

Session 10

Classroom Case Studies, Grades K-2

This is the final session of the *Data Analysis, Statistics, and Probability* course! In this session, we will examine how statistical thinking might look when applied to situations in your own classroom. This session is customized for three grade levels. Select the grade level most relevant to your teaching.

The session for grades K-2 begins below. Go to page 307 for grades 3-5 and page 323 for grades 6-8.

Key Terms for This Session

Previously Introduced

- census
- interval
- population
- quantitative data
- representative sample
- data
- line plot
- qualitative data
- quantitative variables
- sample
- distribution
- mode
- qualitative variables
- random sample
- variation

Introduction

In the previous sessions, we explored statistics as a problem-solving process that seeks answers to questions through data. You put yourself in the position of a mathematics learner, both to analyze your individual approach to solving problems and to get some insights into your own conception of statistical reasoning. It may have been difficult to separate your thinking as a mathematics learner from your thinking as a mathematics teacher. Not surprisingly, this is often the case. In this session, however, we shift the focus to your own classroom and to the approaches your students might take to mathematical tasks involving statistics. **[See Note 1]**

As in other sessions, you will be prompted to view short video segments throughout the session; you may also choose to watch the full-length video for this session.

Learning Objectives

In this session, you will do the following:

- Explore the development of statistical reasoning at your grade level
- Analyze the use of the four-step process for solving statistical problems in your classroom
- Review mathematical tasks and their connection to the mathematical themes in this course
- Examine children's understanding of statistical concepts

Note 1. This session uses classroom case studies to examine how children at your grade level think about and work with data. If possible, work on this session with another teacher or a group of teachers. A group discussion will allow you to use your own classroom, and the classrooms of fellow teachers, as case studies to make additional observations.

The suggested times for this session allow time for personal reflection and group discussion.

Part A: Statistics As a Problem-Solving Process (25 minutes)

A data investigation should begin with a question about a real-world phenomenon that can be answered by collecting data. After the children have gathered and organized their data, they should analyze and interpret the data by relating the data back to the real-world context and the question that motivated the investigation in the first place. Too often, classrooms focus on the techniques of making data displays without engaging children in the process. However, it is important to include children, even very young children, in all aspects of the process for solving statistical problems. The process studied in this course consisted of four components:

1. **Ask a question.**
2. **Collect appropriate data.**
3. **Analyze the data.**
4. **Interpret the results.**

Children often talk about numbers out of context and lose the connection between the numbers and the real-world situation. During all steps of the statistical process, it is critical that students not lose sight of the questions they are pursuing and of the real-world contexts from which the data were collected.

When viewing the video segment, keep the following questions in mind: **[See Note 2]**

- Think about each component of the statistical process as it relates to what's going on in the classroom: What statistical question are the students trying to answer? How were the data collected? How are the data organized, summarized, and represented? What interpretations are students considering?
- How does the teacher keep her students focused on the meaning of the data and the data's connection to a real-world context?
- Thinking back to the big ideas of this course, what are some statistical ideas that these students are beginning to develop?



Video Segment (approximate times: 15:46-20:11): You can find this segment on the session video approximately 15 minutes and 46 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, the teacher, Ellen Sabanosh, applies the mathematics she learned in the *Data Analysis, Statistics, and Probability* course to her own teaching situation by asking her students to analyze and interpret the data they collected earlier. (Each child was given two boxes of raisins; the children then counted and recorded the number of raisins in each box.) The children will now compile their data into a class line plot and discuss the distribution of the data.

Note 2. The purpose of the video segments is not to reflect on the methods or teaching style of the teacher portrayed. Instead, look closely at how the teacher brings out statistical ideas while engaging her students in statistical problem solving.

You might want to review the four-step process for solving statistical problems (Session 1, Part A). What are the four steps? What characterizes each step?

Part A, cont'd.

Problem A1. Answer the questions you reflected on as you watched the video:

- a. What statistical question are the students trying to answer?
- b. How did the students collect their data?
- c. How did they organize, summarize, and represent their data?
- d. What interpretations are the students considering?
- e. How does the teacher keep her students focused on the meaning of the data and the data's connection to a real-world context?
- f. What statistical ideas are these students beginning to develop?

