schools simply does not matter.

Here is a report on the situation in Boston in 1789 [**1**, p. 131]. See if it does not sound familiar today:

> [There was a requirement] that boys aged eleven to fourteen were to learn a standardized course of arithmetic through fractions. Prior to this act, arithmetic had not been required in the Boston schools at all. Within a few years a group of Boston businessmen protested to the School Committee that the pupils taught by the method of arithmetic instruction then in use were totally unprepared for business. Unfortunately, the educators in this case insisted that they were doing an adequate job and refused to make changes in their programs.

Of course the students were unprepared for business, one reason being that it is neither wise nor practicable to try to prepare all students for all possible jobs. Another is that the "applications" in school books were just as phony as ours [**1**, p. 122]:

> Here is a typical word problem, typical in its complexity and in its use of current events to suggest the utility of arithmetic:
>
> > Suppose General Washington had 800 men and was supplied with provision for but two months. How many of his men must leave him, that his provision may serve the remaining five months?
>
> In this particular case the student mechanically applied the Rule of Three, writing $2 : 800 :: 5$ and then dividing 5 into 2×800 to get a final answer of 320. Now, 320 is the number of men who can be fed for five months, not the number who must leave. So Washington's troops would have gone hungry if the schoolboy or his master had been in charge of provisioning.

As Professor Cohen pointed out, if Washington ran short of provisions, he would try to get more instead of telling part of his army to go away.

The conclusion cannot be avoided that school mathematics is not now, and never has been, necessary for jobs. There are a few exceptions, of course, most being for the jobs of teaching the subject. And of course science—both physical and social—cannot advance without a supply of scientists able to use mathematics. But most of these people did not need to be bullied or cajoled into learning the subject.

Even more advanced mathematics turns out to be all too often not needed for work [**8**]:

> Presumably, with degrees in mathematics and statistics [students with mathematical majors] could pursue careers in their disciplines. But, for mathematicians and statisticians who would seek employment in commerce, i.e. in business, industry, or government, this presumption is not presently valid. In fact most, if not virtually all, such mathematical scientists currently employed in commerce do not work in their fields of expertise.

This holds even for those with higher degrees: the National Research Council "reports that at least 90 percent of nonacademically employed mathematical scientists who received master's degrees in 1986 do not work as mathematical scientists" [**8**].

A few years ago I heard an interesting talk at an MAA section meeting on the use of mathematics by employees of the

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Some people believe that the eighteenth century represented a peak of civilization from which we have declined.

How could anyone possibly be opposed to arithmetic?