Session 10

Classroom Case Studies, Grades K–2

This is the final session of the *Number and Operations* course! In this session, we will examine how number and operations concepts from the previous nine sessions might look when applied to situations in your own classroom. This session is customized for three grade levels. Select the grade level most relevant to your teaching.

The session for grades K–2 begins below. Go to page 201 for grades 3–5 and page 217 for grades 6–8.

Key Terms in This Session

New in This Session

- even numbers
- odd numbers

Introduction

In the previous sessions, you explored number and operations as a mathematics learner, both to analyze your own approach to solving problems and to gain some insight into your personal conception of number and operations. It may have been difficult to separate your thinking as a mathematics learner from your thinking as a mathematics teacher—most teachers think about teaching as they are learning and think about learning as they are teaching. In this session, we shift the focus to your own classroom and to the approaches your students might take to mathematical tasks involving number and operations concepts.

As in other sessions, you will be prompted to view short video segments throughout the session; you may also choose to watch the full-length video for this session. **[See Note 1]**

Learning Objectives

In this session, you will do the following:

- Explore the development of number and operations concepts at your grade level
- Examine children's understanding of number and operations concepts
- Explore how you would teach problems involving different number and operations concepts

Note 1. This session uses classroom case studies to examine how children in grades K-2 think about and work with number and operations. If possible, work on this session with another teacher or a group of teachers. Using your own classroom and the classrooms of fellow teachers as case studies will allow you to make additional observations.

Part A: Observing a Case Study (25 min.)

To begin the exploration of what topics in number and operations look like in a classroom at your grade level, watch a video segment of a teacher who took the *Number and Operations* course and then adapted the mathematics to her own teaching situation. When viewing the video, keep the following questions in mind: **[See Note 2]**

a. What fundamental ideas (content) about number and operations is the teacher trying to teach?

b. What mathematical processes does the teacher expect students to demonstrate?

c. How do students demonstrate their knowledge of the intended content? What does the teacher do to elicit student thinking?

Video Segment (approximate time: 3:58-8:37): You can find this segment on the session video 3 minutes and 58 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, Ms. Weiss applies the mathematics she learned in the *Number and Operations* course to her own teaching situation by having her students use counting chips to solve a subtraction problem. The children solve the problem and then discuss their methodology. When they've finished discussing the problem, they must write a mathematical sentence to justify their answers.

Problem A1. Answer questions (a), (b), and (c) above.

Problem A2. At what point(s) in the lesson are the students learning new content? Are they applying what they already know?

Problem A3. Ms. Weiss gave each group of students a set of chips. What were some advantages and disadvantages of using this manipulative?

Problem A4. Ms. Weiss's lesson was based on Session 4 of this course. Discuss the ways in which her lesson was similar to and different from Session 4. What adaptations did she make, and why?

Note 2. The purpose of the video segments is not to reflect on the teaching style of the teacher portrayed. Instead, look closely at the methods the teacher uses to bring out the ideas of number and operations while engaging her students in an activity.

Part B: Reasoning About Number and Operations (40 min.)

Exploring Standards

The National Council of Teachers of Mathematics has identified number and operations as a strand in its *Principles and Standards for School Mathematics.* In grades pre-K through 12, instructional programs should enable all students to do the following:

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- Understand the meaning of operations and how they relate to one another
- Compute fluently and make reasonable estimates

In pre-K through grade 2 classrooms, students are expected to do the following:

- Understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations
- Develop and use strategies for whole-number computations, with a focus on addition and subtraction
- Develop fluency with basic number combinations for addition and subtraction
- Use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators
- Count with understanding, and recognize "how many" are in sets of objects
- Use multiple models to develop initial understandings of place value and the base ten number system
- Connect number words and numerals to the quantities they represent using various physical models and representations

The NCTM Number and Operations Standards state that students should "develop a solid understanding of the base-ten numeration system and place-value concepts by the end of grade 2.... Using concrete materials can help students learn to group and ungroup by tens. For example, such materials can help students express '23' as 23 ones (units), 1 ten and 13 ones, or 2 tens and 3 ones. Of course, students should also note the ways in which using concrete materials to represent a number differs from using conventional notation. For example, when the numeral for the collection [23] is written, the arrangement of digits matters—the digit for the tens must be written to the left of the digit for the units. In contrast, when base-ten blocks or connecting multi-cubes are used, the value is not affected by the arrangement of the blocks" (NCTM, 2000, p. 81).

