

Annenberg/CPB Course Guide

Learning Math

Number and Operations

A 10-part video- and Web-based course for K–8 teachers

Produced by WGBH Educational Foundation

Learning Math: Number and Operations

is produced by
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1-800-LEARNER

P.O. Box 2345
S. Burlington, VT 05407-2345

info@learner.org

www.learner.org

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About This Course

The *Learning Math* Courses

When teachers have a deep conceptual understanding of mathematics, it can help their students develop strong mathematical skills and knowledge. *Learning Math* is a series of five multimedia, college-level courses designed to teach mathematics content to elementary and middle school teachers. Organized around the content standards developed by the National Council of Teachers of Mathematics (NCTM), the courses cover Number and Operations; Patterns, Functions, and Algebra; Geometry; Measurement; and Data Analysis, Statistics, and Probability.

Overall Objectives

- To help teachers better understand mathematics content
- To provide engaging explorations of mathematics using video, interactive activities, and problem solving
- To encourage teachers to view mathematics as more than rote sets of rules and procedures

Overview: *Number and Operations*

Learning Math: Number and Operations, a video- and Web-based course for elementary and middle school teachers, examines the three main categories in the Number and Operations strand of *Principles and Standards of School Mathematics* (NCTM)—understanding numbers, representations, relationships, and number systems; the meanings of operations and relationships among those operations; and reasonable estimation and fluent computation. The course covers the real number system, place value, the behavior of zero and infinity, meanings and models of basic operations, percentages, and modeling operations with fractions, often with the aid of concrete, physical models that enhance understanding. It also examines basic number theory topics, such as factors and multiples, as well as divisibility tests, at both practical and abstract levels. Accordingly, parts of the *Number and Operations* course may be more challenging than other *Learning Math* courses.

The course consists of 10 sessions, each with a half hour of video programming, problem-solving activities provided online and in a print guide, and interactive activities and demonstrations on the Web. Although each session includes suggested times for how long it may take to complete all of the required activities, these times are approximate. Some activities may take longer. You should allow at least two and a half hours for each session. The 10th session (choose video program 10, 11, or 12, depending on your grade level) explores ways to apply the concepts of number and operations you've learned in your own classroom. You should complete the sessions sequentially.

Session 1. What Is a Number System?

Understand the nature of the real number system, the elements and operations that make up the system, and some of the rules that govern the operations. Examine a finite number system that follows some (but not all) of the same rules, and then compare this system to the real number system. Use a number line to classify the numbers we use, and examine how the numbers and operations relate to one another.

Session 2. Number Sets, Infinity, and Zero

Continue examining the number line and the relationships among sets of numbers that make up the real number system. Explore which operations and properties hold true for each of the sets. Consider the magnitude of these infinite sets and discover that infinity comes in more than one size. Examine place value and the significance of zero in a place value system.

Session 3. Place Value

Look at place value systems based on numbers other than 10. Examine the base two numbers and learn uses for base two numbers in computers. Explore exponents and relate them to logarithms. Examine the use of scientific notation to represent numbers with very large or very small magnitude. Interpret whole numbers, common fractions, and decimals in base four.

About This Course, cont'd.

Session 4. Meanings and Models for Operations

Examine the operations of addition, subtraction, multiplication, and division and their relationships to whole numbers and integers. Work with area models for multiplication and division. Explore the use of two-color chips to model operations with positive and negative numbers.

Session 5. Divisibility Tests and Factors

Explore number theory topics. Analyze Alpha math problems and discuss how they help with the conceptual understanding of operations. Examine various divisibility tests to see how and why they work. Begin examining factors and multiples.

Session 6. Number Theory

Examine visual methods for finding least common multiples and greatest common factors, including Venn diagram models and area models. Explore prime numbers. Learn to locate prime numbers on a number grid and to determine whether very large numbers are prime.

