Measurement is the process of quantifying properties of objects. And to do that, we have set procedures that enable us to measure. Measuring helps you to understand how things relate to each other.

Our volume of a sphere actually has a formula of four-thirds pi r-cubed. This course really made me think about how I approach measurement and how I can use measurement every day in the classroom.

In our session today, we are going to be investigating relationships between different types of measures. So far we've really delved into looking at mass or length or area and so today we're going to see: Are there relationships between some of those measures?

In particular, we're going to consider the relationship between the perimeter and the area of a shape and we're also going to look at the relationship between the surface area and the volume of a solid. Our second part of the session, we will actually be looking at how can we use some of these ideas and apply them to the solution of some interesting problems.

Imagine that you have just bought an adorable Highland terrier puppy. Cute little thing, all right? However, you live on a busy street.

You need to have a safe pen for that dog to be outside in. You have limited backyard space and you also have a limited budget so you go out and buy 72 feet
of fencing, all right?

Now, what I'd like us to do is investigate how we can shape that fencing into a rectangular pen.

What would the dimensions of the pen be so that our puppy would have a good area to run around in, okay?

And, in fact, we want to try to get the maximum area so that the puppy can really, you know, take advantage of the time that it's outside.

If we divide it by four, we get 18.

Mm-hmm.

That means a square of 18.

Then we'd square 18 to find the area.

Okay, so 18 x 18 would give us 324 square feet of area.

with the 72 linear feet of perimeter.

Squared.

Yeah.

Now we can try other combinations that would add up to the 72.

so we could just... we could...

instead of making a square, we could make it rectangular.

In our first activity,

we looked at holding a perimeter constant-- in fact, at 72 feet.

And then we looked at what shapes we could build using that perimeter.

Starting first with rectangular shapes,

we found that there was a wide variety of rectangles that would fit a set perimeter of 72
but their area differed.

And the one that had the greatest area was a square.

We started off with our triangle

that had 249 square feet,

did then we went to a hexagon

t hat had 374 square feet.

So maybe we should try a circle now

with a perimeter of 72 feet

and see what our area is.

I then pushed people to think about using other shapes.

What if we considered a hexagon or a triangle or a circle?

What area were we going to get when we still kept the perimeter at 72?

And participants found that the circle gave them the maximum area.

The barn gives us 72 feet of perimeter to work with.

But we don't have to use the whole thing, right?

The 72 feet of fence and cut it in half,

and we get 36 feet of fence

and make an opposite side

of a rectangle that's 36.

That would leave us with 18 feet of fence to complete the rectangle.

18 and 18 gives us an area of 648.

Much larger than the square.

