Measurement is the process of quantifying properties of objects. And to do that, we have set procedures that enable us to measure. Measuring helps you understand how things relate to each other. Our volume of a sphere actually has a formula of four-thirds pi r-cubed. This course really made me think about how I approach measurement and how I can use measurement every day in the classroom. Today in our session, we are going to be looking at some measurement fundamentals. We also are going to consider the role of ratio in measurement. And finally, we will be discussing in a little more detail precision and accuracy. To begin, let's look at some of the fundamentals that underlie how we measure. Unit iteration is the repetition of a single unit. Unit iteration is the repetition of a single unit. So when we are measuring, we are actually taking a unit and repeatedly placing it end to end so that we have a complete, for example, length. Or if we have square units, such as square centimeters, we're placing them side by side so they have a complete covering. Now, what's interesting about measurement is we can continually take our units and subdivide them farther and farther and farther. And this is a very important aspect
34 01:02:24:19 01:02:27:27 of being able to become more and more precise.
35 01:02:27:29 01:02:30:25 Now, let's us experiment what that means,
36 01:02:30:27 01:02:33:00 to take a unit and divide it--
37 01:02:33:02 01:02:36:15 or "partition" it is sometimes the term we use--
38 01:02:36:17 01:02:40:27 into smaller and smaller subunits.
39 01:02:40:29 01:02:43:11 NARRATOR: To explore the idea of partitioning,
40 01:02:43:13 01:02:44:27 the class is given a task:
41 01:02:44:29 01:02:49:28 Find $\frac{17}{48}$ on this unit without the benefit of a measuring tool.
42 01:02:50:00 01:02:54:07 This leads many participants to begin the activity by folding.
43 01:02:54:09 01:02:56:03 Do you want to fold the paper to start...".
44 01:02:56:05 01:02:57:23 Just to see what happens.
45 01:02:57:25 01:02:58:26 Okay.
46 01:02:58:26 01:02:59:24 Because it's equal...
47 01:02:59:26 01:03:01:05 that's equidistant, anyway.
48 01:03:01:07 01:03:02:15 It looks like it's pretty close.
49 01:03:02:17 01:03:07:17 So there's our halfway point.
50 01:03:07:19 01:03:11:02 And it would actually be slightly less than that.
51 01:03:11:04 01:03:13:00 I'm all about approximation.
52 01:03:13:02 01:03:14:21 (both laughing)
53 01:03:14:23 01:03:18:29 Um... I'm not sure how we'd find exactly that point, but...
54 01:03:19:01 01:03:20:00 Then we could fold...
55 01:03:20:02 01:03:21:08 You were talking about doing accordion-style more.
56 01:03:21:10 01:03:22:16 accordion-style more.
57 01:03:22:18 01:03:24:14 We could start folding, and each time we fold
58 01:03:24:16 01:03:26:19 or do an accordion, we'd get a little bit more...
60 01:03:27:21 01:03:28:22 Closer to breaking it down.
61 01:03:28:24 01:03:30:09 Okay.
62 01:03:30:11 01:03:33:28 CHAPIN: In the folding activity, what we were interested in doing
63 01:03:34:00 01:03:36:16 was thinking about
"How do we take a unit and subdivide it, or partition it?"

One of the reasons for partitioning a unit into smaller and smaller subunits is for us to also consider "How does that change the number of units in a measure?"

The units are very small, we are going to have a much larger number. It's kind of inversely proportional, that the smaller the unit, the more the unit it's going to take.

So, then we need to find 17/48. So... this is 24.

We have to find the middle point, right? We have 12 here, right?

Yeah, so... We have 12 here, right?

Yeah, 12, and we need to get up to 17.

We just keep folding it in half, can't we?

This section into halves. Mmm.

This second section?

Yeah, this middle...

Yeah, the second section.

So fold the two creases so they're on top.

Do you see that?

Comme ça.

And then that crease...

18?

18.

Mm-hmm.

There are six in between here, right?

I think I would try to divide in three.

Three and three?

Three, three.

Because now it's easier.

Three and then in three again, in the middle.
See? So I have 12, 13, 14... Quinze, dezesseis... (both laughing) Exactly. And then 17.