Session 7. Fractions and Decimals

Extend your understanding of fractions and decimals. Examine terminating and non-terminating decimals. Explore ways to predict the number of decimal places in a terminating decimal and the period of a non-terminating decimal. Examine which fractions terminate and which repeat as decimals, and why all rational numbers must fall into one of these categories. Explore methods to convert decimals to fractions and vice versa. Use benchmarks and intuitive methods to order fractions.

Session 8. Rational Numbers and Proportional Reasoning

Begin examining rational numbers. Explore a model for computations with fractions. Analyze proportional reasoning and the difference between absolute and relative thinking. Explore ways to represent proportional relationships and the resulting operations with ratios. Examine how ratios can represent either part-part or part-whole comparisons, depending on how you define the unit, and explore how this affects their behavior in computations.

Session 9. Fractions, Percents, and Ratios

Continue exploring rational numbers, working with an area model for multiplication and division with fractions, and examining operations with decimals. Explore percents and the relationships among representations using fractions, decimals, and percents. Examine benchmarks for understanding percents, including percents less than 10 and greater than 100. Consider ways to use an elastic model, an area model, and other models to discuss percents. Explore some ratios that occur in nature.

Session 10. Classroom Case Studies

Explore how the concepts developed in this course can be applied at different grade levels through case studies of K-2, 3-5, and 6-8 teachers (former course participants) who have adapted their new knowledge to their classrooms. In this session, view video 10 for K-2 teachers, video 11 for grade 3-5 teachers, and video 12 for grade 6-8 teachers.

About This Course, cont'd.

Course Components

Each *Learning Math* course consists of 10 two-and-a-half-hour sessions. The first nine sessions are devoted to mathematics content; the 10th session covers classroom applications. Concepts are developed within and across the sessions and the sessions increase in difficulty as they progress. Each session includes reading, problem solving, and group or individual activities that are available on the Web and in print, and a half-hour of video viewing, available on the Web,* on the Annenberg/CPB Channel,** or on videocassette.*** There are additional problems and readings to complete for homework.

* Broadband access is required to view the video on the Web; see Tech Tips, page 5.

** The schedule for broadcasts on the Annenberg/CPB Channel can be found on the course Web site.

*** Purchase videocassettes at www.learner.org or by calling 1-800-LEARNER.

Visit the *Number and Operations* Web site at www.learner.org/learningmath.

About This Course, cont'd.

The following components are in each course:

Key Terms

Key mathematical terms relevant to each session are listed at the beginning of that session. These terms are divided into two parts: terms that are new in that session and terms that were introduced in a previous session. Definitions for key terms may be found in the glossary in the Appendix of this guide.

Notes

Notes can be used by facilitators, study groups, or individuals working alone. They provide extended information about the topics presented in the course, including help for dealing with stumbling blocks that may come up and recommendations for different ways to approach the content.

Problems

Each session contains mathematical problems to be solved individually or by groups. Problems build upon previous concepts and increase in difficulty as the course progresses.

Take It Further

The problems marked "Take It Further" are optional and are not counted as part of the two-and-a-half hour time-frame for each session. These problems are designed for individuals who would like to explore a topic in greater depth. They are often more difficult than the other problems in the session, and they may introduce new information or concepts not previously discussed.

Interactive Activities

Each session in the course includes at least one interactive activity on the course Web site. These activities help you learn new mathematics content or reinforce existing knowledge through hands-on exploration directly on the Web. The interactive activities require the Flash plug-in, which you can download for free from Macromedia's Web site (see Tech Tips, page 6). There are also non-Flash versions of each activity that don't require the Flash plug-in and can be completed offline. If you are working with only the guide, the interactive activities have been adjusted and are included in the guide.

Tips

Tips are available for problems you may find more difficult or need help in getting started. Tips may be found at the end of each session in this guide.

