Session 10

Classroom Case Studies, Grades 3-5

This is the final session of the ***Data Analysis, Statistics, and Probability*** course! In this session, we will examine how statistical thinking might look when applied to situations in your own classroom. This session is customized for three grade levels. Select the grade level most relevant to your teaching.

The session for grades 3-5 begins below. Go to page 291 for grades K-2 and page 323 for grades 6-8.

Key Terms for This Session

Previously Introduced

- bias
- census
- data
- distribution
- line plot
- mean
- median
- mode
- population
- sample
- variation

Introduction

In the previous sessions, we explored statistics as a problem-solving process that seeks answers to questions through data. You put yourself in the position of a mathematics learner, both to analyze your individual approach to solving problems and to get some insights into your own conception of statistical reasoning. It may have been difficult to separate your thinking as a mathematics learner from your thinking as a mathematics teacher. Not surprisingly, this is often the case. In this session, however, we will shift the focus to your own classroom and to the approaches your students might take to mathematical tasks involving statistics. **[See Note 1]**

As in other sessions, you will be prompted to view short video segments throughout the session; you may also choose to watch the full-length video for this session.

Learning Objectives

In this session, you will do the following:

- Explore the development of statistical reasoning at your grade level
- Analyze the use of the four-step process for solving statistical problems in a classroom at your grade level
- Review mathematical tasks and their connection to the mathematical themes in the course
- Examine children's understanding of statistical concepts

Note 1. This session uses classroom case studies to examine how children at your grade level think about and work with data. If possible, work on this session with another teacher or a group of teachers. A group discussion will allow you to use your own classroom, and the classrooms of fellow teachers, as case studies to make additional observations.

The suggested times for this session allow time for personal reflection and group discussion.

Part A: Statistics As a Problem-Solving Process (30 minutes)

A data investigation should begin with a question about a real-world phenomenon that can be answered by collecting data. After the children have gathered and organized their data, they should analyze and interpret the data by relating the data back to the real-world context and the question that motivated the investigation in the first place. Too often, classrooms focus on the techniques of making data displays without engaging children in the process. However, it is important to include children in all aspects of the process for solving statistical problems. The process studied in this course consisted of four components:

1. ***Ask a question.***
2. ***Collect appropriate data.***
3. ***Analyze the data.***
4. ***Interpret the results.***

Children often talk about numbers out of context and lose the connection between the numbers and the real-world situation. During all steps of the statistical process, it is critical that students not lose sight of the questions they are pursuing and of the real-world contexts from which the data were collected.

When viewing the video segment, keep the following questions in mind: **[See Note 2]**

- How do the students in this classroom apply the first two components of the statistical process? What statistical question are the students trying to answer? How were the data collected?
- As the fifth graders move onto the next two components of the statistical process—analysis and interpretation—what issues do you think will come up?
- Thinking back to the big ideas of this course, what are some statistical ideas these students are likely to encounter through their investigation of this situation?

Video Segment (approximate times: 29:48-30:39): You can find this segment on the session video approximately 29 minutes and 48 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, teacher Suzanne L'Esperance applies the mathematics she learned in the *Data Analysis, Statistics, and Probability* course to her own teaching situation. She starts by establishing the context for her students to investigate family size, telling them about her friend who is in construction and how he needs help from the students in the class. The students consider the context and then begin to collect the data.

Note 2. Before examining specific problems at this grade level with an eye toward statistical reasoning, you will watch a teacher (who has also taken the course) teaching in her classroom. The purpose in viewing the video is not to reflect on the teacher's methods or teaching style. Instead, look closely at how the teacher brings out statistical ideas while engaging her students in statistical problem solving.

You might want to review the four-step process for solving statistical problems (Session 1, Part A). What are the four steps? What characterizes each step?

Part A, cont'd.

Problem A1. Answer the questions you reflected on as you watched the video:

a. What statistical question are the students trying to answer?

b. How did the students collect their data?

c. As the students move on to analysis and interpretation of their data, what issues do you think will come up?

d. What statistical ideas are students likely to encounter as they investigate this situation?

