Session 10

Classroom Case Studies, Grades 6–8

This is the final session of the *Number and Operations* course! In this session, we will examine how number and operations concepts from the previous nine sessions might look when applied to situations in your own classroom. This session is customized for three grade levels. Select the grade level most relevant to your teaching.

The session for grades 6-8 begins below. Go to page 189 for grades K-2 and page 201 for grades 3-5.

Key Terms in This Session

Previously Introduced

- algorithm
- base
- divisibility test
- exponent
- percent
- proportion

Introduction

In the previous sessions, you explored number and operations as a mathematics learner, both to analyze your own approach to solving problems and to gain some insight into your personal conception of number and operations. It may have been difficult to separate your thinking as a mathematics learner from your thinking as a mathematics teacher—most teachers think about teaching as they are learning and think about learning as they are teaching. In this session, we shift the focus to your own classroom and to the approaches your students might take to mathematical tasks involving number and operations concepts.

As in other sessions, you will be prompted to view short video segments throughout the session; you may also choose to watch the full-length video for this session. **[See Note 1]**

Learning Objectives

In this session, you will do the following:

- Explore the development of number and operations concepts at your grade level
- Examine students' understanding of number and operations concepts
- Explore how you would teach problems involving different number and operations concepts

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Note 1. This session uses classroom case studies to examine how students in grades 6-8 think about and work with number and operations. If possible, work on this session with another teacher or a group of teachers. Using your own classroom and the classrooms of fellow teachers as case studies will allow you to make additional observations.

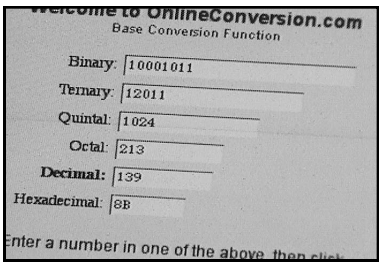
Part A: Observing a Case Study (25 min.)

To begin the exploration of what topics in number and operations look like in a classroom at your grade level, watch a video segment of a teacher who took the *Number and Operations* course and then adapted the mathematics to her own teaching situation. When viewing the video, keep the following questions in mind: **[See Note 2]**

a. What fundamental ideas (content) about number and operations is the teacher trying to teach?

b. What mathematical processes does the teacher expect students to demonstrate? How does this lesson help students achieve reasonable estimations and fluent computations?

c. How do students demonstrate their knowledge of the intended content? What does the teacher do to elicit student thinking?



Video Segment (approximate time: 12:02-17:39): You can find this segment on the session video 12 minutes and 2 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, Ms. Miles applies the mathematics she learned in the *Number and Operations* course to her own teaching situation by leading students through the process of converting from base ten to base five. The students then work in groups to convert base ten numbers to base five.

Problem A1. Answer questions (a), (b), and (c) above.

Problem A2. At what point(s) in the lesson are the students learning new content?

Problem A3. How do students transfer their knowledge of base ten to base five? What is the evidence?

Problem A4. What was the benefit of having students use base five, rather than another base, to examine the concept of bases?

Problem A5. Discuss the role of manipulatives in Ms. Miles's lesson. How did they help deepen the students' knowledge of the content area?

Problem A6. Ms. Miles's lesson was based on Session 3 of this course. Discuss the ways in which her lesson was similar to and different from Session 3. What adaptations did she make, and why?

Note 2. The purpose of the video segments is not to reflect on the teaching style of the teacher portrayed. Instead, look closely at the methods the teacher uses to bring out the ideas of number and operations while engaging her students in activities.

Part B: Reasoning About Number and Operations (40 min.)

Exploring Standards

The National Council of Teachers of Mathematics (NCTM, 2000) has identified number and operations as a strand in its *Principles and Standards for School Mathematics.* In grades pre-K through 12, instructional programs should enable all students to do the following:

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- Understand the meaning of operations and how they relate to one another
- Compute fluently and make reasonable estimates

In grades 6-8 classrooms, students are expected to do the following:

- Develop, analyze, and explain methods for solving problems involving proportions
- Work flexibly with fractions, decimals, and percents to solve problems
- Compare and order fractions, decimals, and percents efficiently, and find their approximate locations on a number line
- Develop an understanding of large numbers, and recognize and appropriately use exponential, scientific, and calculator notation
- Develop meaning for integers, and use them to represent and compare quantities
- Develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results

"In the middle-grades mathematics classrooms, young adolescents should regularly engage in thoughtful activity tied to their emerging capabilities of finding and imposing structure, conjecturing and verifying, thinking hypothetically, comprehending cause and effect, and abstracting and generalizing" (NCTM, 2000, p. 211).