Solutions

A solution is provided for every problem in *Learning Math*, with the exception of a few open-ended questions. When solving a problem with multiple parts, consider writing down your answers to all of the parts on paper first before checking the solution, because the answers to each part of the problem will be visible at once on the solution page. Solutions may be found at the end of each session in this guide.

The following sequence of activities will give you a sense of what you will do as a student using *Learning Math*:

1. Watch the **session video** in its entirety. You can watch the video before you begin the session to become comfortable with the material, or you can view the video after completing the session (so as not to view answers to problems).
2. Do problems sequentially. If you are having difficulty, refer to the **Tip**. If you want a challenge, try a **Take It Further**.
3. Check the **Solutions** at the end of each session.
4. You may want to read **Notes** as you go along to establish a deeper context for the content.
5. Watch **video segments** strategically placed throughout the session, either online by clicking on the "Play Video" button or on videocassette by fast-forwarding to the image and approximate time code that appears in the guide. Zero your VCR clock when the Annenberg/CPB logo appears at the beginning of the program to locate the image using the time code.
6. Do **homework problems** and **readings** (available as PDF files online) at the end of each session to reinforce your learning.

About This Course, cont'd.

Session Videos

Each *Learning Math* session includes viewing a video that is available on the course Web site, on videotape, or on the Annenberg/CPB Channel. The videos feature K-8 teachers working on the *Learning Math* course materials in a workshop with a facilitator. The videos for the nine content sessions show onscreen participants as they are introduced to the concepts, work through the problems, sometimes struggle to reach an understanding, and then reflect on what they have learned. At the end of most videos there is an example of how the content from the session is applied in a “real world” situation. The videos for the 10th session show participants from the videotaped workshops as they apply the content that they have learned back in their own classrooms. You may choose to watch each of these videos before or after you work on the associated course session.

Video Segments

Each session includes short excerpts from the associated video, which you watch (or review) and reflect on to see how the onscreen participants grapple with the same or similar problems and concepts you encounter in the course. Instructions are given to find the segments on videotape. The segments are also available on the course Web site, if you are watching the complete programs online or on the Annenberg/CPB Channel.

Homework

Each session includes approximately 45 minutes of homework problems and/or reflective writing assignments that reinforce the session’s content.

Readings

Readings from journals and books are cited at the end of some sessions. They are available on the *Number and Operations* Web site as downloadable PDF files. Go to www.learner.org/learningmath. If you do not have access to the Internet, call 1-800-LEARNER to obtain a free set of readings.

Using the Videos, Guide, and Web Site

Each *Learning Math* course includes sequentially organized problems, video viewing, interactive activities, readings, and homework. The multimedia elements of the course create an exciting environment for probing mathematics content. The course can be taken entirely on the Web, followed in this print guide, or completed using a combination of Web and print. You can watch the videos online, on the Annenberg/CPB Channel, or on videocassette. If you are watching the programs on the Channel, we recommend taping them so you can look at short video segments when prompted in each session. These segments punctuate concepts developed in the course and create a “virtual” community of learners.

Ways To Take *Learning Math*

Learning Math was flexibly designed for a variety of users and situations. You may choose to work through the sessions on your own, in a study group, or as part of a facilitated, face-to-face, graduate-level course for credit.

Channel-Talk

Join an email discussion group and converse with other teachers taking this course. Go to the course Web site at www.learner.org/learningmath and select Channel-Talk.

Registration and Credit

Go to www.learner.org/4gradcredit for details on receiving graduate credit for *Learning Math*.

About This Course, cont'd.

Taking Multiple *Learning Math* Courses

The five *Learning Math* courses are designed to be independent of one another. You can take just one course, several courses, or all five courses in the order that fits your needs or the needs of your group. The courses also complement one another, with some topics discussed in more than one course but approached differently depending on the focus of that course. Taking several of these courses will increase your own conceptual understanding and ultimately that of your students.