Problem A2. As the students examined the data, Ms. Sabanosh asked several times, "What do you notice?" or "What else do you notice?" What are some reasons for asking open-ended questions at these points in the lesson?

Problem A3. Ms. Sabanosh gave each student two boxes of raisins for data collection. The students counted the number of raisins in each box separately and recorded both data values on the line plot. What were some advantages and disadvantages, mathematically and pedagogically, of her decision to give each student two boxes of raisins?

Problem A4. Ms. Sabanosh asked the students to analyze the data when only about half the data had been compiled onto the class line plot. How might early analysis of partial data, such as in this episode, support students' evolving statistical ideas?

Part B: Developing Statistical Reasoning (40 minutes)

The National Council of Teachers of Mathematics (NCTM, 2000) identifies data analysis and probability as a strand in its *Principles and Standards for School Mathematics*.* In grades pre-K through 12, instructional programs should enable all students to do the following:

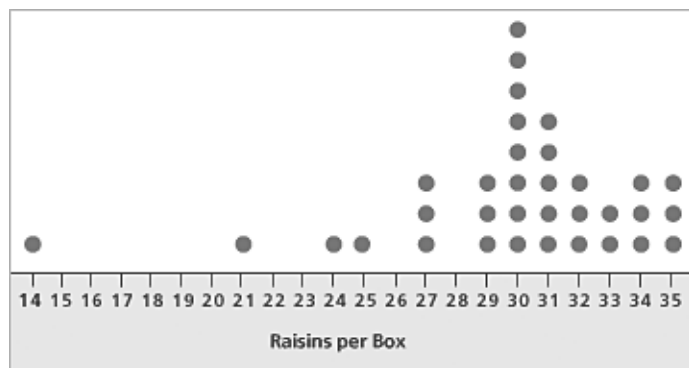
- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Select and use appropriate statistical methods to analyze data
- Develop and evaluate inferences and predictions that are based on data
- Understand and apply basic concepts of probability

In pre-K through grade 2 classrooms, students are expected to do the following:

- Pose questions and gather data about themselves and their surroundings
- Sort and classify objects according to their attributes and organize data about the objects
- Represent data using concrete objects, pictures, and graphs
- Describe parts of the data and the set of data as a whole to determine what the data show

Children at the K-2 grade level readily notice individual data points and are able to describe parts of the data—where their own data fall on the graph, which value occurs most frequently, and which values are the largest and the smallest. A significant development in children’s understanding of data occurs as they begin to consider the set of data as a whole. To focus students’ attention on the shape and distribution of the data, it is helpful to build from children’s informal language to describe where most of the data are, where there are no data, and where there are isolated pieces of data. The words *clusters*, *clumps*, *bumps*, and *hills* highlight concentrations of data. The words *gaps* and *holes* emphasize places in the distribution that have no data. The phrases *spread out* and *bunched together* underscore the overall distribution. Teachers must continually emphasize and help students see that what they notice about the shape and distribution of the data implies something about the real-world phenomena being studied.

The line plot (also commonly referred to as a dot plot in elementary classrooms) below displays the raisin data collected by the students in Ms. Sabanosh’s first-grade classroom: **[See Note 3]**



* *Principles and Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 2000). Standards on Data Analysis and Probability: Grades K-2, 108-115. Reproduced with permission from the publisher. Copyright 2000 by the National Council of Teachers of Mathematics. All rights reserved.

Note 3. Using line plots (dot plots) in elementary school classrooms is a fairly new practice. Consider how you might use this graphical representation of data with your students. For example, when working with pre-kindergarten and kindergarten students, you might have the students begin with pasting the actual boxes on the charts instead of using dots. How does this compare with your current method of presenting data? You may want to refer back to Interpreting a Line Plot in Session 2.

Part B, cont'd.

Problem B1. Imagine yourself in a conversation with the children about this data. A key question you might ask the students is, “What do you notice about the data?” Using informal language (clusters, clumps, bumps, hills, gaps, holes, spread out, or bunched together), write five statements that you hope children would make describing the set of data as a whole.

Too often, children describe data as numbers devoid of context. Another key question you should frequently ask students regarding their observations is, “What does that tell us about the number of raisins in a box?”

Problem B2. For each of the five statements you wrote in Problem B1, indicate what that observation might imply about the real-world context of the number of raisins in a box.

