1	01:00:47:10	01:00:48:20	CHAPIN: Measurement is the process
2	01:00:48:22	01:00:51:23	of quantifying properties of objects.
3	01:00:51:25	01:00:54:20	And to do that, we have set procedures
4	01:00:54:22	01:00:56:21	that enable us to measure.
5	01:00:56:23	01:00:58:13	Oh.
6	01:00:58:15	01:01:00:27	Measuring helps you to understand
7	01:01:00:29	01:01:03:08	how things relate to each other.
8	01:01:03:10	01:01:07:06	Our volume of a sphere actually has a formula
9	01:01:07:08	01:01:09:17	of four-thirds pi r-cubed.
10	01:01:09:19	01:01:13:03	This course really made me think about how I approach measurement
11	01:01:13:05	01:01:16:06	and how I can use measurement every day in the classroom.
12	01:01:21:09	01:01:22:24	In our session today,
13	01:01:22:26	01:01:26:03	we are going to be investigating relationships
14	01:01:26:05	01:01:28:17	between different types of measures.
15	01:01:28:19	01:01:32:00	So far we've really delved
16	01:01:32:02	01:01:36:24	into looking at mass or length or area
17	01:01:36:26	01:01:39:03	and so today we're going to see:
18	01:01:39:05	01:01:43:06	Are there relationships between some of those measures?
19	01:01:43:08	01:01:48:04	In particular, we're going to consider the relationship
20	01:01:48:06	01:01:50:14	between the perimeter and the area of a shape
21	01:01:50:16	01:01:53:07	and we're also going to look
22	01:01:53:09	01:01:58:02	at the relationship between the surface area
23	01:01:58:04	01:02:01:02	and the volume of a solid.
24	01:02:01:04	01:02:05:01	Our second part of the session, we will actually be looking
25	01:02:05:03	01:02:08:16	at how can we use some of these ideas and apply them
26	01:02:08:18	01:02:11:07	to the solution of some interesting problems.
27	01:02:13:21	01:02:16:01	Imagine that you have just bought
28	01:02:16:03	01:02:18:20	an adorable Highland terrier puppy.
29	01:02:18:22	01:02:21:10	Cute little thing, all right?
30	01:02:21:12	01:02:23:21	However, you live on a busy street.
31	01:02:23:23	01:02:29:20	You need to have a safe pen for that dog to be outside in.
32	01:02:29:22	01:02:31:28	You have limited backyard space
33	01:02:32:00	01:02:34:11	and you also have a limited budget
34	01:02:34:13	01:02:40:23	so you go out and buy 72 feet

			of fencing, all right?
35	01:02:40:25	01:02:43:16	Now, what I'd like us to do
36	01:02:43:18	01:02:47:29	is investigate how we can shape that fencing
37	01:02:48:01	01:02:50:11	into a rectangular pen.
38	01:02:50:13	01:02:53:08	What would the dimensions of the pen be
39	01:02:53:10	01:02:58:07	so that our puppy would have a good area
40	01:02:58:09	01:03:01:08	to run around in, okay?
41	01:03:01:10	01:03:04:17	And, in fact, we want to try to get the maximum area
42	01:03:04:19	01:03:08:03	so that the puppy can really, you know, take advantage
43	01:03:08:05	01:03:10:04	of the time that it's outside.
44	01:03:11:21	01:03:12:19	It was 72.
45	01:03:12:21	01:03:13:21	Right.
46	01:03:13:23	01:03:15:24	If we divide it by four, we get 18.
47	01:03:15:26	01:03:17:17	Mm-hmm.
48	01:03:20:07	01:03:25:02	That means a square of 18.
49	01:03:25:04	01:03:26:23	Then we'd square 18
50	01:03:26:25	01:03:30:09	to find the area.
51	01:03:30:11	01:03:32:16	Okay, so 18 x 18
52	01:03:32:18	01:03:37:18	would give us 324 square feet of area
53	01:03:37:20	01:03:40:26	with the 72 linear feet of perimeter.
54	01:03:40:28	01:03:42:18	Squared.
55	01:03:42:20	01:03:43:22	Yeah.
56	01:03:43:24	01:03:45:15	Now we can try other combinations
57	01:03:45:17	01:03:47:01	that would add up to the 72
58	01:03:47:03	01:03:48:19	so we could just... we could...
59	01:03:48:21	01:03:50:01	instead of making a square,
60	01:03:50:03	01:03:51:16	we could make it rectangular.
61	01:03:51:18	01:03:53:17	CHAPIN: In our first activity,
62	01:03:53:19	01:03:56:14	we looked at holding a perimeter constant--
63	01:03:56:16	01:03:57:26	in fact, at 72 feet.
64	01:03:57:28	01:03:59:10	And then we looked
65	01:03:59:12	01:04:02:16	at what shapes we could build using that perimeter.
66	01:04:02:18	01:04:05:18	Starting first with rectangular shapes,
67	01:04:05:20	01:04:08:14	we found that there was a wide variety of rectangles
68	01:04:08:16	01:04:12:00	that would fit a set perimeter of 72

