

Annenberg/CPB Course Guide

Learning Math

Data Analysis, Statistics, and Probability

A 10-part video- and Web-based course for K-8 teachers

Produced by WGBH Educational Foundation

***Learning Math:
Data Analysis, Statistics, and Probability***

is produced by
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About This Course

The *Learning Math* Courses

When teachers have a deep conceptual understanding of mathematics, it can help their students develop strong mathematical skills and knowledge. *Learning Math* is a series of five multimedia, college-level courses designed to teach mathematics content to elementary and middle school teachers. Organized around the content standards developed by the National Council of Teachers of Mathematics (NCTM), the courses will cover Number and Operations; Patterns, Functions, and Algebra; Geometry; Measurement; and Data Analysis, Statistics, and Probability.

Overall Objectives

- To help teachers better understand mathematics content
- To provide engaging explorations of mathematics using video, interactive activities, and problem solving
- To encourage teachers to view mathematics as more than rote sets of rules and procedures

Overview: *Data Analysis, Statistics, and Probability*

Data Analysis, Statistics, and Probability introduces statistics as a problem-solving process. In this course, you will build your skills through investigations of different ways to collect and represent data and to analyze and interpret variation in data. The course consists of 10 sessions, each with a half hour of video programming, problem-solving activities provided online and in this print guide, and interactive activities and demonstrations on the Web. Although each session includes suggested times for how long it may take to complete all of the required activities, these times are approximate. Some activities may take longer. You should allow at least two and a half hours for each session. The 10th session explores ways to apply the statistical concepts you've learned in K-8 classrooms. You should complete the sessions sequentially.

Session 1. Statistics As Problem Solving

Consider statistics as a problem-solving process and examine its four components: asking questions, collecting appropriate data, analyzing the data, and interpreting the results. This session investigates the nature of data and its potential sources of variation. Variables, bias, and random sampling are introduced.

Session 2. Data Organization and Representation

Explore different ways of representing, analyzing, and interpreting data, including line plots, frequency tables, cumulative and relative frequency tables, and bar graphs. Learn how to use intervals to describe variation in data. Learn how to determine and understand the median.

Session 3. Describing Distributions

Continue learning about organizing and grouping data in different graphs and tables. Learn how to analyze and interpret variation in data by using stem and leaf plots and histograms. Learn about relative and cumulative frequency.

Session 4. The Five-Number Summary

Investigate various approaches for summarizing variation in data, and learn how dividing data into groups can help provide other types of answers to statistical questions. Understand numerical and graphic representations of the minimum, the maximum, the median, and quartiles. Learn how to create a box plot.

Session 5. Variation About the Mean

Explore the concept of the mean and how variation in data can be described relative to the mean. Concepts include fair and unfair allocations, and how to measure variation about the mean.

About This Course, cont'd.

Session 6. Designing Experiments

Examine how to collect and compare data from observational and experimental studies, and learn how to set up your own experimental studies.

Session 7. Bivariate Data and Analysis

Analyze bivariate data and understand the concepts of association and co-variation between two quantitative variables. Explore scatter plots, the least squares line, and modeling linear relationships.

Session 8. Probability

Investigate some basic concepts of probability and the relationship between statistics and probability. Learn about random events, games of chance, mathematical and experimental probability, tree diagrams, and the binomial probability model.

Session 9. Random Sampling and Estimation

Learn how to select a random sample and use it to estimate characteristics of an entire population. Learn how to describe variation in estimates, and the effect of sample size on an estimate's accuracy.

Session 10. Classroom Case Studies

Explore how the concepts developed in this course can be applied at different grade levels through case studies of K-2, 3-5, and 6-8 teachers (former course participants), all of whom have adapted their new knowledge to their classrooms.

Course Components

Each *Learning Math* course consists of 10 two-and-a-half-hour sessions. The first nine sessions are devoted to mathematics content; the 10th session covers classroom applications. Concepts are developed within and across the sessions and the sessions increase in difficulty as they progress. Each session includes reading, problem solving, and group or individual activities that are available on the Web and in print, and a half-hour of video viewing, available on the Web,* on the Annenberg/CPB Channel,** or on videocassette.*** There are additional problems and readings to complete for homework.

* Broadband access is required to view the video on the Web; see Tech Tips, page 5.

** The schedule for broadcasts on the Annenberg/CPB Channel can be found on the course Web site.

*** Purchase videocassettes at www.learner.org or by calling 1-800-LEARNER.

Visit the *Data Analysis, Statistics, and Probability* Web site at www.learner.org/learningmath.

About This Course, cont'd.

The following components are in each course:

Key Terms

Key mathematical terms relevant to each session are listed at the beginning of that session. These terms are divided into two parts: terms that are new in that session and terms that were introduced in a previous session. Definitions for key terms may be found in the glossary in the Appendix of this guide.

Notes

Notes can be used by facilitators, study groups, or individuals working alone. They provide extended information about the topics presented in the course, including help for dealing with stumbling blocks that may come up and recommendations for different ways to approach the content.

