Kristen Herbert
and
Rebecca H. Brown

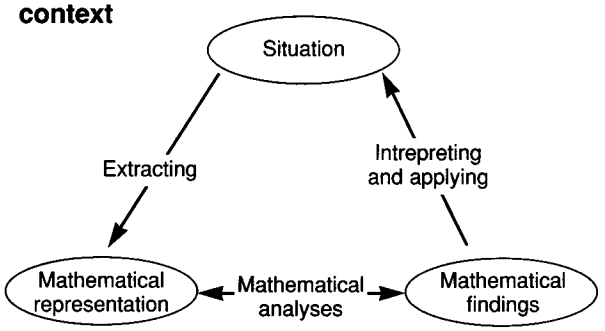
PATTERNS AS TOOLS
FOR ALGEBRAIC REASONING

One teacher made up this story to introduce her sixth-grade students to "Crossing the River," one of the lessons in a unit called Patterns in Numbers and Shapes.

> You and your partner have gone on a long hike with 8 adults. You are very hot and tired. Your group gets to a wide river that you must cross to get home. No one in your group knows how to swim. On the riverbank is a small boat, which can only hold 2 children, or 1 adult, or 1 child. How many one-way trips does it take to get all the people in your group across the river?

After a brief discussion of the story by the class, groups of students sought different tools to solve the problem. Some groups began diagramming the story on paper, others acted it out, and others used cubes of different colors to track the trips across the river. Whatever their strategy, students in each group were following an investigative process that they had learned for finding and generalizing patterns. At the same time, they were building a foundation in algebraic thinking.

FIGURE 1
A framework for algebraic thinking in context



The Patterns in Numbers and Shapes unit was developed as part of the National Science Foundation–funded curriculum MathScape: Seeing and Thinking Mathematically and was field-tested in classrooms across the country. In each lesson, students encounter a problem presented in a context, and they work in pairs or small groups to act out the story, whether kinesthetically, visually by drawing pictures, or manipulatively by modeling the situation with physical objects. They engage in an investigative process to solve the problem: (1) they seek out a pattern in the story, (2) they recognize the pattern and describe it using different methods, and (3) they generalize the pattern and relate it to the story. The Patterns in Numbers and Shapes unit and all the algebra-focused units in the curriculum are intended to provide a new image of how students can develop algebraic-thinking skills. A broad view of algebraic thinking is taken to show students the real-life uses and relevance of algebra. Algebraic thinking is using mathematical symbols and tools to analyze different situations by (1) extracting information from a situation, such as the one described in "Crossing the River"; (2) representing that information mathematically in words, diagrams, tables, graphs, and equations; and (3) interpreting and applying mathematical findings, such as solving for unknowns, testing conjectures, and identifying functional relationships, to the same situation and to new, related situations. The investigative process used in the Patterns in Numbers and Shapes unit is an initial, informal example of this three-part framework (see **fig. 1**).

Kristen Herbert has been a curriculum developer, with particular interests in algebra and performance assessment, at the Education Development Center (EDC) for five years. She is currently teaching middle school mathematics. Rebecca Brown has been a classroom researcher, with particular interests in algebra and technology, at EDC for the past three years. She plans to continue classroom research in mathematics. *The authors are very grateful to the many teachers and students who field-tested this unit and would like to thank their colleagues at the Education Development Center, who helped create this unit, and Charles Lovitt, who provided the initial drafts of many of the problems in the unit.*

This material is based on work supported by the National Science Foundation under grant no. ESI-9054677. The views expressed in this article do not necessarily reflect those of the National Science Foundation.

For the past five years, with funding from the National Science Foundation, Education Development Center (EDC) has been developing a new middle school mathematics curriculum for grades 6–8 called MathScape: Seeing and Thinking Mathematically. The curriculum comprises seven five-week grade-level units that reflect the recommendations of the NCTM's *Standards*. In using these units, students assimilate new information and experiences and construct their own meaning of mathematics through a variety of activities, including hands-on activities and group exercises. The curriculum will be available in 1997 from Creative Publications.