Problem A2. In this video, Ms. L'Esperance establishes a rich and elaborate real-world context to situate the students' investigation of family size. How do you think the class would have responded if she had not constructed a context for the investigation and instead had simply said, "Today we are going to investigate family size; how many people are in your family?" What is the impact on the students' level of engagement?

Problem A3. Too often, students lose the connection between the numbers and the real-world situation once they have gathered their data. How might the richer context provided by Ms. L'Esperance reinforce the connection between the data and the real-world phenomenon being studied, and prevent students from working with mere numbers out of context?

Problem A4. What are some ways in which this richer context will support students' reasoning as they "interpret the results"?

Problem A5. Why do you think Ms. L'Esperance phrased the question about family size as "How many people live in the house that you slept in last night?," as opposed to simply "How many people are in your family?" With your own students, how would you define "family"? **[See Note 3]**

When engaging students in the process of statistical problem solving, students must consider what to measure and how to measure it to ensure accuracy in collecting their data. In this lesson, Ms. L'Esperance defined "family" for her students. But it is also important to give students a chance to form—or to help form—their own definitions for the purpose of their investigations.

Problem A6. How would you facilitate a discussion with your students on what constitutes a "family"? Describe some of the sensitive issues that might arise and how you would handle them.

Note 3. The 2000 United States Census defines a household as one or more persons living in a housing unit. One person who owns or rents the residence is designated as the householder. For the purposes of examining family and household composition, two types of households are defined: family and non-family. A family household has at least two members related by blood, marriage, or adoption, one of whom is the householder. A non-family household can either be a person living alone or a householder who shares the housing unit with non-relatives only—for example, boarders or roommates. The non-relatives of the householder may be related to one another.

Part B: Developing Statistical Reasoning (45 minutes)

The National Council of Teachers of Mathematics (NCTM, 2000) identifies data analysis and probability as a strand in its *Principles and Standards for School Mathematics*.* In grades pre-K through 12, instructional programs should enable all students to do the following:

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Select and use appropriate statistical methods to analyze data
- Develop and evaluate inferences and predictions that are based on data
- Understand and apply basic concepts of probability

In grades 3-5 classrooms, students are expected to use appropriate statistical methods to do the following:

- Describe the shape and important features of a data set and compare related data sets, with an emphasis on how the data are distributed
- Use measures of center, focusing on the median, and understand what each does and does not indicate about the data set
- Compare different representations of the same data and evaluate how well each representation shows important aspects of the data

In grades 3-5, children readily notice individual data points and are able to describe parts of the data—where their own data falls on the graph, which value occurs most frequently, and which values are the largest and smallest. A significant development in children's understanding occurs as they begin to think about the set of data as a whole. Our goal for children is for them to see a data set as a distribution of values with important features, such as center, spread, and shape.

To focus students' attention on the shape and distribution of the data, it is helpful to build from children's informal language to describe where most of the data are, where there are no data, and where there are isolated pieces of data. The words *clusters, clumps, bumps*, and *hills* highlight concentrations of data. The words *gaps* and *holes* emphasize places in the distribution that have no data. The phrases *spread out* and *bunched together* underscore the overall distribution. Teachers must also continually emphasize and help students see that what they notice about the shape and distribution of the data implies something about the real-world phenomena being studied.

In grades 3–5, students learn to use measures of center to summarize a data set. Building on children's informal understanding of what is the most, what is the middle, and what is typical, teachers can help students develop understanding about the mode, median, and the mean. But students need to learn more than simply how to identify the mode or median in a data set and how to find the mean: They need to develop an understanding of what these measures of center tell us about the data, and what each does and does not indicate about the data set. The emphasis in these grade levels should be on the median, with informal exploration of the mean. Children can see where the median is located among the data, but the mean is much more abstract, as it has no clear identity within the data themselves.

When viewing the video segment, keep the following questions in mind:

- Thinking back to the big ideas of this course, what are some statistical ideas that these students are developing?
- What questions could be posed to determine the extent of students' understandings of what the mode, mean, median, and range do and do not indicate about the data set?

Part B, cont'd.