Problems

Each session contains mathematical problems to be solved individually or by groups. Problems build upon previous concepts and increase in difficulty as the course progresses.

Take It Further

The problems marked "Take It Further" are optional and are not counted as part of the two-and-a-half hour time-frame for each session. These problems are designed for individuals who would like to explore a topic in greater depth. They are often more difficult than the other problems in the session, and they may introduce new information or concepts not previously discussed.

Interactive Activities

Each session in the course includes at least one interactive activity on the course Web site. These activities help you learn new mathematics content or reinforce existing knowledge through hands-on exploration directly on the Web. The interactive activities require the Flash plug-in, which you can download for free from Macromedia's Web site (see Tech Tips, page 5). There are also non-Flash versions of each activity that don't require the Flash plug-in and can be completed offline. If you are working with only the guide, the interactive activities have been adjusted and are included in the guide.

Tips

Tips are available for problems you may find more difficult or need help in getting started. Tips may be found at the end of each session in this guide.

Solutions

A solution is provided for every problem in *Learning Math*, with the exception of a few open-ended questions. When solving a problem with multiple parts, consider writing down your answers to all of the parts on paper first before checking the solution, because the answers to each part of the problem will be visible at once on the solution page. Solutions may be found at the end of each session in this guide.

The following sequence of activities will give you a sense of what you will do as a student using *Learning Math*:

1. Watch the **session video** in its entirety. You can watch the video before you begin the session to become comfortable with the material, or you can view the video after completing the session (so as not to view answers to problems).
2. Do problems sequentially. If you are having difficulty, refer to the **Tip**. If you want a challenge, try a **Take It Further**.
3. Check the **Solutions** at the end of each session.
4. You may want to read **Notes** as you go along to establish a deeper context for the content.
5. Watch **video segments** strategically placed throughout the session, either online by clicking on the "Play Video" button or on videocassette by fast-forwarding to the image and approximate time code that appears in the guide. Zero your VCR clock when the Annenberg/CPB logo appears at the beginning of the program to locate the image using the time code.
6. Do **homework problems** and **readings** (available as PDF files online) at the end of each session to reinforce your learning.

About This Course, cont'd.

Session Videos

Each *Learning Math* session includes viewing a video that is available on the course Web site, on videotape, or on the Annenberg/CPB Channel. The videos feature K-8 teachers working on the *Learning Math* course materials in a workshop with a facilitator. The videos for the nine content sessions show onscreen participants as they are introduced to the concepts, work through the problems, sometimes struggle to reach an understanding, and then reflect on what they have learned. At the end of most videos there is an example of how the content from the session is applied in a “real world” situation. The videos for the 10th session show participants from the videotaped workshops as they apply the content that they have learned back in their own classrooms. You may choose to watch each of these videos before or after you work on the associated course session.

Video Segments

Each session includes short excerpts from the associated video, which you watch (or review) and reflect on to see how the onscreen participants grapple with the same or similar problems and concepts you encounter in the course. Instructions are given to find the segments on videotape. The segments are also available on the course Web site, if you are watching the complete programs online or on the Annenberg/CPB Channel.

Homework

Each session includes approximately 45 minutes of homework problems and/or reflective writing assignments that reinforce the session’s content.

Readings

Readings from journals and books are cited at the end of some sessions. They are available on the *Data Analysis, Statistics, and Probability* Web site as downloadable PDF files. Go to www.learner.org/learningmath. If you do not have access to the Internet, call 1-800-LEARNER to obtain a free set of readings.

Using the Videos, Guide, and Web Site

Each *Learning Math* course includes sequentially organized problems, video viewing, interactive activities, readings, and homework. The multimedia elements of the course create an exciting environment for probing mathematics content. The course can be taken entirely on the Web, followed in this print guide, or completed using a combination of Web and print. You can watch the videos online, on the Annenberg/CPB Channel, or on videocassette. If you are watching the programs on the Channel, we recommend taping them so you can look at short video segments when prompted in each session. These segments punctuate concepts developed in the course and create a “virtual” community of learners.

Ways To Take *Learning Math*

Learning Math was flexibly designed for a variety of users and situations. You may choose to work through the sessions on your own, in a study group, or as part of a facilitated, face-to-face, graduate-level course for credit.

Channel-Talk

Join an email discussion group and converse with other teachers taking this course. Go to the course Web site at www.learner.org/learningmath and select Channel-Talk.

Registration and Credit

Go to www.learner.org/4gradcredit for details on receiving graduate credit for *Learning Math*.

About This Course, cont'd.

Taking Multiple *Learning Math* Courses

The five *Learning Math* courses are designed to be independent of one another. You can take just one course, several courses, or all five courses in the order that fits your needs or the needs of your group. The courses also complement one another, with some topics discussed in more than one course but approached differently depending on the focus of that course. Taking several of these courses will increase your own conceptual understanding and ultimately that of your students.

Facilitating the Course

You can prepare for facilitating the course by reading through each session and its "Notes" section prior to meeting with your group. Reading through the material will help you become clear about the activities, plan how much time you need to spend on each one, and pull together necessary materials. The course is designed for use by an individual, but the Notes suggest ways for groups to work through the sessions.