69	01:04:12:02	01:04:13:12	but their area differed.
70	01:04:13:14	01:04:17:20	And the one that had the greatest area was a square.
71	01:04:17:22	01:04:20:25	We started off with our triangle
72	01:04:20:27	01:04:23:10	that had 249 square feet,
73	01:04:23:12	01:04:27:03	and then we went to a hexagon
74	01:04:27:05	01:04:30:10	that had 374 square feet.
75	01:04:30:12	01:04:33:04	So maybe we should try a circle now
76	01:04:33:06	01:04:35:17	with a perimeter of 72 feet
77	01:04:35:19	01:04:37:26	and see what our area is.
78	01:04:37:28	01:04:41:11	CHAPIN: I then pushed people to think about using other shapes.
79	01:04:41:13	01:04:47:11	What if we considered a hexagon or a triangle or a circle?
80	01:04:47:13	01:04:49:03	What area were we going to get
81	01:04:49:05	01:04:51:11	when we still kept the perimeter at 72?
82	01:04:51:13	01:04:52:23	And participants found
83	01:04:52:25	01:04:56:29	that the circle gave them the maximum area.
84	01:04:57:01	01:04:59:12	The barn gives us 72 feet
85	01:04:59:14	01:05:00:22	of perimeter to work with.
86	01:05:00:24	01:05:02:20	But we don't have to use the whole thing, right?
87	01:05:02:22	01:05:06:18	The 72 feet of fence and cut it in half,
88	01:05:06:20	01:05:09:04	and we get 36 feet of fence
89	01:05:09:06	01:05:11:12	and make an opposite side
90	01:05:11:14	01:05:13:19	of a rectangle that's 36.
91	01:05:13:21	01:05:17:14	That would leave us with 18 feet of fence
92	01:05:17:16	01:05:19:27	to complete the rectangle.
93	01:05:19:29	01:05:23:11	18 and 18 gives us an area of 648.
94	01:05:23:13	01:05:26:06	Much larger than the square.
95	01:05:26:08	01:05:30:19	CHAPIN: Finally we pushed that problem to one step further
96	01:05:30:21	01:05:34:14	and looked at using that 72 feet as our perimeter

97	01:05:34:16	01:05:38:10	but that we could have another structure, in this case a barn,
98	01:05:38:12	01:05:41:05	that would form one side of our shape
99	01:05:41:07	01:05:44:21	and again we... we explored what happens
100	01:05:44:23	01:05:46:29	if we build squares off the side of the barn,
101	01:05:47:01	01:05:49:12	if we build trapezoids off the side of the barn,
102	01:05:49:14	01:05:52:13	rectangles and eventually a semicircle.
103	01:05:52:15	01:05:56:24	Ooh, the answer is... 8 1/4.
104	01:05:56:26	01:05:59:24	Oh, so the area is 825... square feet.
105	01:05:59:26	01:06:01:25	So considerably more.
106	01:06:01:27	01:06:02:27	Much, much bigger.
107	01:06:02:29	01:06:05:00	A third more.
108	01:06:05:02	01:06:06:29	Well, that kind of proves us right, then.
109	01:06:07:01	01:06:09:05	That the circle does take up the maximum area,
110	01:06:09:07	01:06:10:13	even if it's a semicircle.
111	01:06:10:15	01:06:11:18	Right.
112	01:06:11:20	01:06:14:19	So we could design a circular fence,
113	01:06:14:21	01:06:16:13	be in the same, exact perimeter,
114	01:06:16:15	01:06:18:15	but if we do it in a circular form,
115	01:06:18:17	01:06:19:23	we give the, uh, puppy
116	01:06:19:25	01:06:21:07	the most area to play in.
117	01:06:24:03	01:06:26:09	Let's take a look at some of these problems
118	01:06:26:11	01:06:28:20	that we were exploring.
119	01:06:28:22	01:06:31:12	When you kept the perimeter at 72--
120	01:06:31:14	01:06:34:03	namely, we sometimes refer to that
121	01:06:34:05	01:06:36:05	as keeping the perimeter constant--
122	01:06:36:07	01:06:38:29	you were forming different rectangular shapes
123	01:06:39:01	01:06:41:06	and looking at their areas.
124	01:06:41:08	01:06:43:24	What conclusions did you come to
125	01:06:43:26	01:06:48:16	in regards to the shape that gives you the greatest area
126	01:06:48:18	01:06:51:24	when the perimeter is held constant?
127	01:06:51:26	01:06:54:02	We discovered that the square

