Harvesting the Fields

A team of farm-workers was assigned the task of harvesting two fields, one twice the size of the other. They worked for the first half of the day on the larger field. Then the team split into two groups of equal number. The first group continued working in the larger field and finished it by evening. The second group harvested the smaller field, but did not finish by evening. The next day one farm-worker finished the smaller field in a single day's work. How many farm-workers were on the team?

Content

Quantities   **N-Q**

Reason quantitatively and use units to solve problems.
   1.   Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
   2.   Define appropriate quantities for the purpose of descriptive modeling.

Creating Equations   **A-CED**

Create equations that describe numbers or relationships.
   1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

Commentary

This is a challenging task, suitable for extended work, and reaching into a deep understanding of units. The task requires students to exhibit MP1, Make sense of problems and persevere in solving them. An algebraic solution is possible but complicated; a numerical solution is both simpler and more sophisticated, requiring skilled use of units and quantitative reasoning. Thus the task aligns with either A-CED.1 or N-Q.1, depending on the approach.

Students who believe that they have found solutions should be encouraged to check their solutions, to see if they work, because by checking their solutions they will understand the problem more clearly.

Although it is not state explicitly, it is assumed that the farm-workers all work at the same rate, harvesting the same area in any given period of time, and that for any period of time, the area cleared by a group of farm-workers is proportional to the number of farmworks working.

A flexible understanding of units simplifies some of the solutions. For example, the second solution is simpler if R= 1, which is achieved by using the rate of a single farm-worker as the unit. And the third solution can be understood in a more sophisticated way as setting the unit for area to the size of one small field.

Solutions
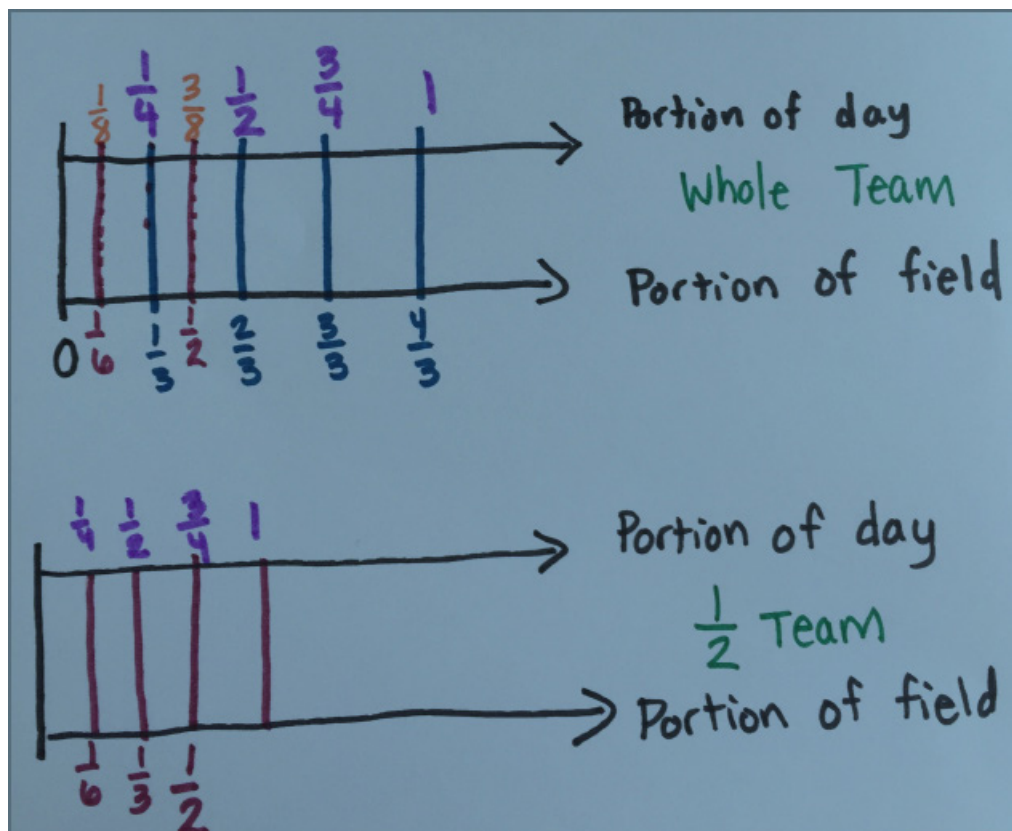**Solution: Harvesting the Fields, method 1: reasoning with rates**

