# 2

# PRIOR EXPERIENCES

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WITH THE REALIZATION that algebra is for everyone comes the increased need for all students to have the prior experiences necessary for success in the formal study of algebra. Identifying these experiences and suggesting when and how they should best be offered are the concerns of this chapter.

No magic point marks the beginning of preparation for the study of algebra. Preparation for algebra begins with the recognition that numbers can represent a wide variety of quantities, that numbers can be classified and related according to their characteristics, and that these relationships can be communicated in a variety of ways. For example, consider the myriad of algebraic concepts involved when a student "discovers" that for any whole-number replacement of $\square$ (or $n$) by $2\square$ (or $2n$), an even number always results.

*Classification:* All whole numbers are either even or odd.

*Reasoning:* Since whole numbers appear to be alternately odd and even, they form a pattern: O, E, O, E, O, E,....Some characteristic of numbers must account for this pattern.

*Number relationships:* An even number consists of pairs; an odd number consists of pairs and one extra.

| Even Numbers | Odd Numbers |
|---|---|
| | 1 x |
| 2 x x | 3 x x x |
| 4 x x x x | 5 x x x x x |
| 6 x x x x x x | 7 x x x x x x x |

*Operation sense:* When an even number is divided by 2, no remainder results; when an odd number is divided by 2, a remainder of 1 always results.

$\frac{24}{2} = 12$ $\frac{25}{2} = 12$ R: 1

$\frac{346}{2} = 173$ $\frac{347}{2} = 173$ R: 1

*Generalization:* Every even number is a multiple of 2.

*Notion of variable:* Every even number is equal to 2 times some number.

*Organization of information:*

| Whole number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | ... |
|---|---|---|---|---|---|---|---|---|
| Corresponding even number | 2 | 4 | 6 | 8 | 10 | 12 | 14 | ... |

*Dynamics of change:* As the whole numbers increase by 1, the corresponding even numbers also increase, but they increase by 2.

*Concept of implication:* If a whole number is even, then it can be expressed as $2n$, where $n$ is a whole number.

*Concept of function:* The set of even numbers can be generated by multiplying members of the set of whole numbers by 2.

*Use of notation:* Even numbers can be represented as $2n$, where $n$ represents any whole number. That is, $f(n) = 2n$.

*Nature of answer:* A mathematical solution is not necessarily expressed as a number. In this example, the solution of how to represent an even number is $2n$.

*Justification:* The solution can be justified either by substituting whole numbers for $n$ to check that $2n$ is always an even number or by showing that since $2n$ is a multiple of 2, it is an even number according to the foregoing generalization statement.

The foundation for these and similar concepts, traditionally considered to be components of a formal study of algebra, is gradually developed throughout the K–8 mathematics curriculum, as recommended by the NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) (hereafter called *Standards*). Thus, an analysis of prior experiences recommended for the study of algebra must encompass the entire spectrum of a student's mathematical experiences prior to the study of algebra.

To identify the most relevant experiences that mold the foundation for algebra, numerous research studies concerning difficulties students encounter in the study of algebra and a variety of algebra prognosis tests and middle school contest examinations were analyzed. Their common expectations are included in this brief summary.

## CONTENT AND PROCESS

Although knowledge of specific content and vocabulary is a necessary ingredient of the foundation for algebra, of at least equal importance is the ability to look beyond the numerical details or dimensions to the essence of a situation. This ability is a learned skill; it is not an inherited talent. Developing this ability requires that instruction focus on process as well as content.

The four process standards, common to each of the grade-level designations (K–4, 5–8, and 9–12) in the NCTM's *Standards* (1989), are problem solving, reasoning, communication, and connections. They permeate instruction of all content and will thus be referenced throughout the discussion of the content areas considered in this summary.

Because mathematics is not a compendium of discrete bits and pieces that can be taught independently, its instruction cannot be classified into neatly defined categories. However, five major content categories were selected for consideration in this summary for the best alignment with current research and a majority of state and district curriculum guides for mathematics. They are patterns, relationships, and functions; number and numeration; computation; language and symbolism; and tables and graphs.