Watch another segment from Ms. Miles's class, and think about how the students are developing this understanding of number and operations.

Video Segment (approximate time: 17:39-24:54): You can find this segment on the session video 17 minutes and 39 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, Ms. Miles leads students through the process of using the division algorithm instead of manipulatives to convert from base five to base ten and from base ten to base five. The students then work in groups to convert base ten numbers to base five.

Problem B1. What reasoning processes are the students using to solve the problems?

Part B, cont'd.

Problem B2. How does working in a different base develop students' sense of number and operations? What does working with different bases tell us about place-value systems?

Join the Discussion! www.learner.org

Post your answer to Problem B2 on an email discussion list, then read and respond to answers posted by others. Go to the *Number and Operations* Web site at www.learner.org/learningmath and find Channel-Talk.

Problem B3. How did the manipulatives help students understand how to use long division to solve the problem?

Problem B4. What are some ways that you see the NCTM Standards being incorporated into Ms. Miles's lesson?

Examining Students' Reasoning

Here are scenarios from two different teachers' classrooms, each involving students' developing ideas about number and operations. Snippets of students' discussions are given for each scenario. For each student, consider the following:

- *Understanding or Misunderstanding:* What does the statement reveal about the student's understanding or misunderstanding of number and operations ideas? Which ideas are embedded in the student's observations?
- *Next Instructional Moves:* If you were the teacher, how would you respond to this student? What questions might you ask so that the student would ground his or her comments in the context? What further tasks and situations might you present for the student to investigate? **[See Note 3]**

Problem B5. Ming Hui and Kenneth were working to translate the base five number 1234 to a base ten number. The teacher has asked them not to use manipulatives. Below is a snippet of their conversation:

Ming Hui: We can't use the tiles this time, so let's try to remember what tiles are put under each place.

Kenneth: Okay, put the 1 tiles under the 4. That's 4.

Ming Hui: And then the 5s are under the 3. That's 15 more.

Kenneth: And then the 25s are under the 2. That makes 50 more.

Ming Hui: So far we have 4 plus 15 plus 50. That's 69.

Kenneth: And then we have one more. That must be the 100s. We've got 169 in all.

Ming Hui: Yes, the base ten number is 169.

a. What methods did the students use to solve the problem? What do these methods tell you about how the students are thinking about the problem? What mistake did they make in their conversion process? Why do you think they made this common error?

b. How would you help the students deal with any misconceptions they have?

Note 3. You may wish to make a two-column chart, with labels "Understanding or Misunderstanding" and "Next Instructional Moves," to help you organize your thinking for each to the next problems. If you are working in a group, these charts could be the basis for a very meaningful discussion about how to assess students' understandings of the concepts of subtraction and the processes for computation.

Part B, cont'd.

Problem B6. Brad and Kent were working to translate the base ten number 342 to a base five number. Below is a snippet of their conversation:

Brad: That's three 25s and four 5s and two more.

Kent: No, we're going the other way.

Brad: Oh, you're right. Then the three 100s are twelve 25s. That's two 125s and two 25s.

Kent: And the four 10s will make eight 5s, or one 25 and three 5s.

Brad: Okay. So far we've got two 125s, three 25s, and three 5s. All we need is two more. So the number is 2332.

a. What methods did the students use to solve the problem? What do these methods tell you about how the students are thinking about the problem?

b. How would you help the students deal with any misconceptions they have?

Join the Discussion! **www.learner.org**

Post your answer to Problems B5 and B6 on an email discussion list, then read and respond to answers posted by others. Go to the *Number and Operations* Web site at www.learner.org/learningmath and find Channel-Talk.

Part C: Problems That Illustrate Reasoning About Number and Operations (55 min.)

As this course comes to a close and you reflect on ways to incorporate your new understanding of number and operations into your teaching, you have both a challenge and an opportunity: to enrich the mathematical conversations you have with your students around number and operations. As you are well aware, some students will readily grasp the ideas being studied, and others will struggle.