Tech Tips

To use all or portions of the *Learning Math* course online, we recommend the following:

Internet Access

A minimum 56K modem connection is required, but an ISDN or high-speed connection is recommended. The slower your connection, the longer it will take to load larger features, such as the Flash activities.

To view the video programs and video segments online, a broadband connection to the Internet (DSL, cable modem, or LAN connection to a T1 line or greater) is required.

Web Browser

You will need Netscape 4.6 (or a higher version) or Internet Explorer 5.0 (or a higher version). Javascript should be enabled, if your browser allows you to disable it. Text fonts and colors, and many features, such as hidden Tips, may not be displayed correctly in older browsers.

Plug-Ins

These plug-ins will allow you to get the most out of the courses. Links to sites where you can download these programs for free are provided on the course Web site.

- Shockwave Flash 5 (or a higher version) for using the Interactive Activities
- Windows Media Player 7 (or a higher version) for watching videos
- Adobe Acrobat Reader for viewing the Readings in the Homework sections

Printing Web Pages

If you are having trouble printing some of the course content pages, you may try doing one or more of the following (from the "Print Preview," "Print...," or "Page Setup..." menu):

- Turn on "Shrink to Fit" mode (IE 5 only)
- Print the page in "Landscape" mode
- Reduce the scale of the printer output

Who's Who

Content Developer/Facilitator



Gary D. Kader

Gary Kader is a professor of mathematical sciences at Appalachian State University. He has been active in the development of educational materials for teaching and learning statistics through his involvement with six statistics education projects funded by the National Science Foundation. These projects developed innovative approaches to teaching and learning statistics and provided professional development opportunities for college faculty. His work in statistics education has focused on activity-based approaches to learning statistical concepts and problem solving. His interest in activity-based learning led him to direct the NSF project "A Laboratory Course in Mathematics for Liberal Arts Majors." He received a Ph.D. in Statistics from the University of Georgia in 1980 and B.S. and M.S. degrees in Mathematics from Georgia Southern University. He is a member of the American Statistical Association and the National Council of Teachers of Mathematics, and he serves as a reviewer for the NCTM journal *The Mathematics Teacher*. He received the Distinguished Alumnus Award from the Department of Mathematics and Computer Science from Georgia Southern University in 1996.

Content Developer

L. Mike Perry

Mike Perry retired in 2002 from Appalachian State University, where he was a professor of mathematical sciences and now works as a consultant in statistics education. His interest in the teaching of statistics led him to direct six projects funded by the National Science Foundation; these efforts developed new approaches to learning statistical ideas, provided teacher education in statistics, and offered professional development to college faculty. His writing focuses on activity-based approaches for learning statistical concepts and problem solving. He was a lecturer at the Institute of Statistics at Texas A&M University (1978-79) and a visiting scholar at the Royal Statistical Society's Centre for Statistical Education at the University of Sheffield (1991) and the University of Nottingham (1997). He currently serves as a consultant for the Advanced Placement Program in Statistics and frequently reviews articles for *The Mathematics Teacher*. He received a Ph.D. in mathematics from North Carolina State University and holds Master's degrees from North Carolina State and Texas A&M universities.

Who's Who, cont'd.

Onscreen Participants



Neuz Aissa de Figueredo
Teacher, Grade 5



Rosalind Agwu-Igbani
Computer Literacy Instructor



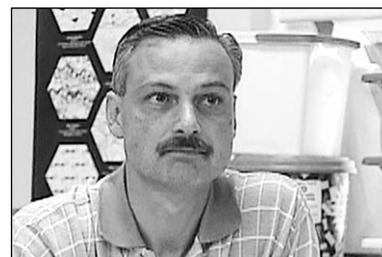
Renee Druke
Special Education Teacher, Grade 8



Kara Farrell
Math Teacher, Grade 7



Mare Fulton, Math, Science, Social
Studies, Language Arts, Reading
Teacher, Grade 6



Larry Greco
Math Teacher, Grade 8



Alma Harris
Teacher, Grade 2



Suzanne L'Esperance
Teacher, Grade 5



Kim Kozak Cook
Teacher, Grade 2

Who's Who, cont'd.



Kimberly Lysaght
Math Teacher, Grade 7



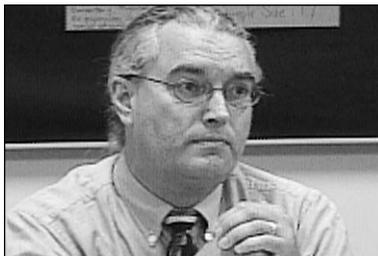
Louise Rijk
Teacher/Supervisor, Grade 6



Georgina Rutherford
Teacher, Grade 5



Ellen Sabanosh
Teacher, Grade 1



Paul Sowden
Math and Social Studies Teacher,
Grade 6



Philip Stameris
Math Teacher, Grade 8



Uzo Ugochukwu
Math and Science Teacher, Middle
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Alma Wright
Teacher, Grades 1-2