The following brief descriptions of the content and process categories illustrate their interdependence, which is further illustrated in a discussion of learning experiences that traces the study of multiples throughout the K–8 curriculum.

## Patterns, Relationships, and Functions

Mathematics is often described as the study of patterns. Students who, from the earliest grades, are encouraged to look for patterns in events, shapes, designs, and sets of numbers develop a readiness for a generalized view of mathematics and the later study of algebra. Recognizing, extending, and creating patterns all focus on comparative thinking and relational understanding. These abilities are integral components of mathematical reasoning and problem solving and of the study of specific concepts, such as percentage, quantitative properties of geometric figures, sequence and limit, and function.

By analyzing and creating tables and graphs of data they have recorded, students develop an understanding of the dynamics of change. By modeling increasing and decreasing relationships, students recognize how change in one quantity effects change in another, which is the essence of proportional reasoning. Consider the following activities.

Students work in groups of four, with a designated duty for each student in the group. One student should record the data, another should be a reporter, and two should be explorers. Each group has several geoboards. Half the groups are given forty pieces of construction-paper squares that each measure 1 geoboard unit on a side and a loop of string whose length is exactly twenty-four of the geoboard units. These groups are to determine how many different rectangles they can make by placing the loop of string around nails of the geoboard and then recording the number of construction-paper squares needed to cover the interior of each rectangle.

The other groups are given a large rubber band and thirty-six construction-paper squares. Their job is to use the paper squares to identify all the possible rectangles whose area is thirty-six square units, and then to record the number of units the rubber band spans to enclose each area.

The results differ with the grade level of the students and their prior experience with cooperative learning. In general, however, the recorded data and final reports resemble the following:

| Perimeter: 24 Units | | |
| --- | --- | --- |
| $l$ | $w$ | Area |
| 6 | 6 | 36 |
| 7 | 5 | 35 |
| 8 | 4 | 32 |
| 9 | 3 | 27 |
| 10 | 2 | 20 |

| Area: 36 Square Units | | |
| --- | --- | --- |
| $l$ | $w$ | Perimeter |
| 6 | 6 | 24 |
| 9 | 4 | 26 |
| 12 | 3 | 30 |
| 18 | 2 | 40 |
| 36 | 1 | 74 |

Students in the first group experience how change in the dimensions of the rectangle affects its area. They recognize that the change is not constant but that it follows a pattern. For consecutive unit increases in the length (and consequent unit decreases in the width), the area decreases by consecutive odd numbers.

Students in the second group also experience the dynamics of change. However, they recognize some dramatic differences. As the length increases, the perimeter also increases. The increases are all even numbers, but a pattern does not appear to be evident, at least not an easily recognizable pattern. The students wonder whether using unit increments of change for the length would clarify the pattern.

| $l$ | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $w$ | 6 | 5.1 | 4.5 | 4 | 3.6 | 3.3 | 3 | 2.8 | 2.6 |
| $P$ | 24 | 24.3 | 25 | 26 | 27.2 | 28.5 | 30 | 31.5 | 33.1 |

The consecutive increases in the perimeter are smaller, but the pattern is still not clear. Perhaps a comparison of the two graphs would help. See figure 2.1. With help, some students will be capable of pursuing the investigation. However, all students will be introduced to algebraic concepts in a problem-solving setting.

## Number and Numeration

The NCTM's *Standards* calls it "number sense"; in *Mathematics Counts (The Cockcroft Report)* (1982), the United Kingdom Committee of Inquiry into the Teaching of Mathematics in the Schools calls it "the sense of number"; Bob Wirtz (1974) has referred to it as "friendliness with numbers." Whatever it is called, research has found that this intuition about numbers and how they are related is an important ingredient of learning and later applying mathematics. The NCTM's *Standards* identifies five characteristics of students with good number sense: they have a broad understanding of (1) number meanings, (2) multiple relationships among numbers, (3) relative magnitudes of numbers, (4) the relative effect of operating on numbers, and (5) referents for measures of common objects and situations in their environment.