In Part C, you'll look at several problems that are appropriate for students in grades 6-8. For each problem, answer these questions:

a. What is the solution to this problem?

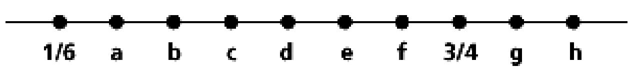
b. What is the number and operations content in this problem?

c. What skills do students need to work through this problem?

d. If students are having difficulty, what questions might help them work through this problem?

e. What questions might extend students' thinking beyond this problem?

Problem C1.



The points are all equally spaced on the number line. Identify the four points that represent the following:

1. 3/4 - 1/6
2. *d* • *f*
3. *e* - *a*
4. *b* ÷ *d*

Problem C2. The yearly changes in the enrollment in your school for the last four years were, respectively, a 20% increase, a 20% increase, a 20% decrease, and a 20% decrease.

1. What is the net change over the four years, to the nearest percent?
2. Would the answer change if the decreases came before the increases?

Problem C3. The sum of nine consecutive integers is 63. What is the smallest positive integer that could be part of the sum?

Problem C4. Lucky Edgar has been hired by a movie star to be his assistant on the set of a new film. For his salary, Edgar has been given a choice: He could get paid either $1,000 per day for 20 days or $1 on the first day, $2 on the second day, $4 on the third day, and so on, doubling each day for 20 days. Which salary should Edgar choose?

Homework

Solutions are not provided for these homework problems, since answers will vary depending on individual experiences.

Problem H1. Assume that you need to report back to your grade-level team or to the entire school staff at a faculty meeting on your experiences and learning in this course. What are the main messages about the teaching of number and operations that you would share with your colleagues? Prepare a one-page handout or an overhead or slide that could be distributed or shown at the meeting.

Problem H2. Look at a lesson or activity in your own mathematics program for your grade level that you think has potential for developing students' reasoning about number and operations. If you were to use this lesson or activity now, after taking this course, how might you modify or extend it to bring out more of the important ideas about number and operations?

Solutions

Part A: Observing a Case Study

Problem A1.

a. Ms. Miles is working with her students on understanding place value and interpreting numbers in different bases. She uses manipulatives and later symbolic expressions and algorithms to help explain the idea that each place in a number stands for a particular power of the base number (i.e., in base five: 5^0, 5^1, 5^2, etc.). In this video segment, students convert base ten numbers to base five numbers using a different process—decomposing the number into powers of 5. They also begin to develop an algorithm.

b. Students need to demonstrate that they can compute fluently and have an understanding of the order of operations and exponents. They need to be able to understand algorithms and translate visual information (e.g., manipulatives) to more abstract forms, such as symbolic representations.

c. The teacher is asking probing questions that help her assess students' understanding and the level of their thinking as well as challenge the students into new understandings. The teacher is also utilizing class discussion, using students' answers to build or scaffold the concepts she is trying to convey and to bring additional clarity.

Problem A2. Students are learning a new base. The new content is the interpretation of place value with a base other than ten.

Problem A3. In the lesson, the teacher is utilizing the students' prior knowledge and familiarity with base ten in order to help them interpret numbers in a new base. She uses both manipulatives and mental math to help them substitute powers of 10 for powers of 5 and make the necessary connections. Sometimes, as shown in this video segment, it is hard for students to separate their thinking about the two bases; an example is Britney's answer about base five, which indicated that she was still thinking in terms of base ten. As they work on the problems themselves, students are clearly developing fluency in converting numbers from base ten to base five.

Problem A4. The teacher chose base five for her students because, since they are familiar with powers of 5, it would be relatively easy for them to work with them. Thus, they can use mental math to convert numbers back and forth more easily, allowing them to focus on making observations about place value rather than getting bogged down in cumbersome calculations. However, working with other bases would be beneficial as well.

Solutions, cont'd.

Problem A4, cont'd.

Additionally, using a different base—for example, base five—is helpful because it makes the underlying concepts of place value more apparent. Students are then able to transfer this knowledge to numbers in other bases—in particular, base ten. In base ten, the same concepts can be harder to notice because of students' familiarity with this base, which may result in their overlooking the particular meanings of a place-value system.

Problem A5. Ms. Miles first uses manipulatives to help students understand the process. She is also preparing them to do the written work after they've learned with manipulatives.