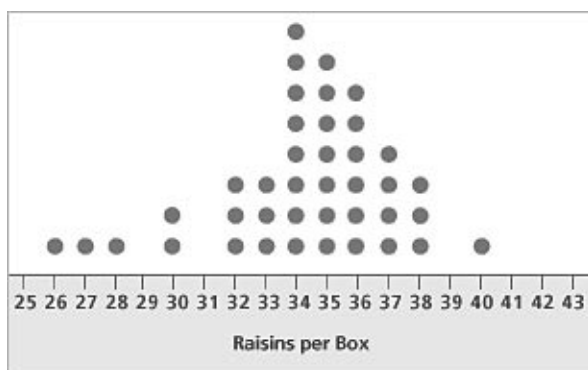
Problem B3. What are some questions about the data that you would pose to the students in Ms. Sabanosh’s first-grade classroom to encourage them to further consider the statistical ideas of outliers, variation, center of a data set, and sampling?

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Post your answer to Problem B3 on an email discussion list, then read and respond to answers posted by others. Go to the *Data Analysis, Statistics, and Probability Web* site at www.learner.org/learningmath and find Channel Talk.

Comparing data sets prompts students to look at and describe a data set as a whole in order to see how the characteristics of one group compare to the characteristics of another. Now we’ll examine data from another class not featured in the video. The line plot below displays the raisin data collected by Mr. Mitchell’s second-grade classroom: [See Note 4]



Note 4. Comparing data sets is also a fairly new practice in elementary school classrooms. Think about why this step might be important for your students. You might also want to review the difference between quantitative and qualitative variables (Session 1, Part B).

Part B, cont'd.

Problem B4. Imagine that Mr. Mitchell's class and Ms. Sabanosh's class compared each other's raisin data to their own data.

- a. What are four or five statements you would anticipate children might make as they compare the two data sets?
- b. Which of these statements compare individual pieces or parts of the data, and which statements compare the data sets as a whole?

Data investigations begin by asking a question that can be answered by gathering data. It's important to formulate a question with an understanding of the type of data you'll need to collect. In this course, you studied two types of data, quantitative and qualitative. Quantitative data, such as the number of hours of television watched each week, are often referred to as numerical data; qualitative data, such as your favorite flavor of ice cream, are referred to as categorical data. The following statistical questions were gathered from pre-K through grade 2 classrooms:

- a. How many raisins are in a box?
- b. How far can you jump?
- c. Who is your favorite author?
- d. How did you get to school today?
- e. Are you 6 years old?
- f. How many people are in your family?
- g. Will you go on the field trip to the zoo?
- h. How many pockets do you have today?
- i. How tall are you?
- j. Do you like chocolate milk or white milk better?
- k. What is your favorite restaurant?
- l. Which apple do you like best: red, green, or yellow?

Problem B5. For each question above, identify the type of data that will be collected and an appropriate way to display the data (e.g., line plot, bar graph).

Problem B6. Formulate 10 questions that would be appropriate for your grade level and of interest to your students, five of which involve collecting qualitative (categorical) data, and five of which involve collecting quantitative (numerical) data.

Part C: Inferences and Predictions (30 minutes)

The NCTM (2000) data analysis and probability standards* state that students should “develop and evaluate inferences and predictions that are based on data.” Appropriate beginnings for the concepts of inference and prediction in pre-K to grade 2 classrooms occur in conversations with children as they consider what the data are telling us, what might account for these results, and whether we’d get the same results in other similar situations.

Inference and prediction are more advanced aspects of working with data, as they require some notion of the ideas of sampling and population. Children’s first experience is often with census data—that is, the population of their class. When they begin to wonder what might be true for other classes in their own school and other schools, they begin to consider that many data sets are samples of larger populations. These considerations are the precursors to understanding the notion of inferences from samples. Children should be encouraged to develop conjectures based on data, consider alternative explanations, and design further investigations to examine their speculations. [See Note 5]

When viewing the video segment, keep the following questions in mind:

- How does Ms. Sabanosh encourage students to make inferences and predictions?
- An expectation for students at this level is that they will “discuss events related to [their] experiences as likely or unlikely” (NCTM, 2000). How is this expectation illustrated in this video segment?