Video Segment (approximate times: 35:35-37:07): You can find this segment on the session video approximately 35 minutes and 35 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, Suzanne L'Esperance selects a group of students to present their findings. Each group of students created a line plot of the class data on family size and determined the mode, median, mean, and range for the data set. Watch as the group of students takes turns presenting the summary information to the class.

Problem B1. Answer the questions you reflected on as you watched the video:

a. What statistical ideas are these students developing?

b. What questions could you pose to determine the extent of students' understanding of what the mode, mean, median, and range do and do not indicate about the data set?

Join the Discussion! **www.learner.org**

Post your answer to Problem B1 on an email discussion list, then read and respond to answers posted by others. Go to the *Data Analysis, Statistics, and Probability* Web site at www.learner.org/learningmath and find Channel Talk.

Problem B2. This line plot (or dot plot) below displays the family-size data collected by the students in Ms. L'Esperance's fifth-grade classroom. Imagine yourself in a conversation with the children about this data. A key question you might ask the students is, "What do you notice about the data?" Using the informal language of clusters, clumps, bumps, hills, gaps, holes, spread out, or bunched together, write five statements that you hope students would make describing the set of data as a whole. **[See Note 4]**

Too often, children describe the data as numbers devoid of context. Another question you should frequently ask students regarding their observations is, "What does that tell us about the family size?"

Note 4. Using line plots (dot plots) in elementary classrooms is a fairly new practice. Consider how you might use this graphical representation of data with your students. How does this compare with your current method of presenting data?

Part B, cont'd.

Problem B3. For each of the five statements you wrote in Problem B2, indicate what that observation might imply about the real-world context of family size.

In another classroom investigation which reveals students' understanding of the notion of "average," students were given the following scenario*:

> We took a survey of the prices of nine different brands of potato chips. For the same-sized bag, the typical or usual or average price for all brands was $1.38. What could the prices of the nine different brands be?

Note that the language used—words like typical, usual, or average—keeps the discussion open to various ways that students might think about the notion of average.

Problem B4. Consider how students might respond to this task and then develop three hypothetical student responses that are each based on a different measure of center—mode, median, and mean. **[See Note 5]**

The potato-chip task was presented to fourth-grade students in individual interviews to research students' understanding of average. Here are some of the students' responses:

a. Some students would put one price at $1.38, then one at $1.37 and one at $1.39, then one at $1.36 and one at $1.40, and so forth.

b. One student commented, "Okay, first, not all chips are the same, as you told me, but the lowest chips I ever saw was $1.30 myself, so, since the typical price is $1.38, I just put most of them at $1.38, just to make it typical, and highered the prices on a couple of them, just to make it realistic."

c. One student divided $1.38 by nine, resulting in a price close to 15¢. When asked if pricing the bags at 15¢ would result in a typical price of $1.38, she responded, "Yeah, that's close enough."

d. When some students were asked to make prices for the potato-chip problem without using the value $1.38, most said that it could not be done.

e. One student chose prices by pairing numbers that totaled $2.38, such as $1.08 and $1.30. She thought that this method resulted in an average of $1.38.

Problem B5. For each response above, was the student reasoning about the "average" as a mode, median, or mean?

Problem B6. Read the article "What Do Children Understand About Average?" by Susan Jo Russell and Jan Mokros from *Teaching Children Mathematics*.

a. What further insights did you gain about children's understanding of average?

b. What are some implications for your assessment of students' conceptions of average?

This reading is available as a downloadable PDF file on the *Data Analysis, Statistics, and Probability* Web site. Go to:

www.learner.org/learningmath

* The potato-chip activity is adapted from *Teaching Children Mathematics*. Copyright 1996 by the National Council of Teachers of Mathematics. Used with permission of the National Council of Teachers of Mathematics.

Note 5. You might want to review the statistical ideas of median and mean (Session 2, Part D and Session 5, Part A).

Part C: Inferences and Predictions (30 minutes)

The NCTM (2000) data analysis and probability standards* state that students should "develop and evaluate inferences and predictions that are based on data." In grades 3-5 classrooms, students are expected to develop and evaluate inferences and predictions, to propose and justify conclusions and predictions that are based on data, and to design studies to further investigate their conclusions or predictions.

