4. The sum of the digits in each multiple of 3 in a given diagonal has the same value—3, 6, or 9—according to the value of the multiple in the first row. (P, N, C, T, r, c)

Appropriate questions lead to the discovery of other relationships (P, N, C, T, r, p, c):

5. All the multiples of 3 that lie in the diagonal line beginning with 9 are multiples of 9, but only every other multiple of 3 that lies in the diagonal line beginning with 6 is a multiple of 6.

6. The numbers in the hundred chart can be rearranged to avoid breaking the diagonal lines on which the multiples of 3 lie. See figure 2.10.

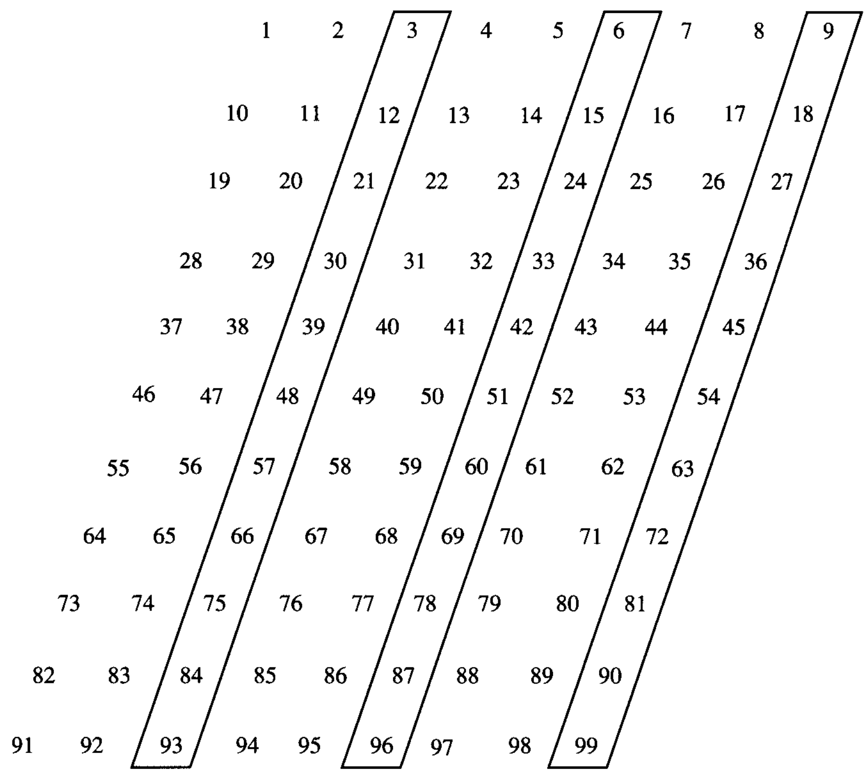

Fig. 2.10. Hundred chart with multiples of 3 on unbroken diagonal lines

7. The numbers in the hundred chart can be rearranged so that the multiples of 3 lie in columns, as in figure 2.11. In this arrangement, the sum of the digits of each of the numbers in the sixth column is 6. In fact, in each column, the sum of the digits of the numbers is equal to the number that heads the column. The

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81
82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99
100								

Fig. 2.11. Hundred chart with multiples of 3 in columns

column headed by 9 is the most interesting. Notice that in addition to the fact that the sum of the digits is 9, the tens and ones digits in each succeeding entry from 9 to 90 increase and decrease by 1, respectively.

In the middle grades, all students should be able to explain why, in successive multiples of 9, the tens digit increases by 1 and the ones digit decreases by 1. All students should also be challenged to explain why the sum of the digits of the numbers in a given column is equal to the number that heads the column. A discussion of the various explanations that groups of students submit affords an excellent opportunity to reexamine the concept of place value. (P, N, C, T, r, p, c, n)

In addition, the relationships listed in the foregoing should be extended by graphing the functions $f(x) = 3x$, $f(x) = 6x$, and $f(x) = 9x$ in the coordinate plane. An examination of the graphs introduces the concepts of linear equations, slope, and such transformations as $(x, y) \rightarrow (x, y + b)$ and $(x, y) \rightarrow (x, ny)$. See figure 2.12.

Eratosthenes' sieve is a hundred chart on which the multiples of 2 through 10, starting with 2 times each of these numbers, are marked in different colors or with different symbols (fig. 2.13). It provides an excellent introduction to prime and composite numbers, prime factorization, and exponents. (P, N, C, L, T, r, c, n)

Further examination by the students of the multiples of 3, 6, and 9 relative to prime- and composite-number characteristics and an extension of their findings to multiples of other numbers lead students to discover divisibility rules and properties of terminating and repeating decimals. (P, N, C, L, T, r, p, c, n)