Video Segment (approximate times: 20:25-22:09): You can find this segment on the session video approximately 20 minutes and 25 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, Ellen Sabanosh asks her students to predict the number of raisins in an unopened box, based on the data they have collected. Watch them discuss both reasonable and unreasonable predictions.

Problem C1. Answer the questions you reflected on as you watched the video:

- a. How does Ms. Sabanosh encourage students to make inferences and predictions?
- b. How is NCTM’s expectation for students (i.e., that they will discuss events related to their experiences as likely or unlikely) illustrated in this video segment?

Problem C2. If you were using this investigation with your own students, what questions could you pose to further elicit and extend student thinking about notions of samples and populations?

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Problem C3. Embedded in the students’ responses about “what would not be a good guess” are early notions of making conjectures and forming hypotheses. How could you extend this conversation to bring out more speculations and then formalize these notions into stated conjectures that could be investigated further? What questions would you ask? How would you expect children to respond? What are some conjectures that might result?

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Note 5. You might want to review the statistical ideas of samples and populations and the meaning of conjecture (Session 1, Part D).

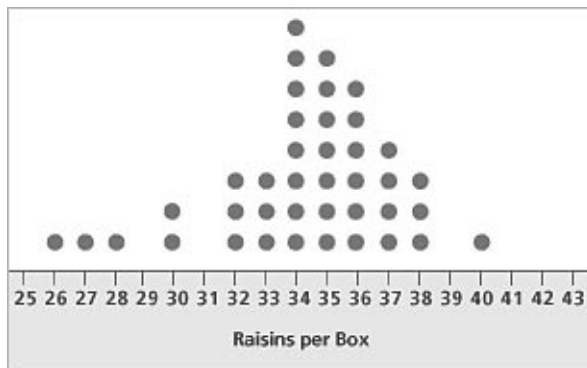
Part D: Examining Children's Reasoning (30 minutes)

As this course comes to a close and you reflect on ways to bring your new understanding of data analysis, statistics, and probability into your teaching, you have both a challenge and an opportunity: to enrich the mathematical conversations you have with your students around data. As you are well aware, some students will readily grasp the statistical ideas being studied, and others will struggle.

The problems in Part D describe scenarios from three different teachers' classrooms, involving young children's developing statistical ideas. Some student comments are given for each scenario. For each student, comment on the following:

- *Understanding:* What does the statement reveal about the student's understanding or misunderstanding of statistical ideas? Which statistical ideas are embedded in the student's observations?
- *Next Instructional Moves:* If you were the teacher, how would you respond to each student? What questions might you ask so that students would ground their comments in the context? What further tasks and situations might you present for the child to investigate? [**See Note 6**]

Problem D1. Mr. Mitchell's second-grade class investigated the number of raisins in a box. The line plot below displays the raisin data collected by the students:



After plotting their data, here is what the students had to say:

- Janet: "That's not fair. I only got 26 raisins. I think we should tell them to put the same number of raisins in each box so that it would be fair."
- Sahar: "I think there will be between 32 and 38 raisins in that [unopened] box because most of our boxes had between 32 and 38."
- Jermaine: "Forty must be an outlier because there's a gap at 39."
- Ramel: "The range is from 25 to 43."
- Ava: "The raisins must all be kind of the same size since most of the data are bunched together in the middle."
- Paul: "I think there will be 34 raisins in that [unopened] box, because 34 had the most."

Note 6. If you're working in a group, make a two-column chart with the labels "Understanding" and "Next Instructional Moves" for recording the group's responses to Problems D1-D3.

Part D, cont'd.

Problem D2. Ms. Jackson asked her kindergarten students to formulate questions. She said, “Each of you will conduct a survey today to find out something about our class. I want you to think of a survey question you can ask your friends that they can answer by saying either yes or no.” The students formulated their questions, wrote them on their papers using words or pictures, and then collected the data. The following conversation took place in a whole-group discussion of the children’s results:

Teacher: Bryce, what was your question?

Bryce: “Do you like dinosaurs?”

Teacher: How many people said yes?

Bryce: Eight.

Teacher: Eight people in your survey said yes. Now what do you think I want you to count?

Bryce: The nos. There were seven nos.