Here. So, here is our 17/48, okay? Obrigado. De nada. (both chuckling)

I notice many of you took the one unit and divided it in half as kind of the first... by folding. Some of you then may have taken the half and cut it up into thirds, thus to get a sixth... or sixths all the way along. From there, we can take each one of those sixths and cut it up into four pieces so that we have 24ths.

And finally, now that we know that this smallest unit is a 24th, if we cut each one of those in half, or divide it, or partition it in half, we have our 48ths, and then we can figure out where along here is 17/48.

CHAPIN: Is a measure an actual accurate number, a specific, or is it more an approximation? Dave?

It's always going to be somewhat approximate, because let's say that we were going to measure something that we could consider to be a foot and we agree that it's a foot and you stick a ruler next to it and it's 12 inches long on the nose. Who's to say that if we could break down that foot into hundredths of a inch

1/2
or thousandths of an inch

136 01:06:30:14 01:06:33:21 that it wouldn't be
one foot and 1/1,000 of an inch?

137 01:06:33:23 01:06:35:03 Or take it even
crazier than that:

138 01:06:35:05 01:06:36:09 You can keep
breaking it down

139 01:06:36:11 01:06:37:26 into smaller and
smaller partitions

140 01:06:37:28 01:06:39:23 and maybe get a greater
degree of accuracy,

141 01:06:39:25 01:06:42:14 but who's to say that that
isn't the perfect accuracy?

142 01:06:42:14 01:06:44:24 And you can break
that hundredth of an inch
down into hundredths of that
143 01:06:44:26 01:06:46:16 144 01:06:46:16 01:06:49:02 and maybe it'll be
a portion of that.

145 01:06:49:04 01:06:51:17 So it's always going to be
a little bit of approximation.

146 01:06:51:19 01:06:53:29 You just kind of have
to take a leap of faith

147 01:06:54:01 01:06:55:23 and accept it
as truth eventually.


149 01:06:56:25 01:06:58:14 Katie?

150 01:06:58:16 01:06:59:20 The purpose will affect

151 01:06:59:22 01:07:01:20 how approximate
you're allowing it to be.

152 01:07:01:22 01:07:04:09 So, you know, he was talking
about "a foot is a foot,"

153 01:07:04:11 01:07:07:14 but if you're trying to,
you know, build something

154 01:07:07:16 01:07:09:16 and create a watertight seal,

155 01:07:09:18 01:07:11:09 how long your foot is
is different

156 01:07:11:11 01:07:13:25 than someone who's just cutting
a foot of string

157 01:07:13:27 01:07:16:00 to be able to play a game with,

158 01:07:16:02 01:07:19:08 or someone who's creating
metalwork that needs to be,

159 01:07:19:10 01:07:22:27 you know, so precise, or medical
instruments, or whatever.

160 01:07:22:29 01:07:26:21 You know, each person in each
of those industries or functions

161 01:07:26:23 01:07:30:16 would have a different sense
of whether they're ready to say,

162 01:07:30:18 01:07:34:15 "Yes, that foot is a foot."

163 01:07:34:17 01:07:37:15 But if we are
measuring something

164 01:07:37:17 01:07:41:18 with the instruments we have,
can I ever say

165 01:07:41:20 01:07:45:14 "That's absolutely
exactly x length long"?

166 01:07:45:16 01:07:47:22 What's it going to depend upon?

167 01:07:47:24 01:07:51:23 It's going to depend on
the measuring tool you use.

And the precision of the unit that we have.

And as a result, the smaller and smaller unit

is going to give us more and more precision,

but we can always think about getting smaller and smaller, okay?

Since all measurements are approximate,

one way to express this is by the maximum possible error.

This is always half of a measuring unit.

For example, if a measurement were made to the nearest centimeter,

the maximum possible error would be one-half centimeter,

or .5 centimeters.

A measure of approximately ten centimeters

would be stated as ten plus or minus .5 centimeters.

This means that the measurements are between 9.5 centimeters

and 10.5 centimeters.

There are different mathematical entities,

and in particular we sometimes separate counts versus measures.