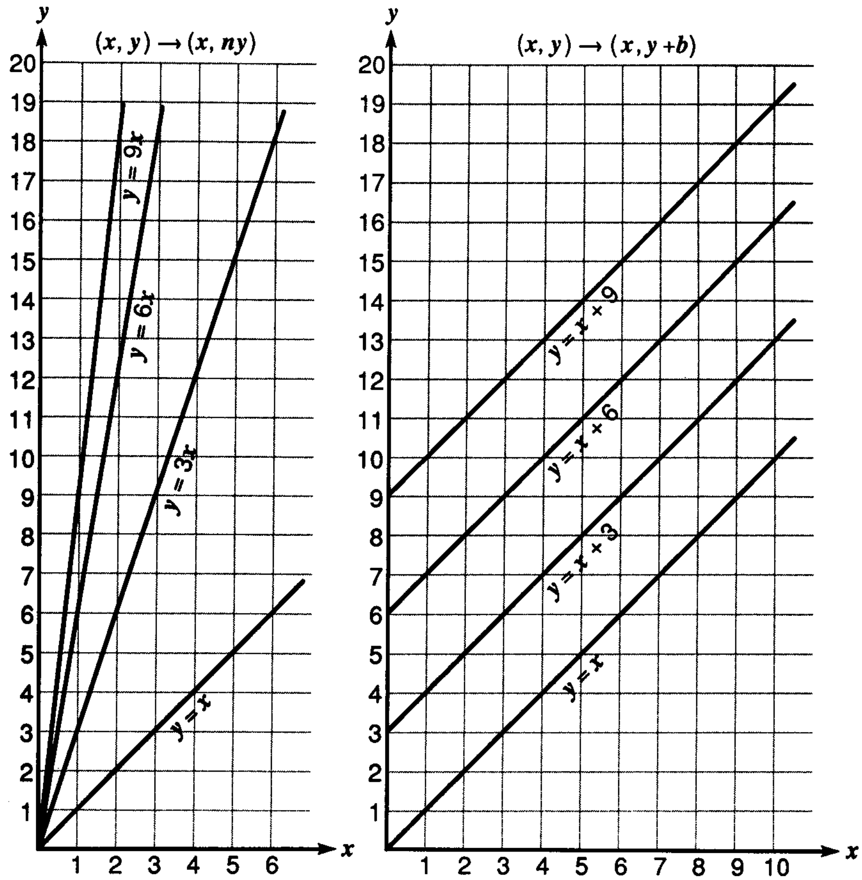

Fig. 2.12. Hundreds chart leads to exploration of linear eqations, slope, and transformations

SUMMARY

Preparation for success in algebra includes much more than computational proficiency. It means focusing on a broad range of mathematical content and processes so that students can become mathematically literate. Prior experiences must present opportunities for students to explore, conjecture, and reason logically so that skills can be acquired in ways that make sense to the students. These opportunities must focus on the development of understandings and on relationships among concepts and between the conceptual and procedural aspects of a problem.

Offering such experiences may require a rethinking of both the curriculum and the roles of teachers and students. Teachers must guide, listen, question, discuss, clarify, and create an environment in which students become active learners who explore, investigate, validate, discuss, represent, and construct mathematics.

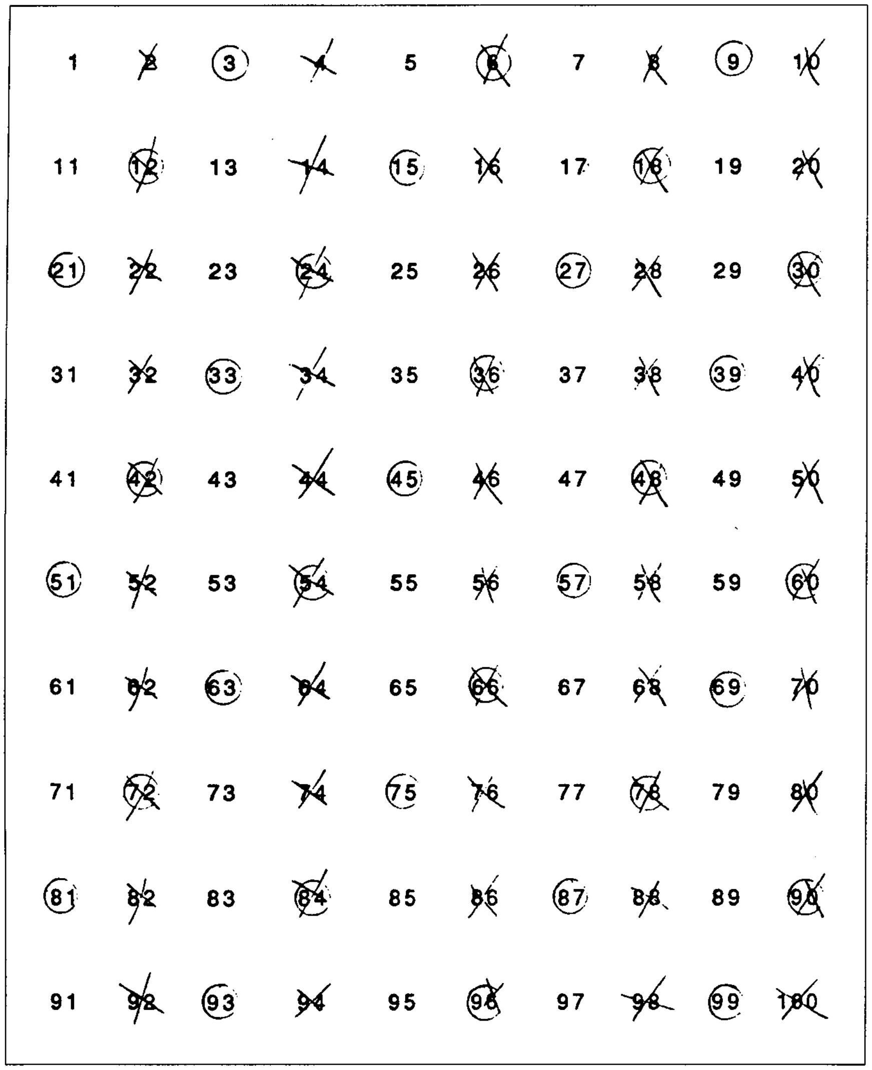

Fig. 2.13. Eratosthenes' sieve

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