128 01:06:54:04 01:06:56:29 gave us the most...
the largest area
129 01:06:57:01 01:07:00:28 and that as we
change our dimensions
on one side,
130 01:07:01:00 01:07:03:14 increase them,
um, by one number
131 01:07:03:16 01:07:04:16 and decrease them
132 01:07:04:18 01:07:06:18 by the same amount
on the other side,
133 01:07:06:20 01:07:08:21 that our area
got smaller.
134 01:07:08:23 01:07:11:27 Great. Now, we then
investigated some other shapes
135 01:07:11:29 01:07:15:11 so if we kept our perimeter
constant with other shapes,
136 01:07:15:13 01:07:19:05 what are the relationships there
in terms of area?
137 01:07:19:07 01:07:22:25 Did anyone find an interesting
shape that gave them more area?
138 01:07:22:27 01:07:24:04 Katy.
139 01:07:24:06 01:07:26:19 We tried, um,
a hexagon
140 01:07:26:21 01:07:30:15 and we tried to look
at an octagon
141 01:07:30:17 01:07:34:21 and then at a 12-sided
dodecagon, um,
142 01:07:34:23 01:07:36:28 and then a circle
and we found
143 01:07:37:00 01:07:39:03 that the more
sides you had,
144 01:07:39:05 01:07:40:19 the more area inside
145 01:07:40:21 01:07:42:24 as you went
toward a circle.
146 01:07:42:26 01:07:44:17 So if you were building
this pen
147 01:07:44:19 01:07:47:00 and really wanted
to have the maximum area,
148 01:07:47:02 01:07:48:20 what would your conclusion be
149 01:07:48:22 01:07:51:06 about what shape
you should have the pen, um,
150 01:07:51:08 01:07:52:28 you should build the pen into?
151 01:07:53:00 01:07:54:26 It should be a circle
if that were practical.
152 01:07:54:28 01:07:57:04 Okay, yeah, and then we get
into practicality.
153 01:07:57:06 01:07:58:09 Can we... can we do that?
154 01:07:58:11 01:08:00:19 Now, the last problem
155 01:08:00:21 01:08:04:17 was looking at using
a side of a barn,
156 01:08:04:19 01:08:12:08 and that barn had one side
that was 70 feet
157 01:08:12:10 01:08:16:20 and so we wanted
to build off that side.
158 01:08:16:22 01:08:18:06 Anyone want to come up
159 01:08:18:08 01:08:21:04 and share one possible pen

that they found?
160 01:08:24:23 01:08:26:13 Laura, come on up.
161 01:08:32:28 01:08:34:23 We found that
we didn't really need
162 01:08:34:25 01:08:36:19 to use the whole side
of the barn.
163 01:08:36:21 01:08:38:07 Okay.
164 01:08:38:09 01:08:42:23 If, um, our best shape
was a square
165 01:08:42:25 01:08:46:21 and we had 72 feet
of fencing,
166 01:08:46:23 01:08:50:03 we took our 72 feet
and just divided it by three
167 01:08:50:05 01:08:51:18 because we only need
168 01:08:51:20 01:08:54:07 to make three more sides
off our barn
169 01:08:54:09 01:08:58:13 and that gave us
a number of 24.
170 01:08:58:15 01:09:05:24 So if we make
each length here 24,
171 01:09:05:26 01:09:07:12 we don't need to use
the whole barn
172 01:09:07:14 01:09:08:26 and I forget
what the area was.
173 01:09:08:28 01:09:11:26 It was 24 x 24
for our area
174 01:09:11:28 01:09:14:24 and that comes up
to be...
175 01:09:14:26 01:09:15:25 CLASS:
576.
176 01:09:15:27 01:09:16:25 Five seventy-six.
177 01:09:16:27 01:09:17:25 And we... we found
178 01:09:17:27 01:09:19:29 that that would be
the best area
179 01:09:20:01 01:09:21:26 for three sides
of a fence.
180 01:09:21:28 01:09:23:12 576 what?
181 01:09:23:14 01:09:24:27 Square feet.
182 01:09:24:29 01:09:26:14 Sorry.
183 01:09:26:16 01:09:28:15 Okay. Great.
184 01:09:28:17 01:09:30:18 And our drawing
is not to scale,
as we can see,
185 01:09:30:20 01:09:32:07 but we can
at least get...
186 01:09:32:09 01:09:34:06 So we used
the idea of a square.
187 01:09:34:08 01:09:36:11 Now, did anyone use
any other ideas
188 01:09:36:13 01:09:39:04 in terms of building
a shape off of the barn
189 01:09:39:06 01:09:43:27 from what we've discovered
about maximum, um, area?
190 01:09:43:29 01:09:46:25 First experimenting with
rectangles determined, um,
191 01:09:46:27 01:09:49:16 that we could actually