Inference and prediction are more advanced aspects of working with data, as they require some notion of the ideas of sampling and population. Students in grades 3-5 are only beginning to develop an understanding of sampling. They often trust their own intuition more than the information they are obtaining from the data. Children begin to develop an understanding of these statistical ideas through conversations as they consider what the data are telling us, what might account for these results, and whether this would be true in other similar situations. Children's early experiences are often with census data, i.e., the population of their class. When they begin to wonder what might be true for other classes in their own and other schools, they begin to develop the skills of inference and prediction. It is not until the later middle grades and high school that students begin to learn ways of quantifying how certain one can be about statistical results. **[See Note 6]**

When viewing the video segment, keep the following questions in mind:

- How does Ms. L'Esperance encourage students to make inferences and predictions?
- What are some of the students' preliminary conclusions?
- How are the ideas of sampling and population embedded in this conversation?

Video Segment (approximate times: 38:33-40:23): You can find this segment on the session video approximately 38 minutes and 33 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, Suzanne L'Esperance facilitates a whole-class discussion as the students consider potential conclusions to the original problem on how large to build the house. Students discuss the variance in their data, the limitations of their small sample, and the need for additional data.

Problem C1. Answer the questions you reflected on above as you watched the video:

a. How does Ms. L'Esperance encourage students to make inferences and predictions?

b. What are some of the students' preliminary conclusions?

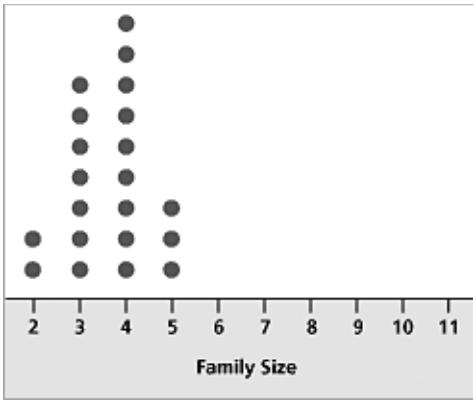
c. How are the ideas of sampling and population embedded in this conversation?

* *Principles and Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 2000). Standards on Data Analysis and Probability: Grades 3-5, 176-181. Reproduced with permission from the publisher. Copyright 2000 by the National Council of Teachers of Mathematics. All rights reserved.

Note 6. You might want to review the statistical ideas of bias in measurement, samples, and populations, and the meaning of "conjecture," possibly providing one or two examples (Session 1, Part D).

Part C, cont'd.

Problem C2. Based on the family-size data gathered by the class and shown in the line plot at right, how would you respond to Ms. L'Esperance's initial question: What size should she tell her friend to build his homes? What reasons can you offer to support this response, and how are they related to the ideas you have studied in this course? Are your reasons based on the data collected, or did you also bring in some of your own judgements?



Problem C3. Children are expected to develop and evaluate inferences and predictions. Evaluate each of the responses below by commenting on the following:

- why the response makes sense (or doesn't) based on the data; and
- the limitations of each response. In other words, what statistical ideas are the children not taking into account?

The children's responses to the question of how big to build the homes were as follows:

a. "He should build homes for four people."

b. "You can tell him to put in a couple of each, because some people live with two people, so he should put more fours and threes, but put some of the other kinds also."

c. "He should build them for three people and four people."

d. "I know some people that have six and eight people in their families, so he should build some larger houses, too."

Problem C4. In thinking about the data that were collected, in what ways might the students' sample be biased? How might you facilitate a discussion with the students about bias in data? What questions would you pose? What issues would you raise?

Problem C5. According the 2000 census, the average size of households in the United States is 2.62 people. How might your students respond to this information in light of their own data? What statistical ideas would you want to surface in this discussion?

Problem C6. If you were teaching this lesson on investigating family size, what questions could you ask students to encourage them to focus on each of these central elements of statistical analysis?