Problem A6. Ms. Miles adapted the lesson to suit her students' grade level and mathematical abilities. Since this was the first time her class was dealing with different bases, she added manipulatives to help her students gain visual understanding, which, initially, was an important element. She then guided them toward working on a more abstract level, using the symbolic expressions as well as algorithms (later in the lesson).

Notice that she did not use the analogy of packaging that was used in the course. This technique may also be helpful in enhancing students' understanding of this type of content.

Part B: Reasoning About Number and Operations

Problem B1. Students are using logical reasoning and their understanding of place value in base ten to help them interpret base five numbers and convert them to base ten. Students are also using mental mathematics and the order of operations to solve the problems. The students realize that they must choose the appropriate place value first, meaning, they must use the exponents to find the greatest power of 5 that is less than the number they are converting.

Problem B2. When students transfer what they know about base ten to another base, their understanding of place value is extended and deepened. Also, when students have an understanding of the base ten place-value system, their ability to do complex computations increases. They are more able to use mental mathematics to solve problems.

Problem B3. Because students can picture what the manipulatives look like, they can understand that they first have to figure out what is the greatest power of 5 that is less than or equal to the number. This translates into dividing by the same power of 5 in the algorithm.

Problem B4. Ms. Miles's lesson focuses on understanding numbers and ways of understanding numbers and number systems. For example, by converting from one base to another and vice versa, the students become more aware of the ways we represent numbers and the meanings underlying those representations, such as place value. They also make connections between the representations and are able to extend them when working with other bases.

Problem B5.

a. These two students imagine using the tiles to solve the problem. They correctly imagine choosing four 1 tiles for the digit on the right, three 5 tiles for the second digit, and two 25 tiles for the third digit. At this point, Ming Hui correctly adds what they have so far. Then Kenneth seems to stop thinking about the tiles and chooses 100, rather than 125, as the value of the next power of 5. Ming Hui appears to be happy with this answer. The students most likely made this error because they inadvertently switched to thinking in base ten, and therefore derived 100 as the next place value instead of 125.

Solutions, cont'd.

Problem B5, cont'd.

b. These students need a reminder of the values of each power of 5. You could suggest that they always make a place-value chart before doing conversions in either direction. For this problem, their chart would look like this:

1	2	3	4
125s	25s	5s	1s

You could then suggest that they work from left to right when converting. This number would be (1 • 125) + (2 • 25) + (3 • 5) + (4 • 1) = 125 + 50 + 15 + 4 = 194.

Problem B6.

a. These students clearly understand place-value systems but are using an unconventional method to translate the numbers. Brad first assumes 342 is a base five number and correctly translates it to base ten. When corrected by Kent, he translates from each base ten value to its base five equivalent. So instead of working with the number 342 from the beginning, he starts with 300. He correctly states that 300 is 12 • 25, which in turn is (2 • 125) + (2 • 25).

b. These students clearly understand the system. However, you might suggest that translating each place of a base ten number separately could become messy and lead to mistakes. These students might also benefit from a base five chart. They could use a chart like the one below to remind them of the powers of 5 that are the place values, and then record and subtract the number of each power of 5 as they keep a running total, as shown:

125s	25s	5s	1s

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 342
-250    2 • 125
----
  92
 -75    3 • 25
----
  17
 -15    3 • 5
----
   2
  -2    2 • 1
----
   0
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This tells us that the number is 2332 in base five.

Solutions, cont'd.

Part C: Problems That Illustrate Reasoning About Number and Operations

Problem C1.

a. Solution:

   1. The answer is *e*. First you need to convert the denominators so that 3/4 equals 9/12, and 1/6 equals 2/12. The difference between 9/12 and 2/12 is 7/12. Since 3/4 is seven points to the right of 1/6, you know that the distance between adjacent points is 1/12. Now you can label all the points on the line: *a* = 3/12 or 1/4, *b* = 4/12 or 1/3 ... and *h* = 11/12. The answer to your subtraction problem, 7/12, is point *e*.
   2. The answer is *b* (*d* • *f* = 1/2 • 2/3 = 1/3 = 4/12).
   3. The answer is *b* (*e* - *a* = 7/12 - 1/4 = 4/12).
   4. The answer is *f* (*b* ÷ *d* = 1/3 ÷ 1/2 = 2/3 = 8/12).