make a rectangular shape
192 01:09:49:18 01:09:50:23 that would be bigger
193 01:09:50:25 01:09:53:06 than making it out
of a three-sided
square--
194 01:09:53:08 01:09:55:07 using three sides
to make a square--
195 01:09:55:09 01:09:57:28 and we're happy
with our conclusions
from there.
196 01:09:58:00 01:10:00:19 And from there
we went into
a semicircle.
197 01:10:00:21 01:10:01:27 We knew
the circumference
198 01:10:01:29 01:10:04:02 of the fencing
that we had to use
was 72 feet,
199 01:10:04:04 01:10:06:22 doubled it--
because we were only
using half a circle--
200 01:10:06:24 01:10:08:10 came out with
a diameter of 45.8--
201 01:10:08:12 01:10:09:16 the exact,
same numbers--
202 01:10:09:18 01:10:11:15 um, cut that in half
to find a radius...
203 01:10:11:17 01:10:12:17 Good.
204 01:10:12:19 01:10:13:22 of 22.9 feet,
205 01:10:13:24 01:10:15:25 and from there we
used the formula
206 01:10:15:27 01:10:17:07 for the area
of a circle--
207 01:10:17:09 01:10:18:26 area equals pi r-squared--
208 01:10:18:28 01:10:21:28 but because we
were only using
half a circle,
209 01:10:22:00 01:10:24:22 we did area equals
pi r-squared over two.
210 01:10:24:24 01:10:27:12 Uh, plugging
in our radius of 22.9,
211 01:10:27:14 01:10:30:27 uh, squaring it, multiplied
by pi and divided by two,
212 01:10:30:29 01:10:32:12 we came up with an area
213 01:10:32:14 01:10:35:28 of approximately,
rounded off,
um, 825 square feet.
214 01:10:36:00 01:10:39:10 The circular
or semicircular pattern seemed
215 01:10:39:12 01:10:41:06 to have maximized area
216 01:10:41:08 01:10:43:19 and it also maximized
the use of the barn.
217 01:10:43:21 01:10:45:29 It used, you know,
45.8 feet of the barn,
218 01:10:46:01 01:10:47:18 as opposed
to the rectangles,

219	01:10:47:20	01:10:50:27	which used 24 or 36 feet of the barn.
220	01:10:50:29	01:10:53:00	So we're maximizing our extra side,
221	01:10:53:02	01:10:54:10	not using our fencing.
222	01:10:54:12	01:10:55:14	Great.
223	01:10:59:17	01:11:03:00	Now, in our next activity, we are going to look
224	01:11:03:02	01:11:06:17	at relationships between volume and surface area
225	01:11:06:19	01:11:10:05	and see if we can, again, start to, um, recognize
226	01:11:10:07	01:11:14:15	when and when there are not relationships that we can use.
227	01:11:14:17	01:11:18:14	In this case, we're going to keep the volume constant.
228	01:11:18:16	01:11:22:18	We are going to start with a volume of 24 cubic units,
229	01:11:22:20	01:11:24:05	which you have here.
230	01:11:24:07	01:11:26:17	CHAPIN: We looked at one activity
231	01:11:26:19	01:11:30:10	where the volume was kept constant at 24 cubic units,
232	01:11:30:12	01:11:34:13	and participants were asked to use the 24 cubic units
233	01:11:34:15	01:11:37:00	to build rectangular prisms.
234	01:11:37:02	01:11:39:20	Then they were to calculate the surface area
235	01:11:39:22	01:11:42:02	of those rectangular prisms
236	01:11:42:04	01:11:46:23	and notice which solids had the greatest surface area
237	01:11:46:25	01:11:49:22	and which had the least surface area.
238	01:11:49:24	01:11:52:21	Well, we noticed as it became more like a cube,
239	01:11:52:23	01:11:54:17	the surface area became less.
240	01:11:54:19	01:11:55:19	Oh.
241	01:11:55:21	01:11:58:17	So the one that... that was very flat--
242	01:11:58:19	01:12:01:03	a long rectangle, a 24 x 1 x 1--
243	01:12:01:05	01:12:02:18	had a surface area of 98.
244	01:12:02:20	01:12:03:27	CHAPIN: 98 what?
245	01:12:03:29	01:12:06:02	Um, 98 units.
246	01:12:06:04	01:12:07:09	Okay, square units.
247	01:12:07:11	01:12:09:00	Square units, that's right.
248	01:12:09:02	01:12:10:11	Um, yet this one
249	01:12:10:13	01:12:12:15	that we just ended up with
250	01:12:12:17	01:12:14:05	with the 4 x 3 x 2,
251	01:12:14:07	01:12:16:09	has only 52 square units.

252	01:12:16:11	01:12:17:26	So it's much less.
253	01:12:17:28	01:12:19:11	It's almost half.
254	01:12:19:13	01:12:22:03	So how could we generalize this
255	01:12:22:05	01:12:26:06	in terms of what type of a rectangular solid
256	01:12:26:08	01:12:28:21	has a smaller surface area
257	01:12:28:23	01:12:32:05	and what type of a rectangular solid
258	01:12:32:07	01:12:34:19	has a large surface area?
259	01:12:34:21	01:12:36:02	Well, a cube would have
260	01:12:36:04	01:12:38:16	the smallest surface area using, um...
261	01:12:38:18	01:12:40:19	but I don't think we can make it
262	01:12:40:21	01:12:42:06	a perfect cube with 24.
263	01:12:42:08	01:12:43:07	But if we could.
264	01:12:43:09	01:12:44:12	But if we could,
265	01:12:44:14	01:12:46:09	it would have the smaller surface area.
266	01:12:46:11	01:12:48:16	Um, and as... as one dimension,
267	01:12:48:18	01:12:49:27	um, stays at one,
268	01:12:49:29	01:12:52:29	that is if we kept the height at one,
269	01:12:53:01	01:12:54:22	then that would give us...
270	01:12:54:24	01:12:57:09	yield the largest surface area
271	01:12:57:11	01:12:59:18	using that... given, um, volume.
272	01:12:59:20	01:13:03:16	CHAPIN: We limited this problem by using only rectangular prisms,
273	01:13:03:18	01:13:05:14	and what participants concluded
274	01:13:05:16	01:13:09:24	was that solids that were more compact, almost more cubelike,
275	01:13:09:26	01:13:11:19	had a smaller surface area
276	01:13:11:21	01:13:15:07	than those rectangular prisms that were very spread out
277	01:13:15:09	01:13:16:28	or elongated.
278	01:13:17:00	01:13:19:11	They had a much greater surface area.
279	01:13:19:13	01:13:21:21	So we need to build a cube
280	01:13:21:23	01:13:22:28	that's 4 x 4 x 4.
281	01:13:23:00	01:13:24:20	So you start at the base.
282	01:13:24:22	01:13:27:05	You start at the base that's 4 x 4.
283	01:13:27:07	01:13:29:18	Now we need to just build up four.
284	01:13:29:20	01:13:32:13	CHAPIN: The next activity looked at cubes