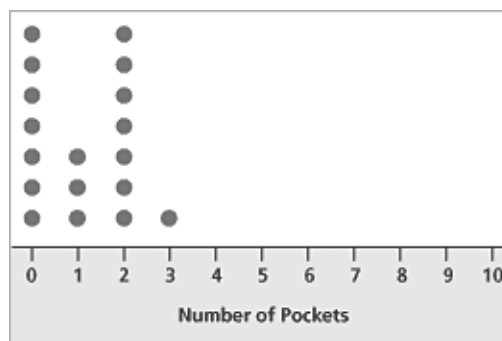
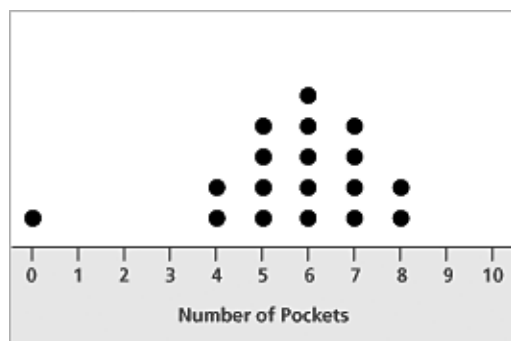
Teacher: Eight people said yes, and seven people said no. What does that tell us about dinosaurs?

Here is how the students responded:

- a. Bryce: “More people like dinosaurs.”
- b. Nicole: “Lots of people like dinosaurs, but lots of people don’t like them, too.”
- c. Wallace: “It’s pretty close—one more, and we would have a tie.”
- d. Maggie: “I think the boys said yes and the girls said no.”
- e. Zula: “I don’t think we can tell if more people like dinosaurs, because two people aren’t here today.”

Part D, cont'd.

Problem D3. Mr. Kettering teaches first grade. Almost every Friday since the beginning of the year, his class has collected pocket data. The children count the number of pockets on their clothes and then record the data on a class line plot. Early in the year, they decided to count only the pockets on the clothes they were wearing at that moment, such as pants, shirts, vests, and sweaters; they did not count pockets on their coats or their backpacks. One day, Mr. Kettering displayed the two line plots below, showing pocket data from two different days, and asked the children to compare the line plots and describe what they noticed about the number of pockets:



Here's how the children responded:

- Ben: "The black dots are a lot more spread out than the gray dots."
- Damon: "I think the gray dots are from the beginning of the school year, when everyone was wearing shorts, and the black dots are probably from December, when it was cold out."
- Emma: "I think that black dot on 0 is someone who forgot we were counting pockets that day."
- Tarra: "With the gray dots, most people only had zero or two pockets, and with the black dots, most people had six."
- Isaiah: "Look at that gap with the black dots—and there aren't any gaps on the gray dots."

For more information about statistics problems for children like the problems in this session:

Burns, Marilyn (1996). *50 Problem-Solving Lessons*. Math Solutions Publications.

Economopoulos, Karen and Murray, Megan (1998). *Mathematical Thinking in Kindergarten*. Dale Seymour Publications.

Economopoulos, Karen; Akers, Joan; Clements, Douglas; Goodrow, Anne; Moffet, Jerrie; and Sarama, Julie (1998). *Mathematical Thinking at Grade 2*. Dale Seymour Publications.

Russell, Susan Jo; Corwin, Rebecca B.; Rubin, Andee, Rubin; and Akers, Joan (1998). *The Shape of the Data*. Dale Seymour Publications.

Homework

Problem H1. Read the Grades Pre-K-2 standard on data analysis and probability that was developed by the National Council of Teachers of Mathematics and is reported in the *Principles and Standards for School Mathematics* (NCTM, 2000).

- After reading this standard, what additional connections do you see between the content you studied in this course and implications for your classroom teaching?
- What are some insights you acquired about the development of children's understanding of data analysis, statistics, and probability from kindergarten through grade 2?
- What are three important ideas you want to remember from the standards when teaching data analysis?

This reading is available as a downloadable PDF file on the *Data Analysis, Statistics, and Probability* Web site. Go to:

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Problem H2. Assume that you need to report back to your grade-level team or to the entire school staff at a faculty meeting on your experiences and learning in this course. What are the main messages about the teaching of data analysis, statistics, and probability you would share with your colleagues? Prepare a one-page handout or an overhead or slide that could be distributed or shown at the meeting.

