Session 10

Classroom Case Studies, Grades 6-8

This is the final session of the ***Data Analysis, Statistics, and Probability*** course! In this session, we will examine how statistical thinking might look when applied to situations in your own classroom. This session is customized for three grade levels. Select the grade level most relevant to your teaching.

The session for grades 6-8 begins below. Go to page 291 for grades K-2 and page 307 for grades 3-5.

Key Terms for This Session

Previously Introduced

- bias
- box plot
- census
- data
- distribution
- Five-Number Summary
- histogram
- interval
- line plot
- mean
- median
- mode
- population
- qualitative data
- qualitative variables
- quantitative data
- quantitative variables
- random sample
- relative frequency
- representative sample
- sample
- scatter plot
- stem and leaf plot
- variable
- variation

Introduction

In the previous sessions, we explored statistics as a problem-solving process that seeks answers to questions through data. You put yourself in the position of a mathematics learner, both to analyze your individual approach to solving problems and to get some insights into your own conception of statistical reasoning. It may have been difficult to separate your thinking as a mathematics learner from your thinking as a mathematics teacher. Not surprisingly, this is often the case. In this session, however, we shift the focus to your own classroom and to the approaches your students might take to mathematical tasks involving statistics. **[See Note 1]**

As in other sessions, you will be prompted to view short video segments throughout the session; you may also choose to watch the full-length video for this session.

Learning Objectives

In this session, you will do the following:

- Explore the development of statistical reasoning at your grade level
- Analyze the use of the four-step process for solving statistical problems in your classroom
- Review mathematical tasks and their connection to the mathematical themes in this course
- Examine children's understanding of statistical concepts

Note 1. This session uses classroom case studies to examine how children at your grade level think about and work with data. If possible, work on this session with another teacher or a group of teachers. A group discussion will allow you to use your own classroom, and the classrooms of fellow teachers, as case studies to make additional observations.

The suggested times for this session allow time for personal reflection and group discussion.

Part A: Statistics As a Problem-Solving Process (20 minutes)

A data investigation should begin with a question about a real-world phenomenon that can be answered by collecting data. After the children have gathered and organized their data, they should analyze and interpret the data by relating the data back to the real-world context and the question that motivated the investigation in the first place. Too often, classrooms focus on the techniques of making data displays without engaging children in the process. However, it is important to include children in all aspects of the process for solving statistical problems. The process studied in this course consisted of four components:

1. ***Ask a question.***
2. ***Collect appropriate data.***
3. ***Analyze the data.***
4. ***Interpret the results.***

Children often talk about numbers out of context and lose the connection between the numbers and the real-world situation. During all steps of the statistical process, it is critical that students not lose sight of the questions they are pursuing, nor of the real-world contexts from which the data were collected.

When viewing the video segment, keep the following questions in mind: **[See Note 2]**

- Think about each component of the statistical process as it relates to what's going on in the classroom: What statistical question are the students trying to answer? How were the data collected? How are the data organized, summarized, and represented? What interpretations are students considering?
- What connections among mathematics topics and across subject-area disciplines are apparent in this data investigation?
- Thinking back to the big ideas of this course, what are some statistical ideas that these students are likely to encounter through their investigation of this situation?

Video Segment (approximate times: 44:33-46:19): You can find this segment on the session video approximately 44 minutes and 33 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, the teacher, Paul Sowden, applies the mathematics he learned in the *Data Analysis, Statistics, and Probability* course to his own teaching situation. He starts by asking his students to think about the relative amount of coins with each type of mint mark. The students then sort the coins into four groupings: Philadelphia, Denver, San Francisco, and no mint mark. They will now begin to analyze and interpret their data.

Note 2. The purpose in watching the video is not to reflect on the teacher's methods or teaching style. Instead, look closely at how the teacher brings out statistical ideas while engaging his students in statistical problem solving.

You might want to review the four-step process for solving statistical problems (Session 1, Part A). What are the four steps? What characterizes each step?

Part A, cont'd.