b. This problem deals with the concept of fractions and computation with fractions. Students need to identify common denominators—in this case, 12. The problem also looks at the placement of rational numbers on the number line. At this level, students need to be able to think about mathematical problems in more abstract terms. There is also some inference and proof, since they need to build conjectures on previous conjectures and justify their answers. They also need to compare the fractions. Notice that at this grade level the complexity of the problem is slightly higher than simply ordering fractions.

c. This problem requires students to understand the concept of fractions. They must be able to place fractions on a number line; add, subtract, multiply, and divide fractions, and estimate the magnitude and reasonableness of such operations; and order fractions.

d. Here are some questions that may help students who are struggling:

   - What is the difference between 3/4 and 1/6? (7/12) How do you know? (By converting the fractions to the same denominator, we get 3/4 - 1/6 = 9/12 - 2/12 = 7/12. At this level, students need to be able to do this type of computation fluently.)
   - Three-fourths is how many points to the right of 1/6? (The answer is seven. Again, after converting the fractions to the same denominator, students need to be able to make such comparisons and understand how they play out on the number line.)
   - How does this relate to the difference between the two numbers? (The difference is 7/12, which corresponds to the seven equal lengths between 3/4 and 1/6. Therefore, the difference between adjacent points must be 1/12. Again, this type of question strengthens students' understanding of fractions and the representation of fractions on the number line. It also connects those understandings with their understanding of operations with fractions.)

e. A question you could ask to extend students' thinking is, "Where would you place 3/8 on this number line?" In order to place it, they need to change 3/8 to 9/24. They would then realize that 9/24 is between 8/24 and 10/24, or 4/12 and 5/12, and use benchmarks to place 24ths halfway between 12ths.

Solutions, cont'd.

Problem C2.

a. Solution:

1. Enrollment has decreased by 8%. Here's how you know:
   - The enrollment at the end of the first year is 120% of the enrollment of the year before; 120% written as a decimal is 1.2.
   - The enrollment at the end of the second year is 120% of the enrollment of the year before; this is (120%) • (120%), or (1.2) • (1.2) times the original enrollment.
   - The enrollment at the end of the third year is 80% of the enrollment of the year before; 80% written as a decimal is 0.8. This is (120%) • (120%) • (80%), or (1.2) • (1.2) • (0.8) times the original enrollment.
   - The enrollment at the end of the fourth year is 80% of the enrollment of the year before; this is (120%) • (120%) • (80%) • (80%), or (1.2) • (1.2) • (0.8) • (0.8) times the original enrollment.

   Now do the math: (1.2) • (1.2) • (0.8) • (0.8) = 0.9216. The enrollment is 92% of what it was four years ago, which means that it has decreased by 8%.

2. The order of the changes does not matter, since the same four numbers will be multiplied, just in a different order. Multiplication is commutative and associative, so the order of the factors does not change the product.

b. This problem deals with percents, computation with percents, and the notion that percent problems can be represented as a proportion of data values compared to a ratio of the percentage to 100. When working with multiple percents, you need to determine the value of the whole, which changes after each computation.

c. Students need to understand the concept of percent, how to represent percent increases and percent decreases, and how to compute with percents.

d. Here are some questions that may help students who are struggling:

- If there were 100 students enrolled in your school to start and there was a 20% increase, how many students would there be at the end of the first year? (120)
- How do you know? (100 • 120% = 100 • 1.2 = 120)
- If there were 120 students enrolled in your school to start and there was a 20% increase, how many students would there be at the end of that year? (144)
- How do you know? (120 • 120% = 120 • 1.2 = 144)
- If there were 144 students enrolled in your school to start and there was a 20% decrease, how many students would there be at the end of that year? (115)
- How do you know? (144 • 80% = 144 • 0.8 = 115.2, which rounds down to 115)
- If there were 115 students enrolled in your school to start and there was a 20% decrease, how many students would there be at the end of that year? (92)
- How do you know? (115 • 80% = 115 • 0.8 = 92)

e. Here are some questions you could ask to extend students' thinking:

- Is the change in enrollment of 40% up and then 15% down the same as 15% up and then 40% down? (The students will need to understand that this is not the same.)
- If the first year was down by 20%, what increase would bring it back to the original enrollment? (The answer is 25%.)

Solutions, cont'd.