- Defining the population
- Defining an appropriate sample
- Collecting data from that sample
- Describing the sample
- Making reasonable inferences relating the sample and the population

Join the Discussion! **www.learner.org**

Post your answer to Problem C6 on an email discussion list, then read and respond to answers posted by others. Go to the *Data Analysis, Statistics, and Probability* Web site at www.learner.org/learningmath and find Channel Talk.

Problem C7. A student commented that the class should "wait until we get more information" before making a recommendation to Ms. L'Esperance's friend. How could you extend this conversation to bring out more predictions and then formalize these notions into stated conjectures that could then be investigated further? What questions would you ask? What are some conjectures that might result? How could these be investigated?

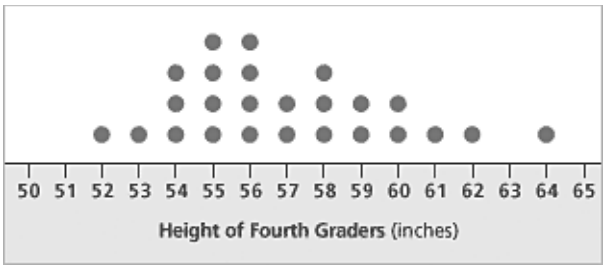
Part D: Examining Children's Reasoning (30 minutes)

As this course comes to a close and you reflect on ways to bring your new understandings of data analysis, statistics, and probability into your teaching, you have both a challenge and an opportunity: to enrich the mathematical conversations you have with your students around data. As you are well aware, some students will readily grasp the statistical ideas being studied, and others will struggle.

The problems below describe scenarios from a classroom case study involving children's developing statistical ideas. Some student comments are given for each scenario. For each student in Problems D1-D3, comment on the following:

- *Understanding:* What does the statement reveal about the student's understanding or misunderstanding of statistical ideas? Which statistical ideas are embedded in the student's observations?
- *Next Instructional Moves:* If you were the teacher, how would you respond to each student? What questions might you ask so students would ground their comments in the context? What further tasks and situations might you present for each child to investigate? **[See Note 7]**

Problem D1. Ms. Johnson's fourth-grade class was examining height. They measured their heights in inches and then displayed their data on the line plot below.



After plotting their data, here's what the students had to say:

a. Damon: "The tallest person is 64 inches, and the shortest person is 52."

b. Juanita: "I think 64 is an outlier, because there's a gap at 63."

c. Asher: "We don't have a mode, because 55 and 56 are the same."

d. Larie: "Most of us are from 54 to 58 inches tall."

e. Michael: "The median is 58 because it's the middle of the range."

f. Ali: "The range from 50 to 65 is 15."

g. Antrell: "I think the median is 57, because it's the middle of our heights."

Note 7. If you're working in a group, make a two-column chart with the labels "Understanding" and "Next Instructional Moves" for recording the group's responses to Problems D1-D3.

Part D, cont'd.

Problem D2. Ms. Johnson then told the class that they were going to measure the heights of the first-grade class. She asked the students, "What do you think will be true about the first graders?"

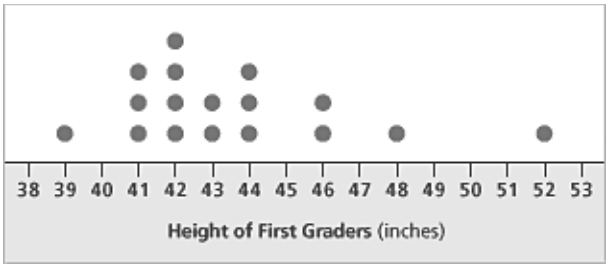
a. Ava: "I think they are going to all be shorter than us because they're only in first grade."

b. Nichole: "Maybe their data will be more bunched together than ours because it seems like lots of first graders are about the same height."

c. Houa: "They're going to be smaller than us so I would say [a typical height is] probably in the 40s."

d. Charles: "I think [a typical height is] maybe three feet, so that would be 36 inches."