Finally we pushed that problem to one step further

and looked at using that 72 feet as our perimeter
but that we could have another structure, in this case a barn, that would form one side of our shape and again we... we explored what happens if we build squares off the side of the barn, if we build trapezoids off the side of the barn, rectangles and eventually a semicircle. Ooh, the answer is... 8 1/4. Oh, so the area is 825... square feet. So considerably more. Much, much bigger. A third more. Well, that kind of proves us right, then. That the circle does take up the maximum area, even if it's a semicircle. Right. So we could design a circular fence, be in the same, exact perimeter, but if we do it in a circular form, we give the, uh, puppy the most area to play in. Let's take a look at some of these problems that we were exploring. When you kept the perimeter at 72-- namely, we sometimes refer to that as keeping the perimeter constant-- you were forming different rectangular shapes and looking at their areas. What conclusions did you come to in regards to the shape that gives you the greatest area when the perimeter is held constant? We discovered that the square
the largest area
and that as we change our dimensions on one side, increase them, um, by one number and decrease them by the same amount on the other side, that our area got smaller.
Great. Now, we then investigated some other shapes so if we kept our perimeter constant with other shapes, what are the relationships there in terms of area? Did anyone find an interesting shape that gave them more area? Katy.
We tried, um, a hexagon and we tried to look at an octagon and then at a 12-sided dodecagon, um, and then a circle and we found that the more sides you had, the more area inside as you went toward a circle. So if you were building this pen and really wanted to have the maximum area, what would your conclusion be about what shape you should have the pen, um, you should build the pen into? It should be a circle if that were practical.
Okay, yeah, and then we get into practicality.
Can we... can we do that? Now, the last problem was looking at using a side of a barn, and that barn had one side that was 70 feet and so we wanted to build off that side.
Anyone want to come up and share one possible pen
Laura, come on up.
We found that we didn't really need to use the whole side of the barn.
Okay.
If, um, our best shape was a square and we had 72 feet of fencing, we took our 72 feet and just divided it by three because we only need to make three more sides off our barn and that gave us a number of 24.
So if we make each length here 24, we don't need to use the whole barn and I forget what the area was.
It was 24 x 24 for our area and that comes up to be...
CLASS: 576.
Five seventy-six.
And we... we found that that would be the best area for three sides of a fence.
576 what?
Square feet.
Sorry.
Okay. Great.
And our drawing is not to scale, as we can see, at least get...
So we used the idea of a square.
Now, did anyone use any other ideas in terms of building a shape off of the barn from what we've discovered about maximum, um, area?
First experimenting with rectangles determined, um,
make a rectangular shape
that would be bigger
of a three-sided square--
to make a square--
and we're happy with our conclusions from there.

And from there we went into a semicircle.

We knew the circumference of the fencing that we had to use was 72 feet,
doubled it-- because we were only using half a circle--
came out with a diameter of 45.8--
the exact, same numbers--
um, cut that in half to find a radius...
Good.
of 22.9 feet,
and from there we used the formula for the area of a circle--
area equals pi r-squared--
but because we were only using half a circle,
we did area equals pi r-squared over two.
Uh, plugging in our radius of 22.9,
uh, squaring it, multiplied by pi and divided by two,
we came up with an area of approximately, rounded off, um, 825 square feet.
The circular or semicircular pattern seemed to have maximized area and it also maximized the use of the barn.
It used, you know, 45.8 feet of the barn, as opposed to the rectangles,
which used 24 or 36 feet of the barn.

So we're maximizing our extra side,

not using our fencing.

Great.

Now, in our next activity, we are going to look at relationships between volume and surface area.

and see if we can, again, start to, um, recognize

when and when there are not relationships that we can use.

In this case, we're going to keep the volume constant.

We are going to start with a volume of 24 cubic units,

which you have here.

We looked at one activity where the volume was kept constant at 24 cubic units,

and participants were asked to use the 24 cubic units to build rectangular prisms.

Then they were to calculate the surface area of those rectangular prisms and notice which solids had the greatest surface area.

and which had the least surface area.

Well, we noticed as it became more like a cube,

the surface area became less.

Oh.

So the one that...

that was very flat-- a long rectangle, a 24 x 1 x 1--

had a surface area of 98.

98 what?

Um, 98 units.

Okay, square units.

Square units, that's right.

Um, yet this one ended up with

with the 4 x 3 x 2,

has only 52 square units.
So it's much less.
It's almost half.
So how could we generalize this in terms of what type of a rectangular solid has a smaller surface area and what type of a rectangular solid has a large surface area?
Well, a cube would have the smallest surface area using, um...
but I don't think we can make it a perfect cube with 24.
But if we could, it would have the smaller surface area.
Um, and as one dimension, um, stays at one, that is if we kept the height at one,
then that would give us...
yield the largest surface area using that... given, um, volume.
We limited this problem by using only rectangular prisms,
and what participants concluded was that solids that were more compact, almost more cubelike,
had a smaller surface area than those rectangular prisms that were very spread out or elongated.
They had a much greater surface area.
So we need to build a cube that's 4 x 4 x 4.
You start at the base, that's 4 x 4.
Now we need to just build up four.
The next activity looked at cubes
that were progressively getting larger and larger

and in each case, I asked the participants
to calculate the volume and calculate the surface area

of the cube

and to look to see what kinds of relationships existed

between surface area and volume.