As you watch another video segment from Ms. Weiss's class, think about how the students are developing this understanding of number and operations.

Part B, cont'd.

Video Segment (approximate times: 16:22-20:54): You can find this segment on the session video 16 minutes and 22 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, two groups of students use Digi-Blocks to solve subtraction problems. **[See Note 3]**

Problem B1.

a. How did the students use the Digi-Blocks to represent the problem?

b. What processes did the students use to group the Digi-Blocks?

c. What subtraction interpretations did the students consider?

Problem B2. How did the Digi-Blocks help students relate their actions to the written algorithm?

Problem B3. What are some ways that you see the NCTM Standards being incorporated into Ms. Weiss's lesson?

Problem B4. Embedded in the children's explanations of solving the subtraction problems are early understandings of place value. How could you extend this conversation to formalize these notions?

Join the Discussion! www.learner.org

Post your answer to Problem B4 on an email discussion list, then read and respond to answers posted by others. Go to the *Number and Operations* Web site at www.learner.org/learningmath and find Channel-Talk.

Note 3. Digi-Blocks are manipulatives that reinforce the concept of grouping by tens. Students place small blocks into rectangular holders that fit exactly 10 blocks. The full holder then becomes a single entity—a group of ten. For more information on Digi-Blocks, go to http://www.digi-block.com/.

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Part B, cont'd.

Examining Children's Reasoning

Here are scenarios from two different teachers' classrooms, each involving young children's developing ideas about number and operations. Snippets of students' discussions are given for each scenario. For each student, consider the following:

- *Understanding or Misunderstanding:* What does the statement reveal about the student's understanding or misunderstanding of number and operations ideas? Which ideas are embedded in the student's observations?
- *Next Instructional Moves:* If you were the teacher, how would you respond to this student? What questions might you ask so that the student would ground his or her comments in the context? What further tasks and situations might you present for the student to investigate? **[See Note 4]**

Problem B5. Second graders Leland, Randy, and Reed are given the following problem: Kent and his dog Nikki weigh 194 pounds when they are together on the scale. Kent weighs 146 pounds. How much does Nikki weigh?

Below is a snippet of their conversation:

Reed: Nikki weighs 48 pounds. I did it in my head. I first added 2 to 194 to get 196. Then I counted backward by tens until I got to 146—196, 186, 176, 166, 156, 146. That's 5 tens, or 50. Then I subtracted the 2 to get 48.

Leland: I didn't do it that way. I counted up by tens—146, 156, 166, 176, 186, 196. That's 50, and 2 less makes 48.

Randy: I wrote it down on a piece of paper, and 6 minus 4 is 2, and 9 minus 4 is 5, and 1 minus 1 is 0. So it's 52.

a. What method did each student use to solve the problem? What does this tell you about how each student is thinking about the problem?

b. Do you think each student is ready to learn a new method? Why or why not?

Problem B6. Second graders Daniel, Tarra, and Mariko are given the following problem: Antonio has 35 marbles. Helen has 52 marbles. How many more marbles does Helen have than Antonio?

Below is a snippet of their conversation:

Mariko: They have 87 marbles all together.

Tarra: Helen has 35, 45, 55—that's 20. Count back 55, 54, 53, 52. She has 17 more.

Daniel: Fifty-two, 42, 32, then 33, 34, 35—that's 23 more.

a. What method did each student use to solve the problem? What does this tell you about how each student is thinking about the problem?

b. Do you think each student is ready to learn a new method? Why or why not?

Join the Discussion! **www.learner.org**

Post your answer to Problems B5 and B6 on an email discussion list, then read and respond to answers posted by others. Go to the *Number and Operations* Web site at www.learner.org/learningmath and find Channel-Talk.

Note 4. You may wish to make a two-column chart, labeled "Understanding or Misunderstanding" and "Next Instructional Moves," to help you organize your thinking for each problem. If you are working in a group, these charts could be the basis for a meaningful discussion on how to assess students' understanding of the concept of subtraction and the processes for computation.

Part C: Problems That Illustrate Reasoning About Number and Operations (55 min.)

As this course comes to a close and you reflect on ways to incorporate your new understanding of number and operations into your teaching, you have both a challenge and an opportunity: to enrich the mathematical conversations you have with your students around number and operations. As you are well aware, some students will readily grasp the ideas being studied, and others will struggle.

In Part C, you'll look at several problems that are appropriate for students in grades K-2. For each problem, answer these questions:

a. What is the solution to this problem?

b. What is the number and operations content in this problem?

c. What skills do students need to work through this problem?

d. If students are having difficulty, what questions might help them work through this problem?

e. What questions might extend students' thinking beyond this problem?