Using this investigative process to solve contextualized problems gives students the kind of informal exploration of algebra discussed in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989, 102). It states, "It is thus essential that in grades 5–8, students explore algebraic concepts in an informal way to build a foundation for the subsequent formal study of algebra." Rather than push students into formal symbolic algebra, the unit emphasizes algebraic thinking by leading students to communicate their thoughts in their own words or their own symbols. In addition, the five-week unit gives students the opportunities and support to develop pattern-seeking skills and an ability to generalize patterns from concrete situations. The approach is also intended to increase students' confidence in themselves as being capable algebra students.

The Investigative Process to Solve Problems

The investigative process consists of three phases: (1) pattern seeking, (2) pattern recognition, and (3) generalization. These three phases are specific components of the general framework shown in **figure 1;** pattern seeking is extracting information, pattern recognition is mathematical analysis, and generalization is interpreting and applying what was learned. Students follow the investigative process for all twelve lessons in the unit. "Crossing the River," one lesson from the unit, will next be used to illustrate the investigative process that students follow. Students are guided through the lesson by the student page shown in **figure 2.** In a teacher's guide, suggestions are made for the use of manipulatives, physical activity, and drawings to help students solve the problems.

Pattern seeking

To begin the "Crossing the River" problem, students model the situation presented in the first step of the task (see step 1 in **fig. 2**) to look for patterns. As suggested in the teacher's guide, teachers give students some type of counters to help solve the task; two counters of one color represent the children, and eight of another color represent the adults (see **fig. 3**).

As students begin to move the boat back and forth across the river, they begin to make comments, such as, "Someone has to row the boat back across the river, maybe a kid can do it" and "The arms on these kids are going to be really tired after all this rowing!"

Students collect and record numerical data as they work through the situation. They develop many different methods of recording the data. Some students record the number of trips either as tally marks or in a list. Others draw a picture or diagram that shows each trip across the river **(fig. 4).**

While gathering and recording their data, students begin to notice recurring patterns. Some students notice a pattern while working with the manipulatives and use their recorded data to check their idea of the pattern. In one class, for example, two girls quickly exclaimed to their teacher, "We found the pattern." Using the manipulatives, they showed their teacher and explained, "First you send two children over, next you bring one child back, then you send an adult over, and the other

The unit leads students to communicate their thoughts in their own words or symbols

FIGURE 2

Student sheet for "Crossing the River"

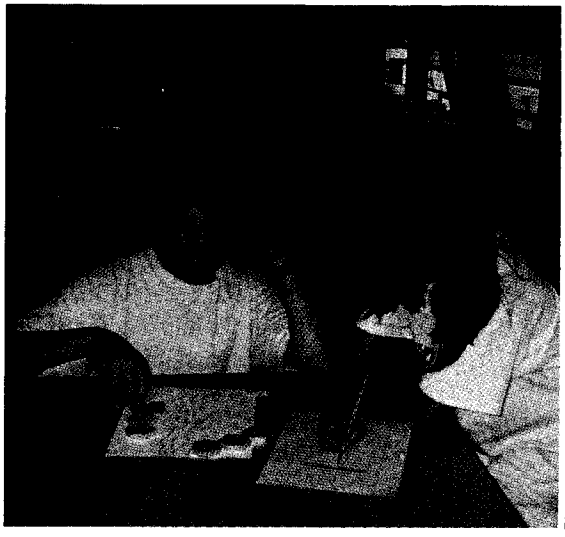
1. A group of 8 adults and 2 children need to cross a river. A small boat is available that can hold 1 adult, or 1 child, or 2 children. Everyone can row the boat.
   How many one-way trips does it take for them all to cross the river?

2. What if there were different numbers of adults? How many trips would be required in the following situations?
   - 6 adults and 2 children?
   - 15 adults and 2 children?
   - 3 adults and 2 children?
   - 100 adults and 2 children?

3. How would you find the number of trips needed for 2 children and any number of adults? Describe the method you would use to solve the problem.

FIGURE 3

Students work through the problem using manipulatives and a river drawn on paper.



Photograph by Rebecca H. Brown; all rights reserved