They were to represent that relationship as a ratio in most reduced form.

So, our surface area... we have each face is 16...

so we have our surface area 96... square units.

Our volume is 4 x 4 x 4... or four cubed, which is...

64, I believe.

So, the ratio would be... (chuckles)

96 divided by 16 would be six to four, or...

three to two?

Three to two.

Again, we were looking at the relationship between these measures;

that it's not a static relationship,

that that relationship changes based on the size of the figure and the shape of the figure.

In the sense of the cubes, that the ratio decreases as the cubes get larger and larger.

And that ratio between surface area and volume decreases as the sides get larger and larger on a cube.

If we do a 4 x 4 x 4...

Six to four reduces down to 1.5 to one.

Now, these ratios are an interesting thing to think about in terms of applications.

One application is when we're building, um, structures and we want to have a large volume but a small surface area.
And so, stores often consider "How can I..."

you know, "What shape can I build this in"

"that will give me that kind of a ratio"

"where it's not going to cost me too much money"

"to put up the outside structure,"

"but it'll give me a large volume inside."

In this next session, we are going to investigate how volume will change as we construct different tanks.

So our problem that we're going to be looking at involves a sheet of metal... okay?

And this sheet of metal is 20 meters by 20 meters.

Now, in our case, it's actually a piece of paper that is 20 centimeters by 20 centimeters, all right?

What we are going to do is cut out squares from each corner and those squares are going to have integer values for the sides.

So we might cut out a 1 x 1 square or a 2 x 2 square, in terms of centimeters.

We then are going to take our... shape.

And here you can see, I've cut out some squares.

In this case, they're 4 x 4.

Might... better here, in terms of the grid.

And then I'm going to take this and form it into a tank.

Often, when we're trying to build something,

if we can use welding, we can fold it up like this.

And our question is:

How is the size of the square that we remove related to the volume of the tank?

We want the tank that has the maximum volume.

So, if we each do
a different one--

you do 2 x 2 out of the corner

and I'll do 3 x 3.

Okay, fine.

And we'll see what the difference in volume is.

In the design-a-water-tank activity,

they were asked to think about what size square should they cut out of the corners.

of a square sheet of paper that would then, when the paper is folded, give them the maximum volume.

I wanted people to realize that sometimes we are using surface area to form a shape that will give us volume, but that some of these relationships may not be exactly the same.

And what did we say before about the cube?

The closer it gets to being like a cube, the more...

We thought that beforehand, that maybe the closer it got to being a cube, the more it would...

the greater the volume, but...

So this one right here is the 7 x 7...

6 x 6 x 7 is the close...

close to a cube, and it's got the least volume.

So that doesn't work.

So far, that doesn't work out.

There are a number of reasons to do that problem, and a lot of insights that I was hoping that they would basically make sense of.

One is that it is not terribly predictable.

So intuitively, the way we often approach this problem
is actually leading us down the wrong path. So by constructing these tanks, calculations using our... what we know about length times width times height to find the volume, we can then come to a pretty close approximation of what looks to be a dimension that will give us the maximum volume. So the 3 x 3 is best one, because it holds the most right here. It's 588 cubic centimeters, which holds more than the 4 x 4 shape did, or when we cut away 3 x 3, so this was our dimensions, right there. Okay. Two centimeters cut off, 512. Cut off two centimeters from each corner... And the third one-- it's the one that we built-- is a 14 x 14 x 3, 588. Mm-hmm, coming up here. It's a little bit larger. And then we start to get smaller again. Uh, 12 x 12 x 4-- the purple one-- was 576. 550... 75... 76... And it's a little bit lower. CHAPIN: I had everyone graph this data because by looking at just the numerical data, many participants concluded that, well, you remove a 3 x 3 square from each corner, that dimension is going to give us the maximum volume. When you graph it, though, and you actually connect the points into a curve,
you realize that the curve is actually going to go up above what your maximum volume number is at that point. And it makes you think, "Hmm, maybe the actual maximum volume is between three and four."