Problem A1. Answer the questions you reflected on as you watched the video:

a. What statistical question are the students trying to answer?

b. How did the students collect their data?

c. How are the data organized, summarized, and represented?

d. What interpretations are students considering?

e. What connections among mathematics topics and across subject-area disciplines are apparent in this data investigation?

f. What statistical ideas are these students likely to encounter as they investigate this situation?

Problem A2. In this video segment, are the students working with quantitative data or qualitative data?

Problem A3. Questions may arise as students examine the nickels in this open-ended investigation. Formulate four statistical questions that students might ask about the nickels that would prompt further investigation.

Problem A4. Why is a circle graph an appropriate way to display this data? What characteristics of data are clearly shown through a circle graph?

Part B: Developing Statistical Reasoning (45 minutes)

The National Council of Teachers of Mathematics (NCTM, 2000) identifies data analysis and probability as a strand in its *Principles and Standards for School Mathematics.** In grades pre-K through 12, instructional programs should enable all students to do the following:

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Select and use appropriate statistical methods to analyze data
- Develop and evaluate inferences and predictions that are based on data
- Understand and apply basic concepts of probability

In the grades 6-8 classroom for data analysis and statistics, students are expected to do the following:

- Formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population
- Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatter plots
- Find, use, and interpret measures of center and spread, including mean and interquartile range
- Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem and leaf plots, box plots, and scatter plots

In grades 6-8, students examine relationships among populations or samples, and they examine two variables within one population, such as comparing arm spans and heights. They learn to use new representations, such as box plots and scatter plots, to help them examine these relationships. Students also use measures of center to summarize and compare data sets. Building on informal understandings of what is typical, what is usual, what is the most, and what is the middle, students develop understanding about the mode, median, and mean. However, students need to learn more than simply how to identify the mode or median in a data set and how to find the mean: They need to develop an understanding of what these measures of center tell us about the data, and what each indicates about the data set. The mean receives increased emphasis in these grade levels, but students also continue to use the median, especially in creating Five-Number Summaries for making box plots.

Problem B1. Consider topics of interest to students in grades 6-8 that involve collecting data about a characteristic shared by two populations. Formulate five questions that involve collecting qualitative (categorical) data and five questions that involve collecting quantitative (numerical) data. For each question, identify the type of data that will be collected and an appropriate way to display the data (e.g., line plot, bar graph, histogram, circle graph, stem and leaf plot, box plot). **[See Note 3]**

---

* *Principles and Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 2000). Standards on Data Analysis and Probability: Grades 6-8, 248-255. Reproduced with permission from the publisher.

Note 3. If you're working in a group, make a three-column chart with the labels "Question," "Type of Data," and "Appropriate Data Display" for recording the group's responses to Problems B1 and B2.

Part B, cont'd.

Problem B2. Consider topics of interest to students in grades 6-8 that involve collecting data about different characteristics within one population, and then formulate five questions that involve collecting quantitative (numerical) data. For each question, identify an appropriate way to display the data (e.g., line plot, bar graph, histogram, circle graph, stem and leaf plot, box plot, scatter plot) and describe how each display would be used to highlight potential relationships between the two characteristics.

Join the Discussion! www.learner.org

Post your answer to Problem B2 on an email discussion list, then read and respond to answers posted by others. Go to the *Data Analysis, Statistics, and Probability* Web site at www.learner.org/learningmath and find Channel Talk.

The next classroom investigation reveals students' understanding of the notion of average. Here's the scenario*:

> We took a survey of the prices of nine different brands of potato chips. For the same-sized bag, the typical or usual or average price for all brands was $1.38. What could the prices of the nine different brands be?

Note that the language used—words like typical, usual, or average—keeps the discussion open to various ways that students might think about the notion of average.