285	01:13:32:15	01:13:35:28	that were progressively getting larger and larger
286	01:13:36:00	01:13:38:29	and in each case, I asked the participants
287	01:13:39:01	01:13:42:28	to calculate the volume and calculate the surface area
288	01:13:43:00	01:13:45:22	of the cube
289	01:13:45:24	01:13:49:14	and to look to see what kinds of relationships existed
290	01:13:49:16	01:13:52:23	in these cubes between surface area and volume.
291	01:13:52:25	01:13:55:25	They were to represent that relationship
292	01:13:55:27	01:13:58:09	as a ratio in most reduced form.
293	01:13:58:11	01:14:01:13	So, our surface area...
294	01:14:01:15	01:14:05:12	we have each face is 16...
295	01:14:05:14	01:14:07:21	so we have our surface area
296	01:14:07:23	01:14:10:16	is 16 times six...
297	01:14:10:18	01:14:14:03	96... square units.
298	01:14:14:05	01:14:17:18	Our volume is
299	01:14:17:20	01:14:20:08	4 x 4 x 4...
300	01:14:20:10	01:14:23:09	or four cubed, which is...
301	01:14:23:11	01:14:26:16	64, I believe.
302	01:14:26:18	01:14:28:29	So, the ratio would be...
303	01:14:29:01	01:14:30:10	(*chuckles*)
304	01:14:33:22	01:14:39:08	96 divided by 16 would be six to four, or...
305	01:14:39:10	01:14:40:14	three to two?
306	01:14:40:16	01:14:41:20	Three to two.
307	01:14:43:09	01:14:45:17	CHAPIN: Again, we were looking at
308	01:14:45:19	01:14:48:12	the relationship between these measures;
309	01:14:48:14	01:14:50:26	that it's not a static relationship,
310	01:14:50:28	01:14:54:01	that that relationship changes
311	01:14:54:03	01:14:57:22	based on the size of the figure and the shape of the figure.
312	01:14:57:24	01:14:59:13	In the sense of the cubes,
313	01:14:59:15	01:15:04:13	that the ratio decreases as the cubes get larger and larger.
314	01:15:04:15	01:15:07:12	And that ratio between surface area and volume decreases
315	01:15:07:14	01:15:11:05	as the sides get larger and larger on a cube.
316	01:15:11:07	01:15:14:08	If we do a 4 x 4 x 4...
317	01:15:14:10	01:15:18:08	Six to four reduces down to 1.5 to one.
318	01:15:18:10	01:15:23:02	Now, these ratios are an interesting thing to think about
319	01:15:23:04	01:15:26:05	in terms of applications.
320	01:15:26:07	01:15:31:07	One application is when we're building, um, structures
321	01:15:31:09	01:15:38:27	and we want to have a large volume but a small surface area.

322	01:15:38:29	01:15:44:09	And so, stores often consider "How can I..."
323	01:15:44:11	01:15:46:11	you know, "What shape can I build this in
324	01:15:46:13	01:15:48:11	"that will give me that kind of a ratio
325	01:15:48:13	01:15:50:14	"where it's not going to cost me too much money
326	01:15:50:16	01:15:51:28	"to put up the outside structure,
327	01:15:52:00	01:15:53:23	but it'll give me a large volume inside."
328	01:15:57:08	01:16:00:21	In this next session, we are going to investigate
329	01:16:00:23	01:16:05:25	how volume will change as we construct different tanks.
330	01:16:05:27	01:16:08:12	So our problem that we're going to be looking at
331	01:16:08:14	01:16:13:12	involves a sheet of metal... okay?
332	01:16:13:14	01:16:18:10	And this sheet of metal is 20 meters by 20 meters.
333	01:16:18:12	01:16:21:16	Now, in our case, it's actually a piece of paper
334	01:16:21:18	01:16:25:02	that is 20 centimeters by 20 centimeters, all right?
335	01:16:25:04	01:16:31:25	What we are going to do is cut out squares from each corner
336	01:16:31:27	01:16:34:18	and those squares are going to have
337	01:16:34:20	01:16:37:01	integer values for the sides.
338	01:16:37:03	01:16:41:00	So we might cut out a 1 x 1 square or a 2 x 2 square,
339	01:16:41:02	01:16:43:27	3 x 3 square, in terms of centimeters.
340	01:16:43:29	01:16:50:13	We then are going to take our... shape.
341	01:16:50:15	01:16:54:12	And here you can see, I've cut out some squares.
342	01:16:54:14	01:16:57:07	In this case, they're 4 x 4.
343	01:16:57:09	01:17:01:20	Might... better here, in terms of the grid.
344	01:17:01:22	01:17:06:13	And then I'm going to take this and form it into a tank.
345	01:17:06:15	01:17:09:22	Often, when we're trying to build something,
346	01:17:09:24	01:17:13:19	if we can use welding, we can fold it up like this.
347	01:17:13:21	01:17:15:03	And our question is:
348	01:17:15:05	01:17:19:13	How is the size of the square that we remove
349	01:17:19:15	01:17:22:21	related to the volume of the tank?
350	01:17:22:23	01:17:26:19	We want the tank that has the maximum volume.
351	01:17:26:21	01:17:28:17	So, if we each do