Problem D3. The line plot shows the first graders' height measurements:



As the fourth-grade students compared their height data to that of the first graders, they made the following comments:

a. Asher: "They are lots shorter than us."

b. Charles: "They are taller than I thought because they are more like three and a half feet tall."

c. Ali: "Most of [the first graders] are 42 inches tall, but most of us are 55 or 56 inches, so we're 13 or 14 inches taller."

d. Nichole: "I think we're 13 inches taller because our median is 56 and their median is 43 ."

e. Tarra: "Wow, I didn't think any first graders would be as tall as us, but that kid is 52 inches tall."

f. Juanita: "Their heights are more spread out."

g. Larie [responding to Juanita's statement]: "I don't really think their heights are more spread out, because most of them are from 41 to 44 inches, and that's only three inches, but most of ours are more from 54 to 60, and that's six inches."

For more information about statistics problems for children like the problems in this session:

Russell, Susan Jo; Corwin, Rebecca B.; Rubin, Andee, Rubin; and Akers, Joan (1998). *The Shape of the Data*. Dale Seymour Publications.

Homework

Problem H1. Read the Grades 3-5 standard on data analysis and probability that was developed by the National Council of Teachers of Mathematics and is reported in the *Principles and Standards for School Mathematics* (NCTM, 2000).

a. After reading, what additional connections do you see between the content you studied in this course and the implications for your classroom teaching?

b. What are some insights you acquired about the development of children's understanding of data analysis, statistics, and probability from grades 3-5?

c. What are three important ideas you want to remember from the standards when teaching data analysis?

This reading is available as a downloadable PDF file on the *Data Analysis, Statistics, and Probability* Web site. Go to:

www.learner.org/learningmath

Problem H2. Assume that you need to report back to your grade-level team or to the entire school staff at a faculty meeting on your experiences and learning in this course. What are the main messages about the teaching of data analysis, statistics, and probability you would share with your colleagues? Prepare a one-page handout or an overhead or slide that could be distributed or shown at the meeting.

Problem H3. Look at a lesson or activity in your own mathematics program for your grade level that you think has potential for developing students' statistical reasoning. If you were to use this lesson or activity now, after taking this course, how might you modify or extend it to bring out more of the important ideas about data analysis, statistics, and probability?

Solutions

Part A: Statistics As a Problem-Solving Process

Problem A1.

a. The question is, "How many people are in a family?"

b. Each child was told to use connecting cubes to show the number of people that live in his or her house.

c. One might expect that the issue of "center" (median) would arise when determining the typical size of families, as well as the issue of variation in the data.

d. Some statistical ideas are the nature of data, quantitative variables, variation, range, measures of center, sampling, making a line plot, and interpreting a line plot.

Problem A2. It is likely that this rich context more fully engages children because there is a clear purpose for investigating family size. The context also increases the authenticity of the task. There is a real-world rationale for why a person in a specific profession would need to know the size of families or households in a particular area. While children might also be curious about the number of people in each of their families, they do not have a reason—other than their interest—to extend the investigation beyond their classroom.

Problem A3. The rich context grounds students more firmly in the situation. With the clear purpose of examining family size and the fact that the data are about them, they more readily analyze the data with the real-world situation in mind than they would if they were just thinking about numbers out of context.

Problem A4. Again, the rich context grounds students in the situation. They are invested in helping the teacher's friend and have a clear purpose for interpreting the data so that they can make an appropriate recommendation. The students are also able to draw on their own knowledge about the neighborhood and about family size. Thus, as they interpret the results, they are more likely to raise issues, think beyond their own classroom sample, and become curious about the larger population.

Problem A5. The teacher's phrase resembles the definition of a "household" as set by the 2000 United States census. Answers to the second question will vary, but might include "all the people you spend holidays with," "the people you're related to," and "your mother, father, sisters, and brothers."

Problem A6. Sensitive issues might involve brothers and sisters who no longer live in the same household, parents or siblings who have died, single- and dual-parent households, same-sex and different-sex guardians, or joint-custody situations. These are all aspects of people's lives, and are good examples of the importance of definitions when collecting data.