First notice that the larger field would have taken the entire team 3/4

of a day to harvest. This is because they worked on it for half a day and then half a team worked on it for another half a day. If the whole team had worked on it for the second half-day, they would have finished it in half the time, that is, 1/4 day. Adding this to the first half-day we get 3/4 day for the whole team.

Since the smaller field is half the size of the larger field, it would have taken the whole team 3/8 of a day to harvest the smaller field. As it was, only half the team worked on the smaller field, so it would have taken them 3/4 of a day to finish. They only worked for 1/2 of a day, so they still had 1/4 of a day's work left, which would have been 1/8 of a day's work for the entire team.

It took one worker a whole day to finish up, so it took one worker a day to do what the entire team could have done in 1/8 day. Therefore the team had 8 workers.

Solution: Harvesting the fields, method 2: setting up an equation

Let x be the number of farm-workers on the team (in units of people per team), and let R be the area one farm-worker harvests in a day (in units of acres per person per day). The area harvested in that first half day was

$$\frac{1}{2} \text{ day} \cdot R \frac{\text{acres}}{\text{person} \cdot \text{day}} \cdot x \frac{\text{people}}{\text{team}} \cdot 1 \text{ team} = \frac{1}{2} \, x \, R \text{ acres.}$$

In the second half day, the area harvested was

$$\frac{1}{2} \text{ day} \cdot R \frac{\text{acres}}{\text{person} \cdot \text{day}} \cdot x \frac{\text{people}}{\text{team}} \cdot \frac{1}{2} \text{ team} = \frac{1}{4} \, x \, R \text{ acres.}$$

So the area of the larger field is

$$A = \frac{1}{2} \, x \, R + \frac{1}{4} \, x \, R = \frac{3}{4} \, x \, R.$$

The area of the smaller field is the area harvested by half the team in half a day added to the area harvested by one person in 1 day:

$$\frac{1}{2} \text{ day} \cdot R \frac{\text{acres}}{\text{person} \cdot \text{day}} \cdot x \frac{\text{people}}{\text{team}} \cdot \frac{1}{2} \text{ team} + R \frac{\text{acres}}{\text{person} \cdot \text{day}} \cdot 1 \text{ person}$$

$$\cdot 1 \text{ day} + \frac{1}{4} \, xR + R \text{ acres .}$$

This is also half of the area of the larger meadow, so

$$\frac{1}{4} \, x + 1 = \frac{3}{8} \, x$$

$$1 = \frac{1}{8} \, x \qquad\qquad x = 8$$

Solution: Harvesting the fields, method 2: choosing a size for the fields

Let's assume that the smaller field is 1 acre and the larger one is 2 acres. Assuming these particular sizes does not change our final answer; if the sizes are different then the rate at which the farm-workers can harvest is different, but the total number of farm-workers stays the same. [A more sophisticated way of saying this would be to say that we are working in units of size one small field. In working with this solution it might make sense to make up a name for this unit.]

Then the larger field would be cleared by the whole team in 3/4 of a day, which means that working together, they clear 8/3 of an acre per day. Then half of them clear 2/3 of an acre in a half day, so for the 1 acre field, this would mean that there is 1/3 of an acre left for the single farm-worker, so he or she clears 1/3 of an acre per day. This is 1/8 of what the entire team can clear, so the entire team is 8 farm-workers.