a different one--
352 01:17:28:19 01:17:30:07 you do 2 x 2
out of the corner
353 01:17:30:09 01:17:31:28 and I'll do 3 x 3.
354 01:17:32:00 01:17:33:21 Okay, fine.
355 01:17:33:23 01:17:36:09 And we'll see what
the difference in volume is.
356 01:17:36:11 01:17:39:02 CHAPIN:
In the design-a-water-tank
activity,
357 01:17:39:04 01:17:40:29 they were asked to think about
358 01:17:41:01 01:17:43:26 what size square should they cut
out of the corners
359 01:17:43:28 01:17:45:12 of a square sheet of paper
360 01:17:45:14 01:17:47:26 that would then,
when the paper is folded,
361 01:17:47:28 01:17:49:20 give them the maximum volume.
362 01:17:49:22 01:17:51:07 I wanted people to realize
363 01:17:51:09 01:17:54:05 that sometimes
we are using surface area
364 01:17:54:07 01:17:57:04 to form a shape
that will give us volume,
365 01:17:57:06 01:17:59:14 but that some of these
relationships
366 01:17:59:16 01:18:03:02 that we looked at before
may not be exactly the same.
367 01:18:03:04 01:18:05:19 And what did we say
before about the cube?
368 01:18:05:21 01:18:08:23 The closer it gets
to being like a cube,
the more...
369 01:18:08:25 01:18:10:13 We thought
that beforehand,
370 01:18:10:15 01:18:12:23 that maybe the closer
it got to being a cube,
371 01:18:12:25 01:18:14:16 the more it would...
372 01:18:14:18 01:18:16:12 the greater
the volume, but...
373 01:18:16:14 01:18:19:08 So this one right here
is the 7 x 7...
374 01:18:19:10 01:18:21:26 6 x 6 x 7 is the close...
close to a cube,
375 01:18:21:28 01:18:24:07 and it's got the least volume.
376 01:18:24:09 01:18:25:25 So that doesn't work.
377 01:18:25:27 01:18:28:05 So far, that
doesn't work out.
378 01:18:28:07 01:18:30:20 CHAPIN:
There are a number of reasons
to do that problem,
379 01:18:30:22 01:18:32:07 and a lot of insights
380 01:18:32:09 01:18:34:27 that I was hoping that they
would basically make sense of.
381 01:18:34:29 01:18:36:26 One is that it is not
terribly predictable.
382 01:18:36:28 01:18:41:08 So intuitively, the way we
often approach this problem

383	01:18:41:10	01:18:44:21	is actually leading us down the wrong path.
384	01:18:44:23	01:18:48:06	So by constructing these tanks, calculations using our...
385	01:18:48:08	01:18:50:28	what we know about length times width times height
386	01:18:51:00	01:18:52:20	to find the volume,
387	01:18:52:22	01:18:55:10	we can then come to a pretty close approximation
388	01:18:55:12	01:18:57:03	of what looks to be a dimension
389	01:18:57:05	01:18:59:07	that will give us the maximum volume.
390	01:18:59:09	01:19:01:17	So the 3 x 3 is best one,
391	01:19:01:19	01:19:05:00	because it holds the most right here.
392	01:19:05:02	01:19:07:18	It's 588 cubic centimeters,
393	01:19:07:20	01:19:10:12	which holds more than the 4 x 4 shape did,
394	01:19:10:14	01:19:12:10	or when we cut away 3 x 3,
395	01:19:12:12	01:19:15:12	so this was our dimensions, right there.
396	01:19:15:14	01:19:16:16	Okay.
397	01:19:16:18	01:19:18:22	Two centimeters cut off, 512.
398	01:19:18:24	01:19:20:17	Cut off two centimeters
399	01:19:20:19	01:19:21:28	from each corner...
400	01:19:22:00	01:19:23:25	And the third one--
401	01:19:23:27	01:19:24:29	it's the one that we built--
402	01:19:25:01	01:19:29:13	is a 14 x 14 x 3, 588.
403	01:19:29:15	01:19:30:16	Mm-hmm, coming up here.
404	01:19:30:18	01:19:31:19	It's a little bit larger.
405	01:19:31:21	01:19:33:15	And then we start to get smaller again.
406	01:19:33:17	01:19:39:05	Uh, 12 x 12 x 4-- the purple one-- was 576.
407	01:19:39:07	01:19:42:04	550... 75... 76...
408	01:19:42:06	01:19:43:19	And it's a little bit lower
409	01:19:43:21	01:19:45:25	CHAPIN: I had everyone graph this data
410	01:19:45:27	01:19:48:28	because by looking at just the numerical data,
411	01:19:49:00	01:19:50:28	many participants concluded
412	01:19:51:00	01:19:54:06	that, well, you remove a 3 x 3 square from each corner,
413	01:19:54:08	01:19:57:06	that dimension is going to give us the maximum volume.
414	01:19:57:08	01:19:58:20	When you graph it, though,
415	01:19:58:22	01:20:01:12	and you actually connect the points into a curve,