Facilitating the Course

You can prepare for facilitating the course by reading through each session and its "Notes" section prior to meeting with your group. Reading through the material will help you become clear about the activities, plan how much time you need to spend on each one, and pull together necessary materials. The course is designed for use by an individual, but the Notes suggest ways for groups to work through the sessions.

Tech Tips

To use all or portions of the *Learning Math* course online, we recommend the following:

Internet Access

A minimum 56K modem connection is required, but an ISDN or high-speed connection is recommended. The slower your connection, the longer it will take to load larger features, such as the Flash activities.

To view the video programs and video segments online, a broadband connection to the Internet (DSL, cable modem, or LAN connection to a T1 line or greater) is required.

Web Browser

You will need Netscape 4.6 (or a higher version) or Internet Explorer 5.0 (or a higher version). Javascript should be enabled, if your browser allows you to disable it. Text fonts and colors, and many features, such as hidden Tips, may not be displayed correctly in older browsers.

Plug-Ins

These plug-ins will allow you to get the most out of the courses. Links to sites where you can download these programs for free are provided on the course Web site.

- Shockwave Flash 5 (or a higher version) for using the Interactive Activities
- Windows Media Player 7 (or a higher version) for watching videos
- Adobe Acrobat Reader for viewing the Readings in the Homework sections

Printing Web Pages

If you are having trouble printing some of the course content pages, you may try doing one or more of the following (from the "Print Preview," "Print...," or "Page Setup..." menu):

- Turn on "Shrink to Fit" mode (IE 5 only)
- Print the page in "Landscape" mode
- Reduce the scale of the printer output

Who's Who

Content Developer/Facilitator



Carol R. Findell, Ed.D.

For more than 30 years, Dr. Findell has been a mathematics educator at the pre-school through college levels. She served as editor of the National Council of Teachers of Mathematics publication *Student Math Notes* and on the editorial board of the *New England Mathematics Journal*, and was author of two *World's Largest Math Events*—*Math Olympics* and *Mathematics: The Language of the Universe*. She was a member and chairperson of the Question Writing Team for the Mathcounts Competition and was head of the writing team for the *Figure This!* national campaign to promote mathematics education reform. Dr. Findell is co-author of many books and a frequent speaker at national conferences. She has participated in several funded projects in mathematics education and has worked with elementary, middle, and high school teachers across the country, primarily in New Hampshire, Massachusetts, and Connecticut.

Onscreen Participants



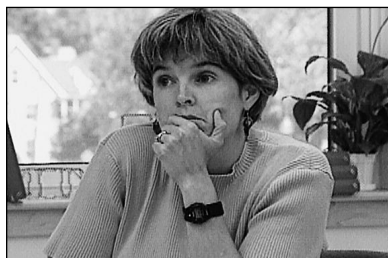
Monique Brinson
Teacher, Grade 3



Ben Carter
Teacher, Grade 6



Rhonda Cherry
Special Education Teacher, Grade 7



Donna Donnell
Math Coordinator, Grades K-5



L.J. Dutton
Teacher, Grade 4



Nancy Gaff
Math Teacher, Grade 8

Who's Who, cont'd.



Andrea Gladkowski
Math Teacher, Grade 8



Elizabeth Hughes
Teacher, Grade 5



Kristen Mann
Teacher, Grade 4



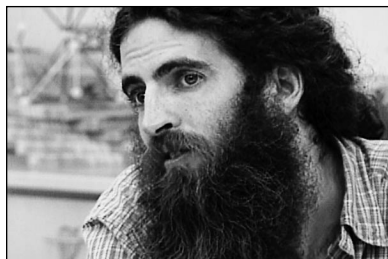
Doug McGlathery
Curriculum Advisor, Grades 9-10



Victoria Miles
Math Teacher, Grade 7



Maria Royston
Teacher, Grade 2



Thomas Szekely
Math Teacher, Grade 8



Rick Tabor
Math Teacher, Grade 8



Jeanne Watts
Math Teacher, Grade 7



Susan Weiss
Math Specialist, Grades K-5