And there's a difference.

We can count, for example, the number of people

and be very exact.

We can count the number of chairs.

We can count the number of apples we eat.

We can also make measurements,

but we need to be aware that those measurements are not exact

because we can continually narrow down

the size of our unit.

And by having smaller and smaller subunits, we can become

more and more accurate, though perhaps never exact

in terms of the actual measurement.

All measurement is a ratio,
a measure to a standard unit.

Now, we then also can set up proportions of where two ratios are equal to each other.

And I know that many of you are very familiar looking at this proportion.

You may be thinking of it in terms of equivalent fractions as well,

because a fraction is a form of a ratio.

All right, now we are going to use proportion to look at scale,

because scale is one area where ratio is used a great deal in measurement.

We make scale models, scale drawings all the time.

Now, if you look in your packet, you have some pictures of some grasshoppers.

And if we have a scale of one to one,

we then have a ratio where the scale drawing, or the model,

is exactly the same size as the original figure.

If we have a scale of one to two,

you can see we have a reduction.

Okay?

And a scale of two to one is an enlargement--

that that grasshopper is twice as long

and twice as high as the original one.

Well, now, we're going to use this

and, thinking about how we use proportional reasoning,

to think about making some scale drawings.

NARRATOR: Professor Chapin gives the class body measurements of herself

to introduce the next activity on scale drawings.

Her height is approximately 68 inches;

head, nine inches;

arms, 28 inches;

and height to navel, 42 inches.
So, how am I going to calculate how long this figure--

I would just multiply 68 inches by 1/8.

And when you do that...

I just did it on my calculator

8½ inches.

So we now can draw

kind of the top of the person and the bottom of the person

which would be 8½ inches.

All right?

How big is my head going to be on this paper?

I would set up a proportion.

We know the scale is one to eight, so we have one ratio,

and then the other side of the proportion,

we know that the actual measurement is nine inches,

so we're going to put that into a fraction.

We are looking for the scale, so we are looking for the numerator,

and the nine inches was the actual measurement,

so that is the denominator.

It sometimes helps to label, in terms of a ratio,

what each thing is representing--

in terms of this is our scale drawing,

and this is the real me,

In terms of just making sense of how we want to set that up.

All right?

So then we can solve this in a lot of different ways.

We can use our cross products and divide.

We can think about going across here...

So, what is the length of my head?

1 1/8 inches.

1 1/8 inches.
All right?

So now we've got--

and I'm just making this...

this isn't accurate here--

that this was the 8½ inches on the paper that might be me.

And then we're going to go down one end,

you know, 1/8 inch and put the head.

All right?

Now, what I'm going to ask you to do is to choose one of you at your table and to sketch out a scale drawing of one of you on your paper.

(people murmuring)

How's this?

Uh... okay.

Okay?

Your head up a little bit.

Put my head up?

It's about nine.

Nine inches?

(people murmuring in background)

Twenty-six and a half.

Okay.

Take the slack out of it.

41.

41?

Yeah.

41 it is.

Well, actually, eight... eight inches.

Measure point to point is 17 inches.

We looked at the idea that a ratio is used in measurement as a scale, as a comparison, or a reduction, and we investigated how we could make a scale drawing of ourselves that would fit on a piece of paper.

and how we could, by using ratios, make the drawing of ourselves proportional.
and so it looked right—that our head was not too big,
and our arms were not too long,
and our legs were about the right length.
I noticed that many of you were able to make a scale drawing,
and, Gin, can you share with us yours?
Take a look here how proportional this person is in terms of the head, the arm length, the leg length, and that it is a reduction.
What was your scale factor that you used?
One to seven, or one-seventh reduction.
All right.
We do want to, though, look at some constants that are involved with ratio.
One that we'll look at in other sessions is pi, but today we will explore what takes place in isosceles right triangles.
You have at your table some sheets of right triangles.
You also have a chart similar to this one,
and what I would like us to do is, using our rulers measuring to the nearest tenth of a centimeter, or a millimeter, we're going to measure the hypotenuse and form a ratio of the side to side on that triangle.
What you may want to do is take that ratio, if you get it, and reduce it using your calculator.
We're measuring this one with the one-inch...
This one is one inch, one centimeter.
And then this is one...
so this would be 13 centimeters--
this would be 1.3 centimeters.
And the other one
is, um...

