Notice in the following dialogue the tentativeness of Ryan's geometric definition of a square number and how continued exploration in this setting enables him to become more consistent in using his geometric understanding of an algebraic equation.

R: No, but I've heard of square numbers.
T: Is 36 a square number?
R: Yes.
T: Tell me why.
R: Because you can make a square with 36 squares.
T: Show me on the chart which are square numbers.
R: [Ryan looks at the numbers in the border column to see if they are square numbers. He tries to build a square of area 8 and cannot.]
T: Where are the square numbers in the table? You told me a square number was one you could make into a square.
R: Right. So all the squares: 9, 16, 25 . . . . Those were the squares!

Ryan's teacher also asked questions about fractions. Even though the fraction questions were new to Ryan, he quickly made a table after the teacher started it. The table shows the fraction of blue squares to the total squares in the first few figures. (See **table 3.**) When asked if the number of blue tiles would be half the total tiles, he said, "Well, it won't be half blue until the border and the number of blues are the same." He looked at the table and noted that it was close in the fifth square (25 out of 49) but not equal. Ryan is beginning to see patterns in equivalent fractions. (See **fig. 5**).

Ryan also is beginning to see that the linear pattern of the white tiles is overtaken by the quadratic pattern of the blue tiles even though he does not know the names for these patterns of growth. Both Grace and Ryan used a lot of computation in the process of looking for patterns. Grace added and counted in multiples, and Ryan multiplied and subtracted.

FIGURE 5 Ryan completes his table.

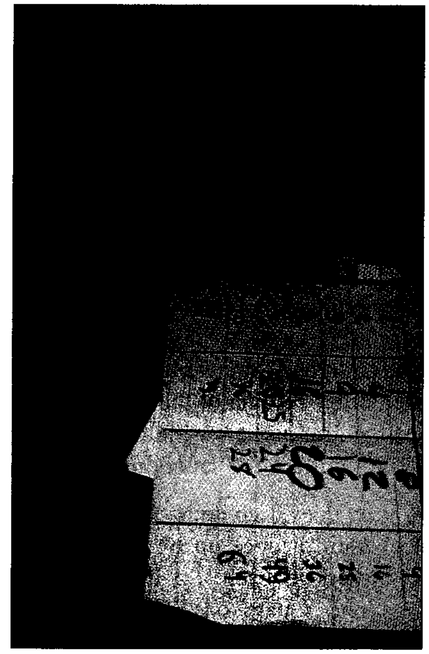


TABLE 3 Ryan's fraction table

1st	1/9
2nd	1/4 (4 out of 16)
3rd	9/25
4th	16/36

Grades 5–6

In grade 5, we can use new ways to represent the relationships between the number of tiles of each color and the number of the square pools. We can begin to make the emphasis on function more explicit. (See **fig. 1.**)

- Make a table showing the numbers of blue tiles for water and white tiles for the border for the first six square pools.
- What are the variables in the problem? How are they related? How can you describe this relationship in words?
- Make a graph that shows the number of blue tiles in each square pool. Make a graph that shows the number of white tiles in each square pool.
- As the number of the pool increases, how does the number of white tiles change? How does the number of blue tiles change? How does this relationship show up in a table and in the graph?
- Use your graph to find the number of blue tiles in the seventh square.
- Can there ever be a border for a square pool

with exactly twenty-five white tiles? Explain why or why not.

Next we can increase the demand of the problem so that students will look for patterns and make gènerializations to help with predicting what will happen in the case of a very large pool. (See **fig 1.**)

- Find the number of blue (white) tiles in the 10th pool. The 25th pool. The 100th pool.
- If there are 144 blue squares, what is the side length of the square pool including the border? How many white tiles are needed for the border?

Looking into the classroom for grades 5 – 6

Some students continue to build the squares using tiles, and they notice relationships between the numbers of blue and border tiles as they build and then record their data in a table. Some students find that using grid paper is helpful. For some students, the act of building or drawing the pools suggests the relationship between the number of blue and the number of white tiles. The number of white tiles is four times the number of blue tiles on a side plus four for the four corners. Some students may use a table to find the number of blue or white tiles in the 10th pool. But some students begin to reason about the patterns. "In the 10th pool, the square formed by the blue tiles is an 8 by 8 square, so there are 64 blue tiles. There are 100 tiles, so 100 total tiles minus 64 blue tiles equals 36 white tiles." Some students may first reason about the number of white tiles, whereas other students may draw the 10th pool and reason from the picture. As they continue to explore these problems, they begin to notice other patterns. Some notice that the number of white tiles will always be a multiple of 4. Some students question whether every multiple of 4 is a white-tile total (see **fig. 6**).

