

“instrumental understanding” and “relational understanding.” Instrumental understanding is categorized as rote learning with little need for explanations. It is basically a collection of rules and routines without reason. It is important because this type of understanding is the goal for most students and teachers in United States classrooms (Skemp 1987). Relational understanding is the goal proclaimed by the recent NCTM *Standards* and the method recommended by current research (NCTM 1989). A clear consensus emerges from the mathematics research and professional teaching and supervising organizations that we should work toward relational understanding, both in algebra and in the preparation for algebra. The claim of this chapter is that such focus will make attainable the goal of algebra for everyone.

It is true that under the present system that advocates instrumental learning, some students find success. It is also true that not all students can be successful under the present system. Not all students can recall, memorize, and maintain the huge collection of rules and routines necessary for instrumental understanding of algebra. Discussing conclusions and implications of current research in mathematics education, Peterson (in Grouws and Cooney [1988] ) claims that the challenge for educators in the next decade will be to improve students’ learning of higher-order skills in mathematics. Recent research and theory suggest that the following classroom processes might facilitate that relational understanding of mathematics:

- (a) Focus on meaning and understanding.
- (b) Encourage students’ autonomy, independence, self-direction, and persistence in learning.
- (c) Teach higher-order processes and strategies.

The findings from naturalistic studies (Grouws and Cooney 1988) of classrooms suggest that teachers do not currently emphasize these processes.

Although the Cockcroft Report (Committee of Inquiry into the Teaching of Mathematics in the Schools 1982) pertained to education in the United Kingdom, the wealth of information in that paper gives us ample suggestions for instructional routines. Mathematics teaching at all levels should include opportunities for—

- exposition by the teacher,
- discussion between teacher and pupils and among pupils,
- appropriate practical work
- consolidation and practice of fundamental skills and routines,
- problem solving, including the application of mathematics to everyday situations, and
- investigative work.

Hoyles (in Grouws and Cooney [1988]) draws from that report to give us specific recommendations for classroom routines that are pertinent for middle school youngsters. First, use mathematics in situations in which its power is appreciated. Then, reflect on the procedures used by various individuals and groups. Finally, attempt to apply these derived procedures to known efficient ones and to new domains and then to make appropriate connections.

We must change the dominating current method of instruction, which reaches for instrumental understanding. Students must be given opportunities to derive rules, make conjectures, and determine patterns. These opportunities come less from teacher direction and more from independent and small-group work. The task of the teacher then becomes relating the derived findings and rules to the appropriate language and algorithms of mathematics. We must teach our students to look for generalizations. Derived formulas mean a greater investment of time, but they require less drill and practice and are more often maintained. It is a longer fight, but a greater pay-off is realized. We must challenge students in the classroom to formulate principles and concepts for themselves.

Grattan-Guinness (1987) claims that “foundations are things we dig down to, rather than up from.” He is stating that we must first produce some mathematics in a variety of ways and contexts before we try to systematize it. In American education, we usually attempt the foundations-up approach, which does not permit investigation, discussion, questioning, and conjecturing by students. This simple quote should tell us much about how to teach mathematics.

### **Pacing**

Is it a compromise to talk about all students getting to algebra, but at different times? Do we weaken the effort of the *Standards* by saying, “Yes we believe all students should take algebra, but some in the eighth grade and some in the tenth grade?” This chapter argues that we do not. We surely do not believe that all students can obtain the same level of understanding of all mathematics. The strong argument presented here is that all students should complete, with success, the content of algebra. Some students will understand with greater meaning than others, and some students will need more time for the preparation for success in algebra. The claim is that consideration of pacing is crucial to reaching that goal.

Consider the previously mentioned levels, or categories, of students determined by achievement ranges from the first five grades of elementary school. Although we could do much to improve their program, the two top groups are not the key concern because they do have the opportunity to take algebra, which can open doors for future education and provide opportunities in the work force. As discussed at the beginning of this chapter, the target group is that lower one-fourth of the present student population who do not succeed in algebra. The content has been defined and instruction schemes suggested, and each of these is probably also acceptable to us, as teachers, for the two top categories. The implications for the lower group are overwhelming. We typically are concerned with managing these students, for whom it is easier to stress instrumental learning and dispensary methods of teaching. We must take the time to implement the recommended curriculum with these students, who require our most expert teaching.

If the content and instruction require student activities, thinking, discussion, and explanation first, then the argument is even more acute for these students at risk of not taking algebra. Recall that the format is for students to derive results and explain what they mean using their own descriptive language, and it is the task of the teacher to translate that effort into the desired technical language and concepts of mathemat-

ics. Successful students can usually meet lesson objectives within one or two classes under the present structure. Less successful students simply require more time. Some students can master the content suggested between arithmetic and algebra in one year. These students already take algebra in grade 8. Other students, the majority according to the Second International Assessment, take the same content over a two-year period. The claim in this chapter is that the third group, which traditionally has been relegated to a repetition of arithmetic with no expectation of enrolling in algebra, could develop relational understanding of the same content over a three-year period.

Such a scheme is not in conflict with the *Standards* and is realistic under the present management of students and curriculum. As John Everett told us in 1932, it would be most logical to limit success in algebra to a few, but such a scheme is abhorrent to our desires. If algebra is a way of solving problems, a plan for organizing data and information, part of a reasoning process, and a key component to thinking mathematically, then every individual should have the chance to succeed in algebra. Perhaps, if we change the emphasis in content and instruction and adjust the pacing, as described in this chapter, for students in the middle grades, then we will reach our goal of “algebra for everyone.”

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