331 01:15:29:27 01:15:33:29  forty... two.
332 01:15:34:01 01:15:34:28  Forty-two?
333 01:15:37:14 01:15:41:17  DAVE: When we did the activity with
the isosceles right triangle,

334 01:15:41:19 01:15:45:08  we had right triangles that had
legs of units one unit long
335 01:15:45:10 01:15:47:04  through units six units long,
336 01:15:47:06 01:15:49:21  and then we're also asked,
with a ruler,
337 01:15:49:23 01:15:52:29  to the nearest tenth of
a centimeter, or millimeter,
338 01:15:53:01 01:15:54:19  to measure the hypotenuse,
339 01:15:54:21 01:15:57:08  and set them up into ratios,
and we found
340 01:15:57:10 01:16:00:02  that each one of those
came out to a number
341 01:16:00:04 01:16:01:16  that was very, very close
342 01:16:01:18 01:16:03:16  to the approximation
of the square root of two.
343 01:16:03:18 01:16:05:03  But then, when you think,
344 01:16:05:05 01:16:07:24  "Well, how did I measure that
being the square root of two?"
345 01:16:07:26 01:16:09:20  That's not something
that I can measure.
346 01:16:09:22 01:16:11:28  The square root of two is
not a measurable number,
347 01:16:12:00 01:16:13:06  because it's irrational,
348 01:16:13:08 01:16:15:04  but yet here it is
right in front of me.
349 01:16:15:06 01:16:18:18  Can I have everybody take a look
up here at this chart,
350 01:16:18:20 01:16:21:29  where I've put some of
the measurements that I made?
351 01:16:22:01 01:16:25:01  You'll probably find
that yours are very similar,
352 01:16:25:03 01:16:26:19  if not exactly the same.
353 01:16:26:21 01:16:29:20  And then what I did was
I started to relate
354 01:16:29:22 01:16:34:06  the ratio of the hypotenuse in
these isosceles right triangles
355 01:16:34:08 01:16:36:00  to the side length,
356 01:16:36:02 01:16:39:00  so in the first one,
I got 1.4 to one;
357 01:16:39:02 01:16:41:09  second one I got 2.9 to two,
358 01:16:41:11 01:16:44:27  which I can then reduce
down to 1.45 to one.
359 01:16:44:29 01:16:47:11  And then we get
4.3 to three...
360 01:16:47:13 01:16:50:04  or does anybody
have a calculator?
361 01:16:50:06 01:16:52:16  What's that going
to reduce down to?
362 01:16:52:18 01:16:55:08  1 43/100 to one.
363 01:16:55:10 01:16:56:23  CHAPIN:
Okay.

Now, if we keep going, what do you notice about each of these numbers?

They're all approximations for the square root of two.

Okay.

So, what kind of a number is the square root of two?

I think it is an irrational number.

Now, can anyone fill us in on what is an irrational number?

How would we define it?

It's a number that you can't write in a fraction form,

like a under b.

Right.

We often represent numbers as a over b where b does not equal zero,

and an irrational number is one that cannot be put into that form.

Likewise, when it is written as a decimal,

it is not a terminating decimal, nor is it a repeating decimal.

It goes on infinitely without repeating.

That brings us to some interesting questions, then,

about finding the actual length of the hypotenuse.

If it is making us think that it's the square root of two,

maybe we would like to think about using the Pythagorean theorem

as rather than directly measuring.

which is what we've just done, to see about deriving the measurement

of some of these isosceles right triangles.