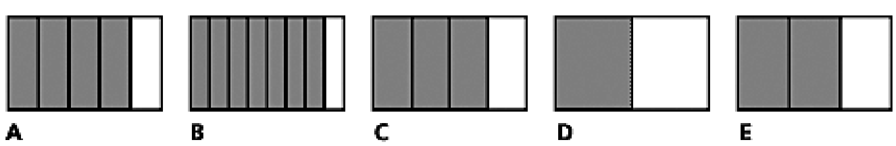
Problem C1. You have the following cards:

0	1	2	3	3	4	5	6

How many different ways can you put two cards in the squares so that their sum equals 6?

$\square + \square = 6$

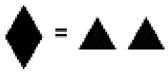
Problem C2. The rectangles are all the same size, and the pieces within each rectangle are all the same size. Which rectangle has the most shaded? Which has the least shaded? How do you know?



Part C, cont'd.

Problem C3.

I.



If you can trade 1 rhombus for 2 triangles, how would you solve the problems below? Explain your reasoning.

1. 
2. 
3. Is 8 triangles a fair trade for 4 rhombuses? How do you know?

II.

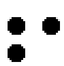
If you can trade 1 trapezoid for 3 triangles, how would you solve the problems below? Explain your reasoning.

1. 
2. 
3. Is 10 triangles a fair trade for 4 trapezoids? How do you know?

Problem C4. Using counters to represent numbers, we can observe the following:

 One is an odd number because one counter has no partner (i.e., it cannot form a pair with another counter).

 Two is an even number because both counters have a partner (i.e., they form a pair).

 Three is an odd number because one counter still has no partner (i.e., it does not form a pair with another counter).

Keeping this in mind, fill in the blanks below:

1. An even number plus an even number gives an _________ number. (How did you decide?)
2. An odd number plus an odd number gives an _________ number. (How did you decide?)
3. An even number plus an odd number gives an _________ number. (How did you decide?)

Problem C5. Consider one of the above problems. What kind of lesson could you generate around this topic?

Homework

Solutions are not provided for these homework problems, since answers will vary depending on individual experiences.

Problem H1. Assume that you need to report back to your grade-level team or to the entire school staff at a faculty meeting on your experiences and learning in this course. What are the main messages about the teaching of number and operations that you would share with your colleagues? Prepare a one-page handout or an overhead or slide that could be distributed or shown at the meeting.

Problem H2. Look at a lesson or activity in your own mathematics program for your grade level that you think has potential for developing students' reasoning about number and operations. If you were to use this lesson or activity now, after taking this course, how might you modify or extend it to bring out more of the important ideas about number and operations?

Solutions

Part A: Observing a Case Study

Problem A1.

a. The content is focused on the meanings of and models for operations—in this case, subtraction. Using different strategies and manipulatives, such as counting chips, gives students some basic insight into the nature of subtraction. They begin doing subtraction in the forms of take-away, comparison, or missing addend, even though they are not aware of it. Indirectly, the students also begin to gain knowledge of place value.

b. The students are using counting strategies to solve the problems. They are also able to count large numbers.

c. The students have determined that they need to subtract to solve the problem, and in most cases they do not analyze how they know they need to subtract. In general, students have trouble understanding that a comparison problem requires subtraction, perhaps because of the wording, which usually contains the phrase "How many more?" Some students count up from the lesser number to obtain the answer. Other students count back from the greater number. Some count out all the chips, take the given amount away, and then recount for the answer.

   To elicit student thinking, Ms. Weiss asks students to justify their answers—to provide a convincing argument that their answer is correct. She insists that they write something on their paper that shows why their answer works. Sometimes this brings to light an error the students made in their computations. In these cases, students often catch the error themselves, self-correct, and then continue their justification as though this new answer were the original one.

Problem A2. These students clearly understand the concept of subtraction and can apply the concept to the solution to a problem. However, there is no evidence that the students are using their knowledge of place value to help them solve the problems. The manipulative does not lend itself to any shortcuts in computation. Most students count each chip separately. Very few students attempt to group the chips so that counting or subtraction is more efficient—this appears to be new content to most of the students.

Problem A3. By having students use this manipulative, Ms. Weiss is trying to help them see the connections between their work with the chips and their work on paper. However, using the chip model does not force any grouping. Most students frequently lost their place, double-counted, or skipped chips as they tried to solve the problem.