Problem B3. Consider how students might respond to this task and then develop three hypothetical student responses that are each based on a different measure of center—mode, median, and mean. **[See Note 4]**

The potato-chip task was presented to students in individual interviews to research students' understanding of average. Here are some of the students' responses:

a. Some students would put one price at $1.38, then one at $1.37 and one at $1.39, then one at $1.36 and one at $1.40, and so forth.

b. One student commented, "Okay, first, not all chips are the same, as you told me, but the lowest chips I ever saw was $1.30 myself, so, since the typical price is $1.38, I just put most of them at $1.38, just to make it typical, and highered the prices on a couple of them, just to make it realistic."

c. One student divided $1.38 by nine, resulting in a price close to 15¢. When asked if pricing the bags at 15¢ would result in a typical price of $1.38, she responded, "Yeah, that's close enough."

d. When some students were asked to make prices for the potato-chip problem without using the value $1.38, most said that it could not be done.

e. One student chose prices by pairing numbers that totaled $2.38, such as $1.08 and $1.30. She thought that this method resulted in an average of $1.38.

Problem B4. For each response above, was the student reasoning about the "average" as a mode, median, or mean?

* The potato-chip activity is adapted from *Teaching Children Mathematics*. Copyright 1996 by the National Council of Teachers of Mathematics. Used with permission of the National Council of Teachers of Mathematics.

Note 4. If you're working in a group, discuss ideas for using different measures of center to solve the potato-chip task. You might also want to review the statistical ideas of median and mean (Session 2, Part D and Session 5, Part A).

Part B, cont'd.

Problem B5. Read the article "What Do Children Understand About Average?" by Susan Jo Russell and Jan Mokros from *Teaching Children Mathematics.*

a. What further insights did you gain about children's understanding of average?

b. What are some implications for your assessment of students' conceptions of average?

c. What would be an example of a "construction" task and an "unpacking" task?

d. Why might you want to include some "construction" and "unpacking" tasks into your instructional program?

This reading is available as a downloadable PDF file on the *Data Analysis, Statistics, and Probability* Web site. Go to:

www.learner.org/learningmath

Russell, Susan Jo and Mokros, Jan (February, 1996). "What Do Children Understand About Average?" Edited by Donald L. Chambers. *Teaching Children Mathematics,* 360-364. Reproduced with permission from *Teaching Children Mathematics.*

Part C: Inferences and Predictions (35 minutes)

The NCTM (2000) data analysis and probability standards* state that students should "develop and evaluate inferences and predictions that are based on data." In grades 6-8 classrooms, students are expected to develop and evaluate inferences and predictions in order to do the following:

- Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken
- Make conjectures about possible relationships between two characteristics of a sample on the basis of scatter plots of the data and approximate lines of fit
- Use conjectures to formulate new questions and plan new studies to answer them

Inference and prediction are more advanced aspects of working with data, as they require an understanding of sampling. Students in grades 6-8 are developing an understanding of the idea of sampling. They often still expect their own intuition to be more reliable than the information they are obtaining from the data. Students begin to develop an understanding of these statistical ideas through conversations as they consider what the data are telling us, what might account for these results, and whether this would be true in other similar situations. Students' early experiences are often with census data—e.g., the population of their class. When they begin to wonder what might be true for other classes and other schools, they begin to develop the skills of inference and prediction. In the later middle grades and in high school, students begin to learn ways of quantifying how certain one can be about statistical results.

When viewing the video segment, keep the following questions in mind:

- How does Mr. Sowden encourage students to make inferences and predictions?
- What are some of the students' preliminary conclusions?
- Which of the students' inferences are based on the data, and which are based on their own personal judgement?

Video Segment (approximate times: 48:33-51:35): You can find this segment on the session video approximately 48 minutes and 33 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, Paul Sowden asks the students to look for patterns in the four line plots and to try to determine where the coins with no marking were minted. Students discuss the variance in the data and speculate on why the coins have no mint marks and about where those coins might have been minted.

Problem C1. Answer the questions you reflected on as you watched the video:

a. How does Mr. Sowden encourage students to make inferences and predictions?

b. What are some of the students' preliminary conclusions?

c. Which of the students' inferences are based on the data, and which are based on their own personal judgement?

* *Principles and Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 2000). Standards on Data Analysis and Probability: Grades 6-8, 248-255. Reproduced with permission from the publisher.

Part C, cont'd.

