a) Evaluate $3x^2y - 4xy^2$ for $x = 2$ and $y = -1$

Solution:

$$\begin{aligned} &3x^2y - 4xy^2 \\ &= 3(2)^2(-1) - 4(2)(-1)^2 \\ &= 3(4)(-1) - 4(2)(1) \\ &= -12 - 8 \\ &= -20 \end{aligned}$$

b) Find the value of k such that the line passing through $(k, 5)$ and $(10, 8)$ has a slope of ¾.

Solution: slope = slope

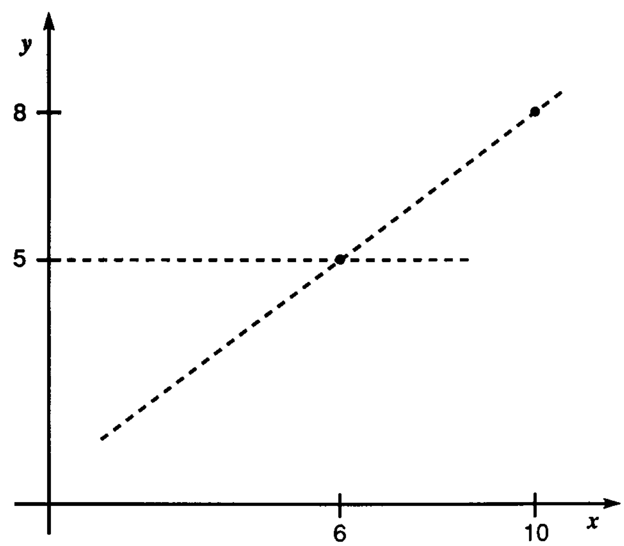
$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3}{4}$$

$$\frac{8-5}{10-k} = \frac{3}{4}$$

$$\frac{3}{10-k} = \frac{3}{4}$$

$$10 - k = 4$$

$$6 = k$$



A sketch of the situation would be part of the expected structure of the solution strategy. The sketch should help the student determine the reasonableness of the solution.

Critical Attributes

Exercise sets can be carefully structured and sequenced so that an important aspect or attribute of a new skill or process can be highlighted. Consider the following sequence of exercises related to combining like terms:

a) $12x + 5x$	*g*) $12x + 5x + 6$
b) $12x + 6x$	*h*) $12x + 5x + 66$
c) $12x + 11x$	*i*) $12x - 3x + 6$
d) $12x - 7x$	*j*) $12x + 3(x + 2)$
e) $12x - 12x$	*k*) $12x + 3(x - 4)$
f) $12x + 5x + 6x$	*l*) $12x + 3(x + 1) + 2x$

In keeping the first term fixed at $12x$ the student is encouraged to concentrate on the critical attributes that change. In addition to furnishing practice in doing the problems, having students write about changes in exercises *a–l* helps students focus on such essentials as distinguishing terms and factors and at the same time gives the teacher information to assess students' understanding and plan future instruction.

Nature of the Task and the Type of Answer Expected

Students perform a great variety of mechanical tasks in algebra, and in so doing they often confuse the various processes. For example, in simplifying the expression $3x - 4 + 6x - 14$, mysteriously the student introduces a zero and comes up with $x = 2$ ($3x - 4 + 6x - 14 = 0$). Practice situations need to anticipate this potential confusion by requiring students to verbalize the nature of the task being performed and the type of answer associated with the task. Consider the following examples:

a) Simplify: $5(x + 2) - 3(2x + 4)$

Nature of the task: Simplifying an expression

Expected result: Another expression.

b) Solve for x: $3x + 7 = 4(x - 2)$

Nature of the task: Solving a linear equation

Expected result: A single value for x that makes the original equation true (assuming the original equation is a typical conditional equation)

c) Solve : $\left\{\begin{matrix} x + 4y = 8 \\ 2x - y = 4 \end{matrix}\right\}$

Nature of the task: Solving a system of two linear equations

Expected results: If the two lines intersect, a unique solution that consists of an ordered pair (x, y) that would make each equation true; if the lines are the same, an infinite set of solutions (x, y) that make each equation true; or if the lines are parallel, no solution.

Discuss and Write Responses

With an emphasis on mechanical tasks, students learning algebra often have difficulty in verbalizing responses to key questions concerning the major skills and concepts being learned. Students often respond, "I have the answer, but I don't know why it works" (or some similar statement). Teachers need to present daily opportunities in which the focus is on developing confidence in verbalizing the concepts. Students need to discuss and write responses. A small-group format can be effective for this type of activity. For example, the student can be asked, "How do you know that -7 is not the solution to the equation $3x + 4 = 6x - 2$?" A typical response might be, "When -7 is substituted for x, you do not get a true statement." Additional sample suggestions could include the following:

a) Can you tell whether the point (3, 4) is on the graph of the line $4x - y = 8$?

b) How do you know whether the line $y = 4x$ is parallel to the line passing through (0, 0) and (1, 8)?

c) Discuss whether the following is an identity: $|x + y| = |x| + |y|$

Anticipation

In formulating practice assignments, it is important for teachers to anticipate skills and formats that students will encounter later. For example, in working with slope, students will need to use the format

$$\frac{(y_2 - y_1)}{(x_2 - x_1)}.$$

Earlier in the school year, in evaluating expressions involving signed numbers, teachers can use the slope formula as a vehicle for some of the practice with signed numbers. A second example involves the quadratic formula. Under normal circumstances a student would not encounter the quadratic formula until the later stages of elementary algebra. However, it is possible to expose students to the kinds of expressions and computations involved by designing exercises with signed numbers and radicals, such as the following:

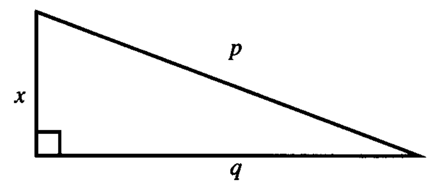
Simplify:

a) -2 ± 4	*e*) $-2 \pm \sqrt{9}$
b) -8 ± 8	*f*) $-2 \pm \sqrt{16}$
c) $\frac{-2 \pm 4}{2}$	*g*) $\frac{10 \pm \sqrt{25}}{3}$
d) $\frac{-4 \pm 8}{12}$	*h*) $\frac{-3 \pm \sqrt{8}}{2}$

INTEGRATING IDEAS

Practice situations need to provide for ongoing review, application, and integration of previously learned ideas. Mixed practice needs to include opportunities for students to transfer learning to new situations. Consider the following example:

In a right triangle, the hypotenuse and the long leg differ by 1 unit. Find an expression for the other leg.



Solution:

Given:

$$p - q = 1$$
$$x^2 + q^2 = p^2$$
$$x^2 = p^2 - q^2$$
$$x = \sqrt{p^2 - q^2}$$
$$x = \sqrt{(p+q)(p-q)} = \sqrt{(p+q) \times 1} = \sqrt{p+q}$$

This problem could be used to integrate several skills and concepts learned earlier, such as factoring and the Pythagorean theorem. Students need opportunities to experiment with the use of algebraic skills and problem-solving strategies without always being told which to employ. Practice that uses applications can be a good arena for this added dimension.

Meaningful practice is essential for success in learning algebra. Students need to view practice as an integral part of the learning process. The major goal should not be to "finish the assignment" but rather to become mathematically powerful. Each assignment should offer students opportunities to verbalize their thoughts, strategies, and understandings and to develop confidence in applying algebraic skills and procedures to help make sense of many different kinds of situations.

REFERENCE

National Council of Teachers of Mathematics, Commission on Standards for School Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: The Council, 1989.