After discussing the patterns in the table, the teacher suggests that the class explore the graphs of these patterns. The graphs suggest that the relationship between the number of the square pool and the number of white tiles can be represented by a straight line, and that the number of the square pool and the number of blue tiles lie on a curved line. In later grades the first pattern is called a *linear function* and the latter pattern is called a *quadratic function.* The students can use the graph to find the number of white tiles (see **fig. 7**) or of blue tiles (see **fig. 8**) given the pool number, and, conversely, given the number of white tiles or blue tiles, they can find the pool number. By graphing the white- and blue-tile patterns on the same grid, students can use the graphs to reason about when the two patterns are equal or when one is greater than the other. (See **fig. 9.**)

FIGURE 6

The number of white tiles is always a multiple of 4.

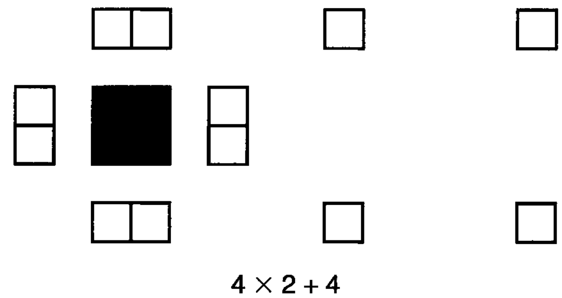
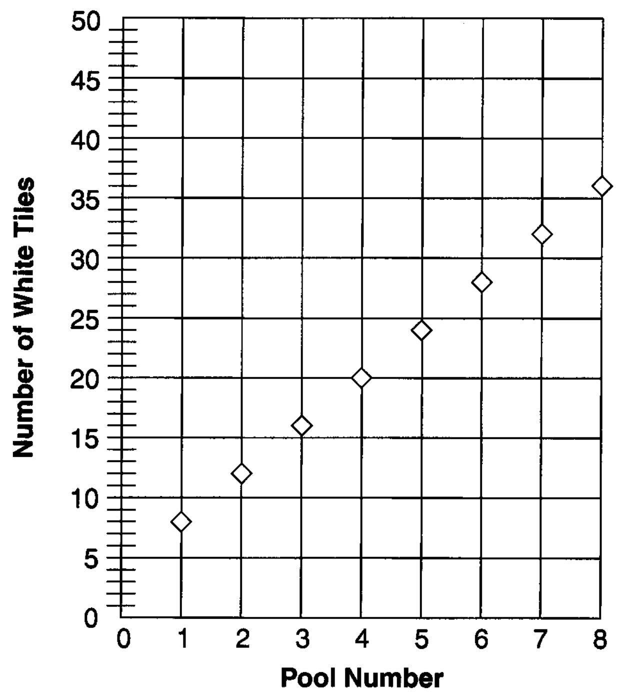


FIGURE 7

A graph of the white tiles for each pool



Where Is the Algebra?

Throughout the grade-level examples of versions of the pool-design problem, children are being challenged to observe patterns in the growth of the numbers of blue tiles and white tiles needed to make the next square pool and to build connections between the physical representations and their verbal descriptions. This activity involves an informal interaction with variables. In the early grades, the focus is on the number of each kind of tile and which is more. Even here the problem has the potential to challenge each child at his or her level of interest and insight. All children can sort and count the blue and white tiles, but the teacher can ask questions that push beyond, for example:

FIGURE 8

A graph of the blue tiles for each pool

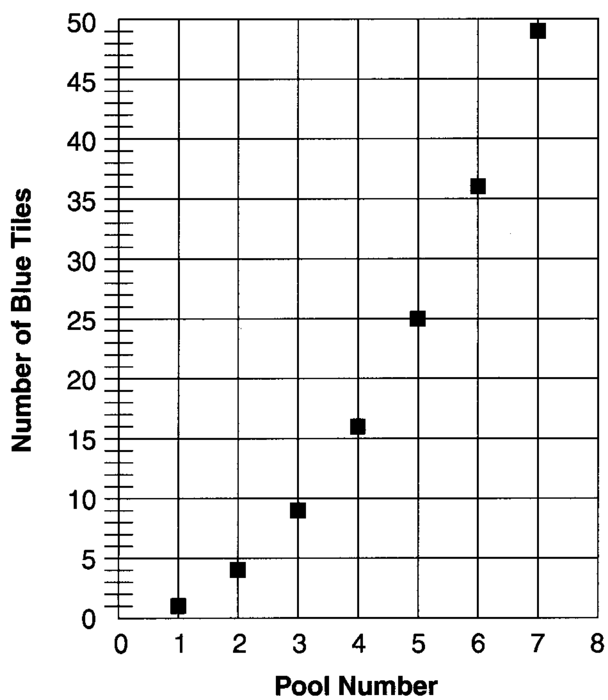
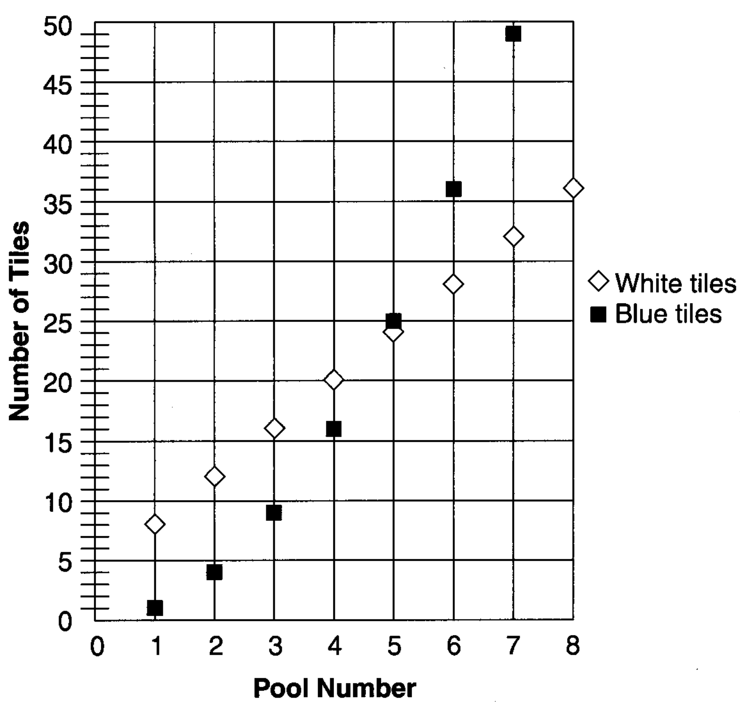


FIGURE 9

Putting the graphs on the same coordinate axis helps to compare the two patterns.



Build your own design out of white and blue tiles. How many blue tiles and white tiles does your design have?

As the problem moves up the grades in elementary school, the questions asked push toward generalization. The children are challenged to find a way to describe the relationship between the number of blue tiles and the number of white tiles and the position in the sequence of pool designs. The ways to represent the change in the variables from one pool to the next also become more varied over time. Verbal descriptions, tables, graphs, and symbolic expressions are all legitimate ways to express the relationships.

Looking for Algebraic Reasoning

Many situations in elementary school mathematics can give teachers an opportunity to generalize and represent mathematical ideas and processes. In this article we offer a geometric setting that illustrates how mathematical ideas can be developed from the study of problems and how algebra emerges as a way to generalize and represent these ideas. Many other settings situated in number, data, and measurement are fruitful sites for developing algebraic reasoning. The following set of questions can serve to organize a classroom discussion in a variety of settings. The wording and choices of representations will vary depending on the experiences of the students.

- What are the variables in this situation? What quantities are changing?
- How are the variables related?
- As one variable increases, what happens to the other variable?
- How can you represent this relationship using words, concrete objects, pictures, tables, graphs, or symbols?
- How can you build connections among representations?
- How can you use this relationship to predict information about the variables?

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