Problem C2. In the video segment from Part A, students considered why this set of coins contained more coins from Philadelphia. One student hypothesized that this was because Philadelphia is the closest U.S. Mint. This student was beginning to think about the sample of coins the students were using. How might you facilitate a discussion with your students about bias in data and the extent to which a data set can be representative? What questions would you pose? What issues would you raise?

Problem C3. If you were teaching this lesson on investigating nickels and their mint marks, what questions might you ask to focus students' attention on each of the following central elements of statistical analysis: **[See Note 5]**

- Defining the population
- Defining an appropriate sample
- Collecting data from that sample
- Describing the sample
- Making reasonable inferences relating the sample and the population

Problem C4. In the video, the students used circle graphs and line plots to examine the variation in their data. The teacher plans to continue analyzing the variation in the data, using different types of representations. What other types of representations might he use to examine the data?

Problem C5. How could you extend the discussion in this video segment to bring out more speculations about the nickels? How might you formalize these notions into stated conjectures that could be investigated further? What are some conjectures that might arise? How could you formulate them into new questions? How could these questions then be investigated?

Join the Discussion! www.learner.org

Post your answer to Problems C3 and C5 on an email discussion list, then read and respond to answers posted by others. Go to the *Data Analysis, Statistics, and Probability* Web site at www.learner.org/learningmath and find Channel Talk.

Note 5. You might want to review the statistical ideas of samples and populations (Session 1, Part D).

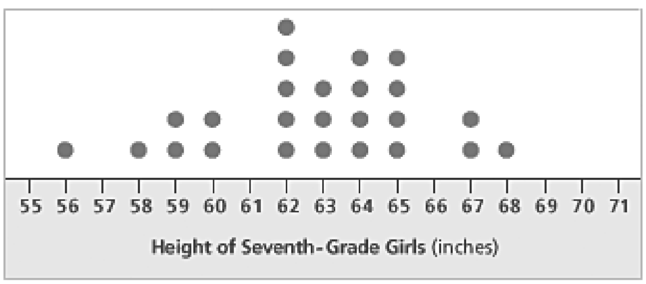
Part D: Examining Children's Reasoning (30 minutes)

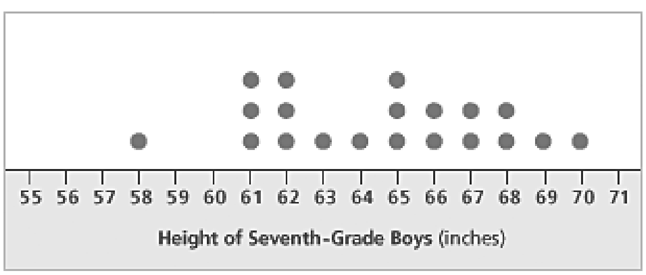
As this course comes to a close and you reflect on ways to bring your new understanding of data analysis, statistics, and probability into your teaching, you have both a challenge and an opportunity: to enrich the mathematical conversations you have with your students around data. As you are well aware, some students will readily grasp the statistical ideas being studied, and others will struggle.

The problems in Part D describe scenarios from a classroom case study involving children's developing statistical ideas. Some student comments are given for each scenario. For each student, comment on the following:

- *Understanding:* What does the statement reveal about the student's understanding or misunderstanding of statistical ideas? Which statistical ideas are embedded in the student's observations?
- *Next Instructional Moves:* If you were the teacher, how would you respond to each student? What questions might you ask so that students would ground their comments in the context? What further tasks and situations might you present for each child to investigate? **[See Note 6]**

Problem D1. Mr. Shapple teaches two sections of seventh-grade math. Students in both groups were trying to determine the typical height of a seventh-grade girl and a seventh-grade boy. They measured their heights in inches, combined their data, and then displayed the data on the line plots below:





After comparing the two data sets, here is what the students had to say:

a. Gregory: "The boys are taller than the girls."

b. Marie: "Some of the boys are taller than the girls, but not all of them."

c. Arketa: "I think we should make box plots so it would be easier to compare the number of boys and girls."

d. Michael: "The median for the girls is 63 and for the boys it's 65, so the boys are taller than the girls, but only by two inches."

e. Paul [reacting to Michael's statement]: "I figured out that the boys are two inches taller than the girls, too, but I figured out that the median is 62 for the girls and 64 for the boys."

f. Kassie: "The mode for the girls is 62, but for the boys, there are three modes—61, 62, and 65—so they are taller and shorter, but some are the same."