Part B: Developing Statistical Reasoning

Problem B1.

a. Statistical ideas included using the mode, median, and mean as measures of center, and the range as an indicator of variation.

b. Answers will vary. Examples of questions would be, "You stated that the range is 3. What does this tell us about the data set?" or "You said that the median is 4. If this is the only information I had asked you to figure out, what wouldn't I know about the data?"

Solutions, cont'd.

Problem B2. Here are some possible statements that children might make:

- There is a bump at 4.
- The data are really bunched together.
- There is a cluster at 3 and 4.
- There's a big gap from 6 to 11.
- The data are not very spread out.

Problem B3.

- The bump at 4 is the size of families that occurred most often for our class.
- Because the data are all bunched together, we know that families in our class are very similar in size.
- The cluster at 3 and 4 indicates that most families in our class have three or four people.
- The gap from 6 to 11 tells us that no families in our class have six, seven, eight, nine, 10, or 11 children.
- The lack of "spread" in our data tells us that our class's families are similar in size.

Problem B4.

- A response based on the mode might be to make the prices of all nine bags exactly $1.38. Another response based on the mode is to price four bags at $1.38 and the others at $1.30, $1.32, $1.36, $1.37, and $1.50. The reasoning is to place more bags at $1.38 than at any other price.
- A response that is based on the median is to make three bags cost $1.38 and the others cost $1.30, $1.30, $1.35, $1.40, $1.47, and $1.49. The reasoning is to put some bags at $1.38 and then to place an equal number of bags at prices lower and higher than $1.38. Here, three bags cost more than $1.38 and three bags cost less than $1.38.
- A response that is based on the mean is to make the bags cost $1.38, $1.37, $1.39, $1.36, $1.40, $1.35, $1.41, $1.34, and $1.42. Since there's an odd number of bags, the reasoning is to place one bag at $1.38 and then add and subtract the same amount to create new prices. Here, 1 cent was subtracted from $1.38 to get $1.37, then 1 cent was added to $1.38 to get $1.39, and so on.

Problem B5.

a. Median

b. Mode

c. Mean

d. Mode or median

e. Mean

Problem B6. Answers will vary. You may want to use the suggestions for action research to assess your own students' understanding of average. How would they respond to the potato chip task?

Solutions, cont'd.

Part C: Inferences and Predictions

Problem C1. Here are some possible answers:

a. Ms. L'Esperance encourages students to make inferences and predictions by focusing their attention on the problem context and asking them to make suggestions regarding what she should tell her friend about how big to build his homes.

b. Many of the children concluded that Ms. L'Esperance should tell her friend to build homes for four people. However, other children took into account the variance in the data and concluded that, while he should build some homes for two people, he should build the most homes for three or four people.

c. When the teacher asks the students to consider the number of data points collected (the sample size), she implicitly encouraged them to consider ideas of sampling and population.

Problem C2. The data make a strong case that homes should be built for families of size two, three, four, and five. You may agree with the students that four is an appropriate conclusion, but you probably also realize that this sample is very small and that more data should be gathered.

Problem C3.

a. This response makes sense in that it is based on the mode; the limitation is that it does not take into account the variation in the data.

b. This response takes into account the variation in the data.

c. This response is based on the two values with the greatest number of responses, so the student does consider variation in a narrow sense but does not take into account the limited sample.

d. In this context, this response doesn't make sense; the student has gone beyond the actual data involved and is considering issues of sampling and population.

Problem C4. The sample is biased in that, as children, they all live in households that contain at least two people; thus, households in which one person lives are not considered. Some questions a teacher might pose include, "Why doesn't our line plot show any families of size one?" and "Does anyone in your neighborhood live in a household with only one person?"

Problem C5. The students are likely to wonder why the average size of households is so much smaller than what their data indicated. You would want students to think about how their sample was collected and the bias or limitations inherent in their sample.

Problem C6. Here are some questions you might ask:

- What should we tell my friend about where this information came from and the part of our city in which he should build homes of this size?
- If my friend decides to build houses in another city, should they be the same size as the houses we think he should build here?