416 01:20:01:14 01:20:04:23 you realize that the curve
is actually going to go up above
417 01:20:04:25 01:20:07:16 what your maximum volume number
is at that point.
418 01:20:07:18 01:20:10:13 And it makes you think, "Hmm,
maybe the actual maximum volume
419 01:20:10:15 01:20:13:00 is between three and four."
420 01:20:13:02 01:20:14:17 It's... well,
if we graphed it well,
421 01:20:14:19 01:20:16:24 it's pretty clear that
this isn't the biggest.
422 01:20:16:26 01:20:18:09 That's definitely
not the biggest,
423 01:20:18:11 01:20:19:16 but it's closer...
424 01:20:19:18 01:20:22:17 But it's closer
than this one.
425 01:20:22:19 01:20:24:14 But, um, we didn't
have to build a lid.
426 01:20:24:16 01:20:26:13 We don't need
a top on it, so...
427 01:20:26:15 01:20:28:03 Well, that's true.
428 01:20:28:05 01:20:30:27 So, that, uh, sort
of argues in favor
of a flatter surface
429 01:20:30:29 01:20:33:23 so that we
can take advantage
of a larger missing lid.
430 01:20:33:25 01:20:35:09 We could have, yeah,
431 01:20:35:11 01:20:37:00 because a good
amount of surface
432 01:20:37:02 01:20:39:22 doesn't have to be
here on the top.
433 01:20:39:24 01:20:42:15 So we can use much, much
more of it on the bottom
434 01:20:42:17 01:20:47:26 and end up with a shallower
but much broader tank.
435 01:20:47:28 01:20:53:15 And what do we notice
about the maximum volume?
436 01:20:53:17 01:20:56:24 Here is our value,
588 cubic centimeters,
437 01:20:56:26 01:21:01:09 which goes with, when we remove
a square that's 3 x 3.
438 01:21:01:11 01:21:05:27 But notice the curve seems
to kind of go a little higher.
439 01:21:05:29 01:21:07:26 We can see by the curve here
440 01:21:07:28 01:21:12:01 that there's some other values
that maybe we can determine.
441 01:21:12:03 01:21:15:24 However, our scale here
is not terribly accurate--
442 01:21:15:26 01:21:19:13 it's going up by 50 cubic
centimeters each time.
443 01:21:19:15 01:21:23:11 So, again, we wouldn't
want to use the graph--
444 01:21:23:13 01:21:25:04 get an approximate
measure here,
445 01:21:25:06 01:21:27:24 but we need to know

that it's only approximate.
446 01:21:27:26 01:21:30:15 CHAPIN:
I hope that participants
will reconsider
447 01:21:30:17 01:21:33:12 how they think about their
lesson planning--
448 01:21:33:14 01:21:35:13 that they will first reflect
449 01:21:35:15 01:21:38:12 on what are the important
mathematical ideas
450 01:21:38:14 01:21:39:29 in regards to measurement.
451 01:21:40:01 01:21:42:20 They know more
about measurement,
452 01:21:42:22 01:21:46:29 and that will inform them
and will help them to ask
453 01:21:47:01 01:21:50:01 very articulate
and focused questions
454 01:21:50:03 01:21:53:05 that will help their students
get at the mathematics as well.
455 01:22:02:14 01:22:06:01 NARRATOR:
A lion-fish from the tropics...
456 01:22:06:03 01:22:09:01 a penguin from South Africa...
457 01:22:09:03 01:22:12:25 a turtle from the Amazon...
458 01:22:12:27 01:22:16:25 and a sand tiger shark
from the oceans of the world.
459 01:22:16:27 01:22:19:10 These are just some
of the many species
460 01:22:19:12 01:22:23:24 that call the New England
Aquarium home.
461 01:22:23:26 01:22:28:11 WOMAN:
The New England Aquarium has
been here in Boston since 1969.
462 01:22:28:13 01:22:30:09 It's a very diverse aquarium.
463 01:22:30:11 01:22:33:25 Oftentimes aquariums
will focus on just local areas,
464 01:22:33:27 01:22:35:18 whereas the New England Aquarium
465 01:22:35:20 01:22:37:23 has saltwater tanks,
freshwater tanks,
466 01:22:37:25 01:22:40:07 cold marine tanks,
tropical tanks
467 01:22:40:09 01:22:42:18 and tanks of varying sizes.
468 01:22:44:28 01:22:49:09 NARRATOR:
The wide variety of exhibits
educates, amazes and delights
469 01:22:49:11 01:22:51:26 over 1.3 million visitors
each year.
470 01:22:51:28 01:22:53:26 Look at him, Ma.
471 01:22:53:28 01:22:56:09 NARRATOR:
One of the most
popular attractions
472 01:22:56:11 01:22:57:28 is the giant ocean tank
473 01:22:58:00 01:23:01:21 that spirals up several stories
from the penguin pool.
474 01:23:01:23 01:23:05:27 WOMAN:
Our giant ocean tank at the
Aquarium is our main exhibit.
475 01:23:05:29 01:23:08:19 What's special