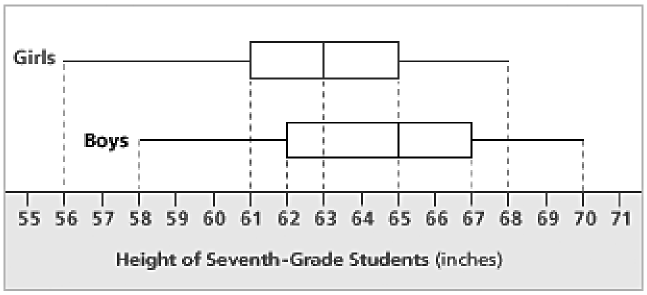
g. DeJuan: "But if you look at the means, the girls are only 62.76 and the boys are 64.5, so the boys are taller."

h. Carl: "Most of the girls are bunched together from 62 to 65 inches, but the boys are really spread out, all the way from 61 to 68."

Note 6. If you're working in a group, make a two-column chart with the labels "Understanding" and "Next Instructional Moves" for recording the group's responses. You might also want to review the process of making a box plot (Session 4, Part D).

Part D, cont'd.

Problem D2. The class then made box plots, building on Arketa's suggestion. This new representation gave them another way to compare and discuss the variance between the two data sets as they analyzed the heights of seventh-grade boys and girls. Here are the box plots they made:



As they compared the box plots, students made the following comments:

a. Arketa: "There is a lot of overlap in heights between the boys and girls."

b. Michael: "We can see that the median for the boys is higher than for the girls."

c. Monique: "It looks like just 12.5% of the boys are taller than all of the girls, and maybe about 10% of the girls are shorter than the shortest boy."

d. Gregory: "The boys are taller than the girls, because 50% of the boys are taller than 75% of the girls."

e. Morgan: "You can see that the middle 50% of the girls are more bunched together than the middle 50% of the boys, so the girls are more similar in height."

f. Janet: "Why isn't the line in the box for the boys in the middle like it is for the girls? Isn't that supposed to be for the median, and the median is supposed to be in the middle?"

Problem D3. To encourage students to discuss ideas of sampling and population, Mr. Shapple asked them to think about what they could say about other classes of seventh graders. Here are some of their responses:

a. Kassie: "I think we would get similar results if we collected data from all the seventh graders in the school district."

b. Nichole: "I think our data would be spread out more if we got data from seventh graders from all over the country, because then there would be more short kids and more taller kids; we're probably more in the middle."

c. Charles: "I think the boys would still be taller, on average, than the girls."

d. Carl: "I think our data would be similar to other seventh graders in our country, but I don't think we can say much about seventh graders in other countries."

Homework

Problem H1. Read the Grades 6-8 standard on data analysis and probability that was developed by the National Council of Teachers of Mathematics and is reported in the *Principles and Standards for School Mathematics* (NCTM, 2000).

a. After reading this standard, what additional connections do you see between the content you studied in this course and implications for your classroom teaching?

b. What are some insights you acquired about the development of students' understanding of data analysis, statistics, and probability from grades 6 through 8?

c. What are three important ideas you want to remember from the standards when teaching data analysis?

This reading is available as a downloadable PDF file on the *Data Analysis, Statistics, and Probability* Web site. Go to:

www.learner.org/learningmath

Principles and Standards for School Mathematics (Reston, VA: National Council of Teachers of Mathematics, 2000). Standards on Data Analysis and Probability: Grades 6-8, 248-255.

Problem H2. Assume that you need to report back to your grade-level team or to the entire school staff at a faculty meeting on your experiences and learning in this course. What are the main messages about the teaching of data analysis, statistics, and probability you would share with your colleagues? Prepare a one-page handout or an overhead or slide that could be distributed or shown at the meeting.