Solutions, cont'd.

Problem A4. The lesson touches on the concepts covered in Session 4 of this course. Ms. Weiss presents different meanings of subtraction in an introductory manner, where the students are gaining different experiences and making their own observations about those meanings. Unlike Session 4, you will see that this lesson focuses on comparing different manipulatives and induces students' initial understanding of the efficiency of counting in groups (of 10). The lesson deals only with whole numbers.

Part B: Reasoning About Number and Operations

Problem B1.

a. The Digi-Blocks forced students to group in tens and hundreds. Most students started with the larger number of blocks. Students then took apart tens to obtain more ones, or hundreds to obtain more tens, as they completed the subtraction processes.

b. These blocks are automatically grouped in tens and hundreds. Students "see" the connections between the place value and the digits they are writing.

c. In this case, most students started with the larger collection and took away the smaller collection.

Problem B2. With this manipulative, the symbols students write on the paper match the actions they took with the blocks.

Problem B3. Ms. Weiss's lesson is structured around understanding the meaning of operations, in particular subtraction and addition. The students utilize a variety of methods and tools to solve problems. They also use multiple manipulatives, such as Digi-Blocks, which are particularly helpful in strengthening students' ability to compute fluently and understand place value. The students using Digi-Blocks are also learning to count with understanding and recognize "how many" in sets of objects.

Problem B4. Ask students to talk about what they did with the blocks and what they wrote down on their papers. Listen for statements like this one: "I needed to take away one of the ones, and I didn't have any ones. So I had to take apart one of the tens." Students should be talking about taking apart and putting together units of ten.

Problem B5.

a. Reed understands the concept and has good mental math skills, so he can do the computation in his head. He is well able to think of shortcuts to make mental computation easier.

   Leland also has a good understanding of the concept and has the mental math skills to think of shortcuts for mental computation.

   Randy does not understand the paper-pencil algorithm for subtraction with regrouping. He heard both Reed and Leland discussing their answers, but it didn't seem to bother him that his answer didn't match theirs.

b. Reed is probably ready to work on and understand other methods. For example, he could work on a written algorithm to match his informal understanding. We do not know how well he could do this with paper and pencil.

   Leland is probably ready to work on and understand other methods. He also needs to work on a written algorithm to match his informal understanding. You can test his ability to do this type of computation with paper and pencil.

   Randy is probably not ready to move on to new methods. He needs to work on some mental math skills and sense-making for subtraction with regrouping.

Solutions, cont'd.

Problem B6.

a. Mariko has entirely misunderstood the problem. She has found the sum of the two numbers rather than their difference. This may indicate that she is using key words without really reading the problem. She might have interpreted the key word "more" to mean addition.

   Tarra counted up by tens, starting from 35, until she got close to the desired number, and then counted backward by ones until she reached the goal. She knew to count backward instead of forward here because she had overshot the goal. This is sophisticated thinking. She appears to have a good understanding of the concepts and procedures.

   Daniel counted backward by tens until he got close to the number, and then counted up by ones until he reached the desired goal. Daniel thinks that since he is now at 32, he must count up three to get to 35. He does not realize that he passed 35 on the way to 32.

b. Mariko is probably not ready to move on to new methods. She needs some hands-on practice with subtraction interpretations with smaller numbers. She does not appear to know that comparison requires subtraction. She should be discouraged from choosing an operation based solely on key words and encouraged to read and reread until the problem makes sense to her.

   Tarra has a good understanding and is probably ready to move on to new methods. She now needs to be given more complicated problems, to test whether she can do subtraction using paper and pencil.

   For Daniel, new methods may or may not be helpful. Although he understands the concept of subtraction, he still needs to practice with smaller numbers to strengthen his procedures for mental subtraction.

Part C: Problems That Illustrate Reasoning About Number and Operations

Problem C1.

a. Solution: There are seven different ways:

   0 + 6, 1 + 5, 2 + 4, 3 + 3, 4 + 2, 5 + 1, 6 + 0

b. The number and operations content in this problem is addition of whole numbers. There is also an exposure to equations and the symbols involved, such as the equality symbol.

c. Students need to be able to recognize the numerals for the numbers 0 through 6 and to be able to add whole numbers with sums to 6. They also need to understand the equality and addition symbols. And they need to use logical reasoning and have the ability to make an organized list.

d. For students who are having difficulty, give them six counters and ask them to arrange the counters in the two squares, starting with all six in the right square. Ask them to record this number fact: 0 + 6 = 6. Then ask them to move one counter to the left square and record a new number fact: 1 + 5 = 6. Have them continue until all the counters are in the left square.

e. Choosing a sum other than 6 can extend this problem. A nice extension activity would be to ask students to make a table with all the ways to get each sum from 1 to 10:

Number	Sums	Number of Ways
1	0 + 1; 1 + 0	2
2	0 + 2; 1 + 1; 2 + 0	3
. . .		
10	0 + 10; 1 + 9; 2 + 8; 3 + 7; 4 + 6; 5 + 5; . . . 10 + 0	11

Solutions, cont'd.