Problem H3. Look at a lesson or activity in your own mathematics program for your grade level that you think has potential for developing students' statistical reasoning. If you were to use this lesson or activity now, after taking this course, how might you modify or extend it to bring out more of the important ideas about data analysis, statistics, and probability?

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Post your answer to Problem H3 on an email discussion list, then read and respond to answers posted by others. Go to the *Data Analysis, Statistics, and Probability* Web site at www.learner.org/learningmath and find Channel Talk.

Solutions

Part A: Statistics As a Problem-Solving Process

Problem A1.

- The question is, “How many raisins are in a box?”
- The students collected the data by counting the number of raisins in each of the boxes of raisins they were given.
- Students organized and represented their data by placing blue dots on a class line plot, and they summarized their data by finding the mode.
- Students interpreted their data by reasoning that smaller numbers meant that they had bigger raisins.
- The teacher asked the students to interpret their results by relating them back to the context.
- Some statistical ideas the students touched on are the nature of data, quantitative variables, variation, range, mode as a summary measure of a data set, sampling, and making and interpreting a line plot.

Problem A2. Asking open-ended questions gives students more opportunities to engage in statistical problem solving and to construct their understanding of statistical ideas.

Problem A3. The main advantage is that giving students two boxes of raisins enlarged the sample, making the results slightly more representative of the population than if students had only been given one box. However, the overall sample size is still relatively small. One disadvantage in giving students two boxes of raisins is that the teacher and students had to carefully determine ways to organize their work environment so that each box was counted and recorded separately.

Problem A4. The early analysis of partial data encouraged students to begin thinking and making predictions about how the data might evolve.

Part B: Developing Statistical Reasoning

Problem B1. Here are some possible statements that children might make:

- There is a clump of data from 29 to 35.
- I can see a bump at 30.
- There is big gap from 15 to 20.
- There are four holes in the data.
- The data are spread out from 14 to 35.

Problem B2.

- The clump from 29 to 35 tells us that lots of boxes had this number of raisins.
- The bump at 30 is the number of raisins that occurred most often when we were counting how many raisins were in a box.
- The gap from 15 to 20 tells us that no boxes had 15, 16, 17, 18, 19, or 20 raisins in them.
- The holes in the data tell us that at each point where there’s a hole, there are no boxes with that number of raisins.
- The “spread” of the data tells us that the smallest number of raisins (the “minimum”) in a box is 14, and the largest number (the “maximum”) is 35.

Solutions, cont'd.

Problem B3. To get at the idea of “outliers,” you might ask students whether there are any unusual data points on the graph. To focus on the variation in the data, you might ask students to talk about intervals where the data are clustered or concentrated, or where they are spread out. To introduce the concept of the median, you could ask students to try to find the center of the data set.

Problem B4.

- a. A sample answer is that the first graders are likely to notice that the mode for Mr. Mitchell’s class is 34, whereas their mode was only at 30. They are also likely to point out that they had a “really low number” of raisins at 14, which was “a lot lower” than the lowest number from the second-grade class, 26. Students might also notice that their raisin data is more spread out with a larger range than the raisin data from Mr. Mitchell’s class.
- b. The first observations compare individual pieces of the data. The second observation compares the whole data sets to each other.

Problem B5. As data may be displayed in many ways, answers will vary; the representations listed here are only one option:

	Type of Data	Data Displays
a. How many raisins are in a box?	quantitative (numeric)	line plot
b. How far can you jump?	quantitative (numeric)	line plot
c. Who is your favorite author?	qualitative (categorical)	bar graph
d. How did you get to school today?	qualitative (categorical)	bar graph
e. Are you 6 years old?	quantitative (numeric)	bar graph
f. How many people are in your family?	quantitative (numeric)	line plot
g. Will you go on the field trip to the zoo?	qualitative (categorical)	bar graph
h. How many pockets do you have today?	quantitative (numeric)	line plot
i. How tall are you?	quantitative (numeric)	line plot
j. Do you like chocolate milk or white milk better?	qualitative (categorical)	bar graph
k. What is your favorite restaurant?	qualitative (categorical)	bar graph
l. Which apple do you like best: red, green, or yellow?	qualitative (categorical)	bar graph

Problem B6. One example of a question that involves collecting quantitative (numeric) data and that often interests students in first or second grade is, “How many teeth have you lost?” An example of a question that involves collecting qualitative (categorical) data is, “What is your favorite pizza topping?”

