Workshop 4
Quadratic Functions

Overview

Description

In this workshop, teachers will learn ways to give students experience in graphing quadratic functions from equations written in vertex form, and formulating equations, in vertex form, based on graphs of quadratic functions.

- Part I: Tremain Nelson and his students use a basketball toss as a launching point to learn how the constants in the equation $y = a(x - h)^2 + k$ transform the parent function $y = x^2$.
- Part II: Tremain Nelson and his students use the information that they learned in the previous lesson to model several bounces of a ball dropped below a motion detector.

Featured Software

- *Cognitive Tutor: Algebra 1,* Carnegie Learning, Inc., Pittsburgh, Pennsylvania

Featured Educator

- A. Tremain Nelson, Hastings High School; Houston, Texas

Featured Commentator

- Anthony Piccolino, Montclair State University; Verona, New Jersey

Learning Objectives

In these activities, you will learn how to help students:

- Learn how to graph quadratic functions written in the form $y = a(x - h)^2 + k$, and understand how the constants a, h, and k transform the parent function of $y = x^2$.
- Develop mathematical models of real-world events.
- Write equations in the form $y = a(x - h)^2 + k$ when given a graph of a parabola.
- Identify the x-intercept(s), y-intercept, and axis of symmetry of a parabola.

Workshop Session (On-Site)

Part I: Graphing Quadratic Equations

Getting Ready (15 minutes)

The equation of a quadratic function written in vertex form is $y = a(x - h)^2 + k$. Describe how the constants a, h, and k transform the parent function $y = x^2$. Talk about the problems that students have in learning these concepts.

Watching the Video (30 minutes)

Watch Part I: Graphing Quadratic Equations.

Going Further (15 minutes)

Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- What classroom management strategies did Tremain use to keep the class engaged and on task?
- What are some ways that teachers can help students understand the effects of the constant h in the equation written in vertex form? What are the strengths and weaknesses of the approach Tremain used?
- Discuss Tremain's use of "table talk" as a teaching strategy.
- Analyze the different types of assessments Tremain used during this lesson.
- Discuss the way in which the class developed an assessment tool for the presentations. What were the pros and cons of this technique?
- Do you think a computer lab can be an effective tool for individualized assessment? Discuss ways you might assess your students individually without such hardware and software.
- What evidence did you see that this class was a "community of learners"?

Part II: Modeling With Quadratic Functions

Getting Ready (15 minutes)

You observe a person drop a racquetball from a height of 6 feet above the ground. The ball bounces several times. Sketch the following two graphs:

- the actual path of the bouncing ball as observed from 5 feet away
- the height of the ball as a function of time, starting from the moment it was dropped

Discuss the differences and similarities in the two graphs. Which graph is the one we most often study in mathematics? What problems or misconceptions might students have when looking at either of these graphs?

Watching the Video (30 minutes)

Watch Part II: Modeling With Quadratic Functions.

Going Further (15 minutes)

Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- Discuss the effectiveness of the technique that Tremain used to demonstrate how to graph the quadratic function $y = -(x - 3)^2 + 4$, using one transformation at a time.
- Do you agree that the students need only find the vertex in order to find the equation for one bounce of the ball?
- Discuss whether or not it is worthwhile to use "canned" data, as Tremain did in his classroom. Should students have been told that they all had the same data?
- Talk about how Tremain brought his class to the point of doing the presentations, and how he elicited feedback from the rest of the class.
- Design another experiment for students to model something using a quadratic function.

Between Sessions (On Your Own)

Homework Assignment

Reflect on ways that you can teach students to better understand the effects of the constants in a quadratic function written in the form $y = a(x - h)^2 + k$, and how these constants transform the parabola. Share your thoughts on Channel-Talk or write them in your online journal.

Ongoing Activities

You may want to carry on these activities throughout the course of the workshop:

Keep a Journal

Read the Teaching Strategies for Workshop 4 and answer the journal prompts. Include thoughts, questions, and discoveries from the workshop itself and learning experiences that take place in your own classroom. You are encouraged to use the online journaling tool at www.learner.org/channel/workshops/algebra.

Web Site: www.learner.org/channel/workshops/algebra

Investigate the Resources section to deepen your understanding of the Teaching Strategies for this workshop.

Share Ideas on Channel-Talkinsights@learner.org

Share your thoughts on what math teachers can do to create a community of learners in the classroom.

Video Teacher Reflections

Tremain Nelson

Below are Tremain Nelson's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

When watching Workshop 4, Parts I and II, what did you notice about your teaching strategies and student thinking?

The only requirement for students to participate in the taping was that they have parental permission and not be failing more than one class. Therefore, I was happy to see that the tape reflects my actual classroom environment. I use varied methods for delivering new content to students (e.g., direct teaching, self-discovery, round table discussions). For this lesson, the discussion method was chosen because the students were introduced to quadratics in a previous lesson. Therefore, they were more likely to be engaged in the discussion. It is important that they communicate the mathematics verbally and use the appropriate terms in these discussions. Most students seemed willing to communicate their ideas without much prompting; however, there were one or two students that seemed to dominate some of the discussions. It can be difficult at times to muffle the enthusiasm of one student in order to get feedback from a less participatory student.

Discuss the process involved in getting your class to function as a community.