Problem C1 (e), cont'd.

Another extension would be to allow more than two addends. Ask, "How many different ways can you add counting numbers to get 6?" (Notice that 0 is not allowed in this problem.) Solution: There are 31 different ways to get 6 by adding counting numbers. (Remember that 2 + 1 + 2 + 1 is different from 1 + 2 + 2 + 1, etc., so there are six different ways to write these four numbers.)

Problem C2.

a. Solution: Rectangle B has the most shaded because it has the greatest number of parts. As the number of parts increases, the size of the parts decreases. Rectangle B is only missing one part out of eight parts. The others have fewer parts. Rectangle D has the least shaded because it is divided into the fewest number of parts, so the missing part is greater than any of the others.

b. The number and operations content is the basic concept of fractions and the inverse relationship between the number of parts and the size of the parts.

c. Students need to be able to make sense of visual representations, extract information from a visual representation, and make observations about what they see.

d. Students who are having difficulty with this problem need to have concrete materials to represent the fractions. They need to be able to compare the size of 1/3 to the size of 1/4 and so on.

e. This problem can be extended by asking students to order all five of the rectangles from least shaded to most shaded. You could also include more fractions and not use the pictures.

Problem C3.

a. Solution:

I.

1. You can trade 2 rhombuses for 2 sets of 2, or 4, triangles.
2. There are 3 sets of 2 triangles, so you can trade for 3 rhombuses.
3. Yes. Eight triangles is 4 sets of 2, so you can trade for 4 rhombuses.

II.

1. There are 3 trapezoids, so there should be 3 sets of 3 triangles.
2. There are 2 sets of 3 triangles, so there should be 2 trapezoids.
3. No. There should be 3 triangles for each trapezoid. Three sets of 3 triangles is 9 triangles, and 4 sets of 3 triangles is 12 triangles. That means that 10 triangles is not a fair trade for 4 trapezoids.

b. The number and operations content of this problem is the concept of fractions and proportional reasoning. It also involves basic exposure to equations.

c. Students must be able to add numbers to 9, count by twos and threes, and count backward by twos or threes. They need to understand the equality symbol. Students will also be asked to make observations based on visual representations and to use logical thinking.

d. If students are having trouble, give them pattern blocks to use. Have them cover each rhombus with 2 triangles and each trapezoid with 3 triangles.

e. One extension of this problem is to use more complex relationships. For example, if 1 hexagon can be traded for 2 trapezoids, how many triangles can it be traded for? How many rhombuses?

Solutions, cont'd.

Problem C4.

a. Solution:

   1. Even. Every counter has a partner.
   2. Even. The two counters without partners can be partners, so every counter has a partner.
   3. Odd. The counter without a partner still has no partner.

b. The number and operations content of this problem is number theory (odd and even numbers) and addition.

c. Students must understand odd and even numbers and be able to add one-digit numbers. Logical thinking and working with word problems are also present.

d. If students are having difficulty, give them counters. Ask them to choose two odd numbers and make as many partners as they can for each number. They will see that one is left over in each number. Ask them what happens when the two numbers are added. Do the same with the other problems.

e. One extension of this problem is to ask students to think about numbers that are multiples of 3. Describe these numbers as numbers you say when you count by threes. What happens when you add two numbers that are multiples of 3?

Problem C5. Answers will vary, but here is one example:

Many students at this level work with pattern blocks. A lesson could be structured around exploring pattern blocks and the relationships that exist among different shapes. Once students gain a familiarity with these, this knowledge can be further extended into writing up those relationships as simple equations. For example, for Problem C3, students could begin by using the triangular pattern blocks to "make" a rhombus before they work on the written equations. This way, students can use manipulatives to reason about and justify their answers. By writing these simple equations, students are not only learning about the equality symbol, but also gaining some early groundwork on fractions, multiples, and addition (for example, one half of a rhombus is a triangle, and three triangles make one trapezoid).