Problem H3. Look at a lesson or activity in your own mathematics program for your grade level that you think has potential for developing students' statistical reasoning. If you were to use this lesson or activity now, after taking this course, how might you modify or extend it to bring out more of the important ideas about data analysis, statistics, and probability?

Solutions

Part A: Statistics As a Problem-Solving Process

Problem A1.

a. The question the students are trying to answer is, "What is the relative frequency of each type of mint mark?"

b. The teacher brought a collection of nickels to the class so that students could examine the coins' mint marks.

c. The students organized their data into a circle graph.

d. The students are developing conjectures about the relative frequency of each mint mark.

e. The students are using their knowledge of fractions as they explore this problem. The investigation of mint marks involves connections to social studies.

f. Some statistical ideas are the nature of data, qualitative variables, variation, relative frequency, sampling, making a circle graph, and interpreting a circle graph.

Problem A2. The students are working with qualitative (categorical) data.

Problem A3. Answers will vary. One question might be, "Is this sample of nickels representative of the population of nickels?"

Problem A4. A circle graph is an appropriate way to display categorical data. Circle graphs show the fractional relationship of each category or part of data to the whole data set.

Part B: Developing Statistical Reasoning

Problem B1. Answers will vary. Two questions that involve qualitative (categorical) data that could be displayed with bar graphs and might interest students are, "What is your favorite type of music?" and "What is your favorite musical group?" Two questions involving quantitative (numerical) data that could be displayed on line plots are, "How many hours do you spend per week in chat rooms?" and "How much money do you spend on CDs each month?"

Problem B2. One such question might be, "What is the relationship between grade point average and the number of hours a student studies?" A scatter plot would be an appropriate way to display this data.

Problem B3.

- A response based on the mode might be to make the prices of all nine bags exactly \$1.38. Another response based on the mode is to price four bags at \$1.38 and the others at \$1.30, \$1.32, \$1.36, \$1.37, and \$1.50. The reasoning is to place more bags at \$1.38 than at any other price.
- A response that is based on the median is to make three bags cost \$1.38 and the others cost \$1.30, \$1.30, \$1.35, \$1.40, \$1.47, and \$1.49. The reasoning is to put some bags at \$1.38 and then to place an equal number of bags at prices lower and higher than \$1.38. Here, three bags cost more than \$1.38 and three bags cost less than \$1.38.
- A response that is based on the mean is to make the bags cost \$1.38, \$1.37, \$1.39, \$1.36, \$1.40, \$1.35, \$1.41, \$1.34, and \$1.42. Since there's an odd number of bags, the reasoning is to place one bag at \$1.38 and then add and subtract the same amount to create new prices. Here, 1 cent was subtracted from \$1.38 to get \$1.37, then 1 cent was added to \$1.38 to get \$1.39, and so on.

Solutions, cont'd.

Problem B4.

a. Median
b. Mode
c. Mean
d. Mode or median
e. Mean

Problem B5. Answers will vary. You may want to use the suggestions for action research to assess your own students' understanding of average. How would they respond to the potato-chip task?

Part C: Inferences and Predictions

Problem C1. Here are some possible answers:

a. The teacher asks the students to consider reasons why some coins do not have mint marks. Then he pushes them to think about which mints can likely be eliminated based on the information they see across the four line plots.

b. Some students think the coins with no mint marks were just mistakes. Others think that those coins are likely to be from Philadelphia, because so many nickels contain the Philadelphia mint mark. When thinking about which coins could be eliminated, the students eliminate Denver first and then San Francisco.

c. At first, students are reasoning more from their own personal judgement, such as when one student says that the missing mint marks are a mistake because the machines were working too fast. Other students' comments were more grounded in the data.

Problem C2. Two questions you might ask are, "Where do you think I got these coins?" and "How might that affect our results?"

Problem C3. Answers will vary. Here are two examples:

- How well do you think our sample of nickels represents the population of nickels in this country?
- If we lived near San Francisco and collected nickels, how might our results be different? How about if we lived near Denver?

Problem C4. The teacher might have the students make box plots for each category of coin mint mark.