The students were in my regular algebra classroom for almost a year prior to the videotaping. Therefore, much work [had already been done] to develop a sense of community in the classroom. The process began at the beginning of the year when students were actively engaged in social behavior exercises, mathematics learning teams, and group presentations. Students were asked to perform tasks, complete assignments, and prepare for presentations that could not be completed without the help of someone else. These activities helped to foster a sense of togetherness and made it easier for me to focus on the application of concepts instead of the concepts themselves. There is a heterogeneous mix of abilities, personalities, and willingness to participate in classroom activities in my class. Positive reinforcement and motivation techniques were used to encourage stronger students to help weaker ones, to overcome cultural differences, and to maintain a sense of focus on the common goal. For example, when there was conflict between team members, every effort was made during my consultation with them to help them see the benefits of staying together and working as a team. The last resort is to move individuals out of a group into another one. During cooperative learning activities, high-achieving students are told that their work is not complete until everyone on the team has mastered the covered skill as they have.

Reflect on the role of the students who are not active participants in a table talk. How do you hold them accountable, and how do you manage behavior problems?

The challenge in the lecture style of teaching is that it limits the teacher's ability to monitor which kids are engaged in the lesson and which are not. The table talk discussions increase teachers' ability to monitor learning; however, this does not ensure that all students will participate in the discussion. This is why table talk is used in conjunction with cooperative learning teams. During the table talk, one member is chosen from each team to sit closest to the teacher during the discussion. The other members are invited to gather around, but are not expected to participate as much as those "team leaders" closest to the teacher. The team leaders are expected to gather enough understanding of the topic during the table talk to be able to go back to their teams and explain the concept or task to the other team members. Those surrounding the leaders are there to support them and to make sure that they also have an understanding because ultimately the entire team will be responsible for presenting their work to the class. As seen in the video, the leaders from each team change regularly so that different sets of students surround the teacher during our discussions. Behavior problems are handled as they should be in any classroom—with redirection and consequences for actions. However, I have noticed that when the class decides to function as a community of learners, the misbehaving child seems like an anomaly rather than the norm.

Discuss how you got the students to give better-than-superficial feedback on the graph posters in Part I and on the presentations in Part II.

Students present on a regular basis (at least once a week) over the course of a school year. The presentations are a key component of the curriculum because they give me the feedback I need in order to adjust or modify my lessons to ensure proper knowledge transfer. Presentations can have many forms, however, the goal of every presentation is to determine the level of understanding of the students in the classroom. Students begin the year by presenting information

Video Teacher Reflections, cont'd.

with which they are comfortable or on questions for which they already know their answers are correct. An example of this type of presentation is the gallery tour seen in Part I. During the tour, the students use the rubric I provided to grade other students' solutions to similar problems. After the grading, one or two exhibits are chosen randomly for presentation. These initial presentations are intended to help build confidence as well as model teacher expectations for more difficult presentations to come. As the year progresses, the scaffold is slowly removed until students are given the responsibility of organizing and presenting information that is new to them and unfamiliar to their classmates as seen in Part II. Students must understand that it is not satisfactory to simply state their solutions to a given problem during a presentation; they must also defend them.

Give some background on the students' prior understanding of quadratic relationships before these two lessons.

Quadratic functions were covered earlier in the year for these students. Part I was a follow-up to an introduction to parabolas, where the students participated in a basketball shooting contest and then used their imagination and knowledge of quadratic functions to model different occurrences of parabolas in nature (such as rainbows, bridges, half moons, etc.). They had no experience with the vertex form of the quadratic equation prior to the taping. However, they were familiar with key terms such as vertex, axis of symmetry, and x- and y-intercepts.

How does the computer lab work relate to and support the topics of regular instruction? How does it enhance student learning?

The students spend 60% of their time in regular instruction and 40% of their time in the computer lab. In the computer lab, students work with the Cognitive Tutor™ software program, which works much like a human tutor. The software is able to monitor and track each student and create problems based on individual needs. Therefore, students may be at varying points in the computer lab depending on their mathematical abilities. The software spirals the content for them and provides multiple representations of various algebraic concepts to assist students in achieving a profound understanding of mathematics. The problems in the classroom and in the lab are scenario-driven. That is, the mathematics is discovered in the context of real-life problems that allow students to use their prior knowledge in problem solving. Students who were not able to grasp the concepts of the ball-bouncing activity, were not willing to participate in the table talk discussions, and/or may have trouble presenting in front of the class now have an opportunity to master the same skills covered in the classroom using an unobtrusive, yet effective, personal cognitive coach.

What are the strengths and weaknesses of the different methods of assessment you use in your classroom?

Presentations, table talks, and cooperative learning teams work well as methods to introduce new material, develop a deeper understanding of mathematics, and assess student knowledge. As assessment tools, they allow teachers to receive oral feedback from the student that is necessary to assess their understanding. During these activities, the students are given the opportunity to communicate the mathematics to the teacher and/or their peers, investigate multiple problem scenarios, and re-evaluate their own thinking. The downfalls of using these assessment tools are that time does not often allow for every student to be assessed based on their oral communication and it is easier for students to choose not to participate in these assessments.

Another major assessment tool is the Cognitive Tutor™ software. The computer software provides immediate feedback to the student and the teacher concerning the level of mathematical proficiency obtained during the completion of each unit. Each set of skills is monitored and displayed for both the student and the teacher to review daily. As the areas of need are identified, the system displays a message to the student with that regard and saves the information in a database to use in the creation of new problems or for future reference by the instructor. Unfortunately, the Cognitive Tutor is most effective when there are enough computers in a school to allow for a class to have access to the software on a regular basis, which is a disadvantage for those schools that can not support this level of availability.