The development of these characteristics should be an ongoing focus throughout the curriculum to include experiences with whole numbers, fractions, decimals, integers, and irrational numbers. In the early grades, experiences with manipulatives illustrate equivalent forms of numbers. See figure 2.2. In the intermediate grades, explorations with calculators extend this understanding:

$$64 = 8^2, 4^3, \text{ or } 2^6$$

$$\sqrt{75} \text{ is between 8 and 9 because } \sqrt{75} \text{ is between } \sqrt{64} \text{ and } \sqrt{81}$$

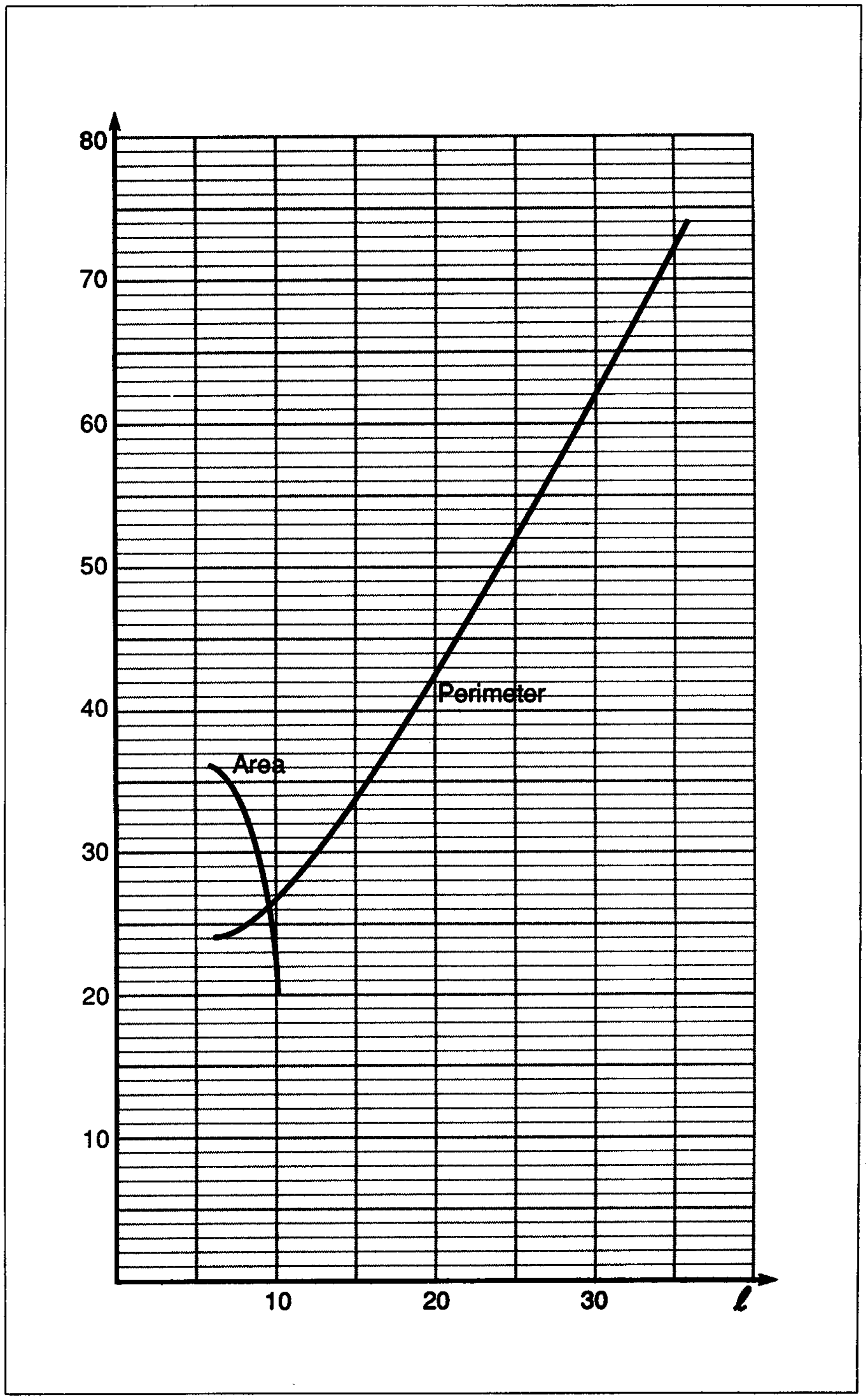


Fig. 2.1. Comparison of collected data

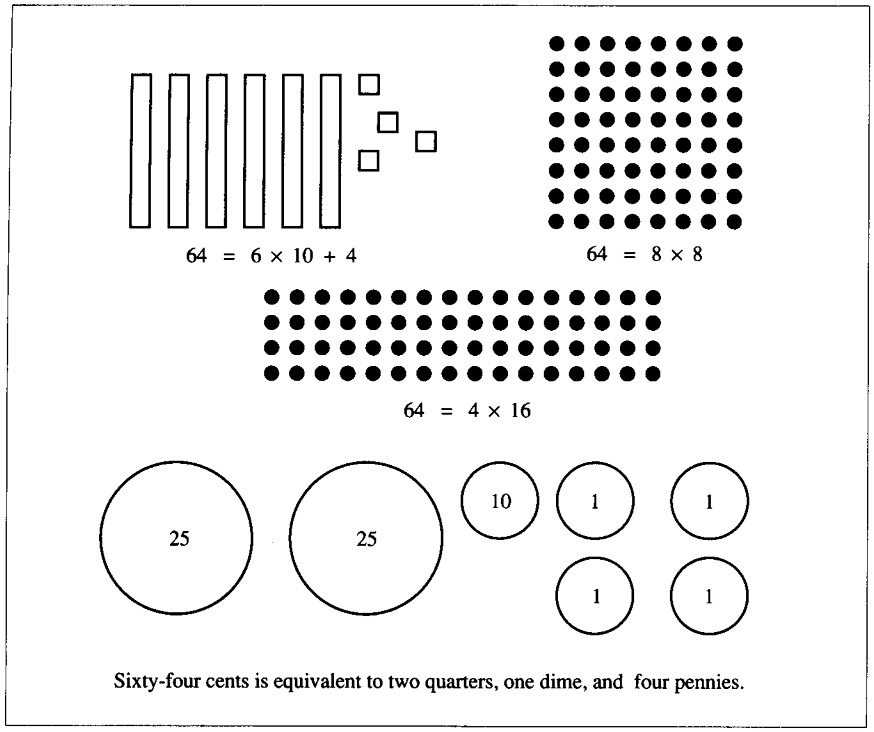


Fig. 2.2. Manipulative models of equivalent forms of numbers

In "Incredible Equations," an activity used in the *Box It and Bag It* program (Burk, Snider, and Symonds 1988), students spend a few minutes each day expressing the day of the month in as many ways as they can devise. The representations become more complex at each grade level as students incorporate into their work new knowledge and experience with numerical operations and symbolism.

As each new number system is studied, students should be given opportunities to acquire a "feel," or "sense," of the numbers in the system; the symbols used to represent them; and their role in the real world; and to discover how operations on them compare with, and differ from, previously studied sets of numbers.

## Computation

As reported in *The Mathematics Report Card* (Dossey et al. 1988), the report of the 1986 National Assessment of Educational Progress, nearly one-half of the students at grades 7 and 11 agreed that mathematics is mostly memorizing. More than 80 percent of students at these grade levels viewed mathematics as a rule-bound subject. The fact that other recent research substantiates these findings explains, at least in part, another finding. In a comparison with earlier assessments, the level of students' performance on questions that require application of concepts and problem solving has decreased despite an increase in performance that requires