Problem C5. One example of a conjecture that students might make is "Philadelphia produces more nickels than the other mints." This could be formulated as a new question to be investigated: "Does Philadelphia produce more nickels than the other mints?" The students could investigate this question in several ways. They might want to just enlarge their own sample of nickels, with each student collecting nickels over the next week for further analysis. This could also evolve into an Internet project in which your students contact students in other parts of the country, especially those who live closer to the other mints. Each group of students could collect and analyze a sample of nickels. They could then compare across samples and finally combine them into one larger sample.

Solutions, cont'd.

Part D: Examining Children's Reasoning

Problem D1.

a. Gregory does not quantify his statement and may only be looking at the upper extreme value. The teacher could ask Greg to determine "how much taller" the boys are than the girls.

b. Marie is comparing the variation in the data and notices overlap in the values. The teacher might ask her to quantify her response.

c. Arketa is considering other representations that might make certain patterns and relationships between the data sets more apparent. The teacher could ask the class to consider additional ways to represent the data that would make some comparisons more visible.

d. Michael correctly determines the median for each data set and quantifies "how much taller" the boys are than the girls by comparing the medians of the data sets. The teacher might ask the other students to react to Michael's statement and then consider why it can be useful to compare the medians of two data sets.

e. Paul quantifies "how much taller" the boys are than the girls by comparing what he thinks are the medians of the data sets; what he found, though, was the middle of each *range* and not the middle of the data. This is an opportunity for the teacher to review the meaning of median as well as ways to find the median of a set of ordered data.

f. Kassie believes that she is comparing the modes of the data sets, but when three or more values have the same number of data points, such as the boys, the data is considered not to have a mode. The teacher can review the meaning of mode and ask the students to speculate as to why statisticians say that a data set doesn't have a mode when three or more values have the same number of data points.

g. DeJuan correctly calculates the means and quantifies "how much taller" the boys are than the girls by comparing the means of the data sets. The teacher could now have the students compare the medians and means of the two data sets. What does each tell us about the data? In this situation, is one comparison more appropriate than the other one? Why or why not?

h. Carl is comparing intervals of the two data sets that contain the most data. The teacher could take this opportunity to focus further attention on the importance of examining intervals in considering how the data are spread out or bunched together.

Problem D2.

a. Arketa is comparing the variation by looking at the range of each data set. The teacher might ask her to quantify her response.

b. Michael compares the data sets by looking at the medians. The teacher could ask Michael to point to the median on each box plot and then review that 50%, or half, of the data box plot is on each side of the median.

c. Monique incorrectly reasons that one can further subdivide the lines (or boxes) and that a fractional part of a line reflects a fractional part of the data. The teacher should ask Monique how she arrived at those percentages and then show this same finding on the line plot to see if she recognizes the discrepancy.

d. Gregory is correctly reasoning about the box plots with quartiles. The teacher might ask the rest of the class to evaluate Gregory's statement for its accuracy.

e. Morgan is correctly reasoning about the spread of the middle 50% of the data on the box plots. The teacher might ask the rest of the class to evaluate the accuracy of Morgan's statement.

f. Janet does not understand how the box represents quartiles of the data. The teacher could go back to the line plots of the data and actually draw the box plot directly over the data so that Janet can see the distribution of the data within the quartiles of the box plot.

Solutions, cont'd.

Problem D3.

a. Kassie thinks that her class's sample is representative of the district if the district were defined as the population. The teacher could ask the other students to give reasons for supporting or refuting Kassie's conjecture.

b. Nichole uses personal judgement about her class's data probably being "more in the middle," but she is correct in thinking that a larger sample would increase the range of the data, as a larger sample might reveal that her classmates are more homogenous in comparison to other seventh-grade classes, if the population were defined as the country. The teacher might ask Nichole why she thinks her class is "more in the middle," and then ask the rest of the class to react to her conjecture.

c. Charles is thinking about measures of center. The teacher might ask Charles to explain further what he means by "on average." Is he referring to the mode, the median, or the mean?

d. Carl thinks that their sample is representative of the country if the country were defined as the population, but not if the population included seventh graders from other countries. The teacher might use this as an opportunity to discuss further reasons for defining the population being investigated.

Notes