about the giant ocean tank
476 01:23:08:21 01:23:11:04 is that it's
a cylinderlike tank,
477 01:23:11:06 01:23:13:23 and it's 23 feet deep,
40 feet across.
478 01:23:13:25 01:23:17:05 It holds about
200,000 gallons of water.
479 01:23:21:09 01:23:25:11 CUTLER:
The giant ocean tank is home
to 700 individual animals,
480 01:23:25:13 01:23:27:05 about 135 different species.
481 01:23:27:07 01:23:29:04 Within those species,
482 01:23:29:06 01:23:31:16 we have bony fish,
cartilaginous animals--
483 01:23:31:18 01:23:33:02 which are
the sharks and the rays--
484 01:23:33:04 01:23:35:20 and our sea turtles,
which are the reptiles.
485 01:23:37:28 01:23:42:02 NARRATOR:
This large saltwater tank
was the first of its kind
486 01:23:42:04 01:23:44:22 when initially built
over 30 years ago.
487 01:23:44:24 01:23:48:07 With a volume
of nearly 29,000 cubic feet
488 01:23:48:09 01:23:51:21 and a surface area of 5,400
square feet,
489 01:23:51:23 01:23:54:29 its cylindrical shape
maximizes capacity
490 01:23:55:01 01:23:57:03 while allowing
ample viewing areas
491 01:23:57:05 01:24:00:10 for visitors to observe
the marine life inside.
492 01:24:00:12 01:24:03:03 CUTLER:
Typically you'll see aquariums,
493 01:24:03:05 01:24:06:18 usually they're square,
they're set into a wall,
494 01:24:06:20 01:24:10:00 and you're only seeing the fish
from one dimension.
495 01:24:10:02 01:24:12:06 The idea to have a round tank
496 01:24:12:08 01:24:14:26 that people could go
from the very bottom level
497 01:24:14:28 01:24:17:11 to the very top level,
and see animals living
498 01:24:17:13 01:24:19:28 at different water columns
within the exhibit
499 01:24:20:00 01:24:21:24 seemed like
a very exciting idea.
500 01:24:21:26 01:24:24:08 NARRATOR:
The size and shape of the tank
501 01:24:24:10 01:24:26:29 not only gives people
a unique perspective,
502 01:24:27:01 01:24:30:10 but it's also beneficial
to the fish flourishing within.
503 01:24:30:12 01:24:34:13 CUTLER:
The circular structure is really

great for creating a current.
504 01:24:34:15 01:24:36:25 A current
is important to the fish
505 01:24:36:27 01:24:39:23 because as they swim
against the current,
506 01:24:39:25 01:24:42:04 it rushes water over their gills
507 01:24:42:06 01:24:45:11 and maximizes
their ability to obtain oxygen.
508 01:24:47:19 01:24:49:25 NARRATOR:
The surface area within the tank
509 01:24:49:27 01:24:52:08 is increased by
the addition of a substrate,
510 01:24:52:10 01:24:55:07 an authentic replica
of a Caribbean coral reef.
511 01:24:55:09 01:24:58:00 WU:
We have an artificial reef
in the middle,
512 01:24:58:02 01:25:00:03 and we can host
a lot more animals,
513 01:25:00:05 01:25:03:09 they have more hiding spaces,
a lot more surface area,
514 01:25:03:11 01:25:05:06 whereas that rectangular tank,
515 01:25:05:08 01:25:06:29 all the substrates
are in the back
516 01:25:07:01 01:25:09:10 and the fish can only hide
in the substrate.
517 01:25:09:12 01:25:11:03 They can't go around
the substrate
518 01:25:11:05 01:25:13:02 like we have
in the giant ocean tank.
519 01:25:15:03 01:25:18:21 NARRATOR:
While the giant ocean tank
has inspired the development
520 01:25:18:23 01:25:20:29 of similar--
and even grander displays--
521 01:25:21:01 01:25:22:27 all over the world,
522 01:25:22:29 01:25:25:12 it's spiraling vistas
and exotic seascapes
523 01:25:25:14 01:25:28:02 will always provide
a fascinating window
524 01:25:28:04 01:25:30:05 into the mysteries
of marine life
525 01:25:30:07 01:25:32:28 for visitors
to the New England Aquarium.
526 01:25:37:16 01:25:40:19 Captioned by
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