Problem C3.

a. Solution: The smallest positive integer that could be part of the sum is 3. Students need to know that with nine consecutive numbers, the center number will be 63 ÷ 9. Any time you are adding an odd number of integers, the middle is the average of all of them. Since 63 is the sum, the average is 63 ÷ 9, or 7, meaning that the string of consecutive integers starts with 3 (3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = 63).

b. This problem deals with number theory, consecutive integers, divisibility, odd and even numbers, patterns, and logical reasoning.

c. Students would need to understand odd and even numbers and some divisibility tests. Logical and systematic thinking are also necessary skills. Students would need to think about the behavior of sums of consecutive integers. They also need to reason through the pattern by trying out a couple of cases and thinking about similarities in those situations.

d. If students are struggling, you could review the behavior of sums of consecutive integers, as shown above in (a). Students should experiment, using specific examples to test these conjectures. For example, to prove that the sum of three consecutive integers is divisible by 3, they might try the following: 1 + 2 + 3 = 6; 2 + 3 + 4 = 9; 3 + 4 + 5 = 12. This should convince them that the sum of three consecutive integers is always divisible by 3. It is important that they can reason out what the pattern has to be by trying a couple of cases and thinking about the symmetry of the situation. You can also do a picture like this:

   □ - 3, □ - 2, □ - 1, □, □ + 1, □ + 2, □ + 3

   Ask what happens if they add up these numbers. Then ask what happens if they have 9, 11, or another odd number of numbers. Ask why they think this trick doesn't work for summing an even number of numbers.

e. Here are some questions you could ask to extend students' thinking:

   - Could 63 be the sum of two consecutive integers? (Yes, 31 and 32. Any odd number can be obtained by adding two consecutive integers.)
   - Could 63 be the sum of three consecutive integers? (Yes, 20, 21, and 22. Any number that is divisible by three can be obtained by adding three consecutive integers. The middle number of the consecutive integers can be found by dividing the original number by 3.)
   - Could 63 be the sum of four consecutive integers? (No, 63 is not divisible by 4. Any four consecutive integers, when added together, will yield an even number.)
   - If 63 is the sum of six consecutive integers, what is the largest number that could be part of the sum? (It's 13.)
   - The sum of five consecutive integers is 5. What is the smallest of the five integers? (Recognizing that consecutive integers also "go the other way" is important; ... -3, -2, -1, 0, ... are all part of the sequence we care about. You might begin by dividing the sum, 5, by 5 (the number of consecutive integers). This yields 1 as the middle number. So the integers will be -1, 0, 1, 2, 3. The -1 and 1 cancel each other out, so the sum is 5.)

Solutions, cont'd.

Problem C4.

a. Solution:

	Option 1		Option 2	
Day	**This Day's Pay**	**Total**	**This Day's Pay**	**Total**
1	$1,000	$1,000	$1	$1
2	$1,000	$2,000	$2	$3
3	$1,000	$3,000	$4	$7
4	$1,000	$4,000	$8	$15
n	$1,000	$1,000*n*	2^{n-1}	$(2 \cdot 2^{n-1}) - 1$
20	$1,000	$20,000	2^{19} = $524,288	$1,048,575

The second option is clearly better!

b. The content addressed by this problem is exponential growth, computation with large numbers, reasonable estimation, and the graphing of linear and exponential equations. (To learn more about exponential growth, go to *Learning Math: Patterns, Functions, and Algebra* at www.learner.org/learningmath and find Session 7, Part B.)

c. Students need to understand powers, as well as how to compute with exponents, how to organize data, and how to graph data. However, they will likely find that their estimation skills do not extend to situations dealing with exponential growth. We do not often think about problems of exponential growth, so our intuition can be faulty.

d. For students who are struggling, you might ask, "At what point in this process do you catch up? In other words, when do the two options cross each other?" It might be helpful for the students to fill out the table with more values before they jump to Day 20. They may also need assistance in recognizing the patterns, particularly in the exponential function.

e. For Option 2, students will likely write each day's pay by doubling the previous day's pay. After they have done five days, ask students to determine the pay on Day 10 without doing the numbers in between. You could try to have the students notice that each day's pay is 2 to the exponent that is one less than the day number.

   If they are filling in a chart, you can try to have students notice that the totals in the "Total" column are always one fewer than the next day's pay.

   You can also have students graph the functions on one coordinate system to compare the two different types of growth: linear and exponential.

Notes