Problem C7. Two conjectures that might result are, "The typical family size in our area is four people" and "You will not find families in our area that have 10 people." These could be formulated as new questions to be investigated: "What is the typical family size in our area?" and "What is the range of family size in our area?" The students could investigate this question in several ways. They might want to survey students in other classes and grade levels in their school on family size, they might want to have each student survey 10 neighbors, or they might want to locate census data for their community.

Solutions, cont'd.

Part D: Examining Children's Reasoning

Problem D1.

a. Damon is focusing on the extreme values and the range. The teacher could reinforce the meaning of range.

b. Juanita incorrectly reasons that 64 is an outlier because there is no data point at 63. However, with 64 separated by only one inch from the other values, it is not unusual enough to be considered an outlier. The teacher might ask the rest of the class to discuss further the meaning of an outlier.

c. Asher incorrectly reasons that there can be a mode only when one value has more data points than any other value. In this example, 55 and 56 are both considered modes. The teacher might use this as an opportunity to discuss the meaning of mode.

d. Larie is looking at an interval of the range where most of the data is concentrated. The teacher might ask Larie to explain how she reached this conclusion and then ask the class to consider why it can be helpful to look at these smaller intervals of concentrated data.

e. Michael incorrectly reasons that 58 is the median by finding the middle of the range from 52 to 64 instead of the middle of the data points. The teacher might begin by reviewing the meaning of median and then ask all the children to line up from shortest to tallest and, using themselves, find the median. Now the class would need to discuss the discrepancy between the median Michael proposed and the one the class found.

f. Ali incorrectly thinks the range comprises all the numbers shown on the line plot, even when the numbers do not contain any values. This error arises if students are accustomed to seeing line plots that almost always begin and end with values that contain data. The teacher could use the following questioning to get Ali to focus on the data points: "What is the smallest height of someone in our class? Point to it on the line plot with your left hand. What is the tallest height of someone in our class? Point to it with your right hand. This distance from the lowest to the highest number is what we call the range."

g. Antrell correctly reasons about the median and finds it accurately. The teacher might want Antrell to show the class how he found the median and then ask the class to consider why it is important to find the median of a data set.

Problem D2.

a. Ava is thinking about the whole set of data about first graders in comparison to the fourth graders' set of data. The teacher might ask other students whether they agree and to explain why or why not.

b. Nichole is thinking about the spread of the data the class will collect. The teacher could ask the students to consider the spread of their own height data and to quantify their predictions for the range of height data of the first graders.

c. Houa is thinking about the range of the data the class will collect. The teacher might ask the students to get out some measuring tapes and mark on the wall the heights from 40 to 50; once the children have looked at these heights, the teacher can ask them to react to Houa's statement.

d. Charles is thinking about a specific value, most likely the mode. The teacher might ask Charles if he thinks all first graders, a few of them, or most of them will be three feet tall.

Solutions, cont'd.

Problem D3.

a. Asher does not define "lots," nor does she mention that one of the first graders is as tall as a fourth grader. The teacher could ask Asher to explain what she means by "lots" and ask whether all the first graders are shorter than all the fourth graders.

b. Charles is reasoning by focusing on the mode of 42 inches. The teacher could reinforce the meaning and use of the term mode.

c. Ali is quantifying "how much taller" the fourth graders are than the first graders by comparing the modes of the data sets. The teacher might ask Ali why she compared the modes and then ask the rest of the class to consider the advantages and disadvantages of this approach.

d. Nichole is quantifying "how much taller" the fourth graders are than the first graders by comparing the medians of the data sets. The teacher might ask Nichole why she compared the medians and then ask the rest of the class to consider the advantages and disadvantages of this approach as compared to using the modes.

e. Tarra is comparing the data sets and thinking about outliers as she reasons that it is unusual for a first grader to be 52 inches tall. This would be an opportunity to discuss further the meaning of outlier.

f. Juanita is comparing the range of the two data sets. The teacher could ask Juanita to explain how she reached this conclusion and what this tells us about the heights of first graders.

g. Larie is thinking about intervals of the data that contain most of the data. The teacher might use this as an opportunity to focus further attention on the importance of examining intervals when considering how the data are spread out or bunched together.