Solutions, cont'd.

Part C: Inferences and Predictions

Problem C1. Here are two possible answers:

- Ms. Sabanosh encouraged students to make inferences and predictions by asking them to think about how many raisins would be in one of the unopened boxes.
- When the students discussed what would not be good guesses about the number of raisins in a box, they clearly suggested numbers that were not likely.

Problem C2. Answers will vary. One example of a question is, "If I went to the store and bought a different brand of raisins, but the same size box, how many raisins do you think would be in that box?"

Problem C3. Two conjectures that might result are, "A small [half-ounce] box of raisins cannot contain 100 raisins" and "A small box of raisins will contain between 14 and 35 raisins." These could be formulated as the following question to be investigated: "What is the smallest and largest number of raisins that a box can hold?" The students could investigate this question by obtaining additional half-ounce boxes of raisins of the same brand, then counting the number of raisins in each box, and finally adding this information to their line plot. An extension would then be to count and analyze the number of raisins in half-ounce boxes of other brand names.

Part D: Examining Children's Reasoning

Problem D1.

- Janet is not willing to accept that there is variation in the raisin data. The teacher might point out to Janet that the raisins are packaged by weight, then have her examine the various sizes of the raisins more closely, and finally ask her to think about why the raisins are packaged by weight and not by number.
- Sahar is able to consider an interval of the data that is most representative of the data. The teacher could ask the class to speculate on the likelihood of her prediction.
- Jermaine reasons incorrectly that 40 is an outlier because it is the largest number separated from the others by a gap. The gap, however, is only one raisin, so based on this data, 40 is not unusual enough to be an outlier. The teacher might ask the rest of the class to discuss further the meaning of an outlier.
- Ramel is looking at the range of numbers written on the horizontal axis rather than at the span of the data points. The teacher could use the following questioning to get Ramel to focus on the data points: "What is the smallest number of raisins that we found in a box? Point to it with your left hand. What is the largest number of raisins that we found in the box? Point to it with your right hand. This distance from the lowest to the highest number is what we call the range."
- Ava reasons about an interval of the data that contains the most data points and provides an interpretation of raisin size based on this observation. This would be an opportunity to write out Ava's conjecture and ask the other children to evaluate it. Do they agree or disagree? Would they want to modify the conjecture in any way?
- Paul is reasoning with the mode as a summary statistic that describes a measure of center. The teacher might ask the rest of the class to react to Paul's prediction. How many of them agree, and why? How many disagree, and why? You could also ask Paul and the other children to comment on the likelihood that an unopened box contains 34 raisins.

Solutions, cont'd.

Problem D2.

- a. Bryce is noticing the mode of the data. The teacher could ask Bryce to think about the closeness of the numbers seven and eight and acknowledge that eight is more.
- b. Nicole is considering the variance of the data and the whole data set. The teacher might ask the rest of the class to react to the accuracy of Nicole's statement.
- c. Wallace is noticing that there is not much variation in the data. The teacher could ask Wallace to further explain what he means by "one more" and by a "tie."
- d. Maggie is formulating a conjecture or hypothesis about the data. This would be an opportunity for the class to plan a follow-up data investigation to test Maggie's conjecture.
- e. Zulay is thinking about the sample of students in attendance today and the population of the class. The teacher might ask the rest of the class to consider Zulay's comment and to then predict how they think the two children who are absent would respond to the question about dinosaurs.

Problem D3.

- a. Ben is most likely reasoning by looking at the range of each data set. The teacher could ask the students to pretend that the data value at 0 is not there and to then think about Ben's statement.
- b. Damon is providing an interpretation of the data. The class could be asked to evaluate Damon's conjecture and to propose other reasons for the differences in the data.
- c. Emma is developing a hypothesis about the outlier 0. The teacher could ask the class to formulate other conjectures about the outlier 0.
- d. Tarra is reasoning about the data by looking at the modes of each data set. The teacher could introduce or reinforce the meaning of the term mode.
- e. Isaiah is interested in examining areas with no data as well as areas with data. He also notices that the gray dots are more concentrated with little variation. The teacher might use this as an opportunity to focus further attention on the importance of examining gaps and considering how the data are spread out or bunched together.

Notes