Just as a quick review,

the Pythagorean theorem is that in a right triangle,

we can say that a-squared plus b-squared equals c-squared...
where either of the sides are a and b and the hypotenuse is always known as c. We happen to be working with triangles where the two sides are exactly the same, right? Now, let's explore that and see what happens. if we put in some values for the sides, not measuring but using the Pythagorean theorem. If we have our side length of one, we can then put one-squared plus one-squared is going to give us two. And so we know that c-squared equals two. Put that over here. And so if we want to find the actual length of c, we know that it's going to be... the square root of two. Here we were able to use a-squared plus b-squared equals c-squared to find the length of the hypotenuse. and write it as a value times the square root of two-- not terribly practical when we need to actually go out and make a measurement. or actually find a length, but on the other hand, very accurate when we are saying, "How long is this?" realizing that we have got a constant-- we are multiplying by the square root of two times the side length each time. Well, we've had a lot of things that we've covered today.

We have looked at some of the fundamental ideas of measurement and especially the unit and how we can cut that unit up into as many subunits as we want. We've looked at the role of ratio.
and how important ratio is to measurement. And finally, we're starting to explore some of the role of irrational numbers in measurement and how that is going to impact our interpretation of the actual measure. (playing Prelude from Bach's Suite No. 1 for Cello Solo)

Cellist Owen Young is playing a Bach prelude on an 18th-century cello made by master craftsman Gennaro Gagliano. It comes from an age renowned for its hand-crafted string instruments, and is today highly valued for its exceptional tone and beauty. In the shadow of the Old North Church in Boston, there is a school that is carrying on this tradition of craftsmanship. The North Bennet Street School was started in 1888 to train immigrants to become productive workers in the United States, and it continued on in that capacity serving community needs for probably a good part of the next 75 years, and then it gradually evolved into a school that provided training in traditional crafts, violin making being one of those. As a teacher at the North Bennet Street School, David Polstein provides his students with the hands-on training they need to master the intricacies of making an instrument such as a violin. Over time, the violin has become very standardized to certain measurements,
and the degree of accuracy

An example of accurate measuring would be checking the thicknesses of the top or back of an instrument. We’re using a dial caliper, which is specifically used to measure thicknesses.

There’d be a determination of what the appropriate thickness of a top would be, and then check everywhere with a gauge like this to make sure that you had accurately reached the measurements you set out to get.

On all violin-family instruments, a brace is put into the top--glued in and fit--and the location of that brace is determined in part by some proportional relationships. One would determine where the center of the instrument was, and then measuring from the center at the widest part of the upper and lower bout, and that measurement would then be divided into seven parts.

The divider would be at a narrower dimension, and then the inclination of this brace relative to the center line would be determined by this one-seventh measurement.

A lack of precision can have a steamrolling effect. If one thing is wrong, it leads to another thing being slightly wrong, and it grows exponentially.

For example, the width of this thing might not be that critical, but for example, if the sides are not actually square to each other,
any layout that you do might lead to a compounding error--
the relationship between the two sides would tend to be different.
A number of relationships are important in laying out the basic parts of the violin.
There's a relationship between the length of the neck and what's called the body stop, which is a measurement taken from the side of the neck to the center of the bridge, and that measurement should be three parts to two parts the length of the neck. Another ratio relationship would be the length of the fingerboard, which is determined by the overall length of the strings, so you take the string length, and the fingerboard would be five-sixths of that length. Another ratio measurement would be this measurement from the tailpiece to the bridge, and that measurement is usually one-sixth of the total string length. Because a violin-family instrument does not have frets, the player has to know where to put their fingers, and there are certain cues in terms of the shaping of the neck, and the location of particular parts.
As the player shifts, when they reach certain cues like this part of the neck or the body, they know they're going to be at a certain note. and if the proportion is wrong, then when they get to that point, they're going to be at the wrong note. Setups of most old instruments
don't correspond
to what the modern settings are.
Baroque settings--
the original settings,
say, on an instrument
made by Stradivari,
would be slightly different.
It's quite possible
the neck would have been
a little shorter.
So these modern ratios
have evolved
to meet the specific playing
demands of modern players
and the evolution
of the quality of strings
and the fact that you would be
playing in a much larger hall
to larger audiences.
( prelude continues )
Making a stringed instrument
has an interesting reward,
because not only
when you're done
you have possibly
a visually satisfying object,
but presumably,
you have something
that can be used
to make beautiful music
and will give an individual
lasting pleasure in its use
over many, many years.
(piece concludes )
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