It's... well, if we graphed it well, it's pretty clear that this isn't the biggest. That's definitely not the biggest, but it's closer... But it's closer than this one. But, um, we didn't have to build a lid. We don't need a top on it, so...

Well, that's true. So, that, uh, sort of argues in favor of a flatter surface so that we can take advantage of a larger missing lid.

We could have, yeah, because a good amount of surface doesn't have to be here on the top. So we can use much, much more of it on the bottom and end up with a shallower but much broader tank.

And what do we notice about the maximum volume? Here is our value, 588 cubic centimeters, which goes with, when we remove a square that's 3 x 3. But notice the curve seems to kind of go a little higher. We can see by the curve here that there's some other values that maybe we can determine. However, our scale here is not terribly accurate--it's going up by 50 cubic centimeters each time. So, again, we wouldn't want to use the graph--get an approximate measure here, but we need to know
I hope that participants will reconsider how they think about their lesson planning—
that they will first reflect on what are the important mathematical ideas in regards to measurement.
They know more about measurement, and that will inform them and help them to ask very articulate and focused questions that will help their students get at the mathematics as well.

A lion-fish from the tropics...
a penguin from South Africa...
a turtle from the Amazon...
and a sand tiger shark from the oceans of the world.
These are just some of the many species that call the New England Aquarium home.
The New England Aquarium has been here in Boston since 1969.
It's a very diverse aquarium.
Oftentimes aquariums will focus on just local areas, whereas the New England Aquarium has saltwater tanks, freshwater tanks, cold marine tanks, tropical tanks and tanks of varying sizes.
The wide variety of exhibits educates, amazes and delights over 1.3 million visitors each year.
One of the most popular attractions is the giant ocean tank that spirals up several stories from the penguin pool.
Our giant ocean tank at the Aquarium is our main exhibit.
What's special
about the giant ocean tank

476 01:23:08:21 01:23:11:04  is that it's
477 01:23:11:06 01:23:13:23  and it's 23 feet deep,
478 01:23:13:25 01:23:17:05  It holds about
479 01:23:21:09 01:23:25:11  CUTLER:

The giant ocean tank is home
to 700 individual animals,

481 01:23:27:07 01:23:29:04  Within those species,
482 01:23:29:06 01:23:31:16  we have bony fish,
cartilaginous animals--
483 01:23:31:18 01:23:33:02  which are
the sharks and the rays--
484 01:23:33:04 01:23:35:20  and our sea turtles,
which are the reptiles.
485 01:23:37:28 01:23:42:02  NARRATOR:

This large saltwater tank
was the first of its kind

486 01:23:42:04 01:23:44:22  when initially built
over 30 years ago.
of nearly 29,000 cubic feet
488 01:23:48:09 01:23:51:21  and a surface area of 5,400
square feet,
489 01:23:51:23 01:23:54:29  its cylindrical shape
maximizes capacity
490 01:23:55:01 01:23:57:03  while allowing
ample viewing areas
491 01:23:57:05 01:24:00:10  for visitors to observe
the marine life inside.
492 01:24:00:12 01:24:03:03  CUTLER:

Typically you'll see aquariums,
493 01:24:03:05 01:24:06:18  usually they're square,
they're set into a wall,
494 01:24:06:20 01:24:10:00  and you're only seeing the fish
from one dimension.
495 01:24:10:02 01:24:12:06  The idea to have a round tank
496 01:24:12:08 01:24:14:26  that people could go
from the very bottom level
497 01:24:14:28 01:24:17:11  to the very top level,
and see animals living
498 01:24:17:13 01:24:19:28  at different water columns
within the exhibit
499 01:24:20:00 01:24:21:24  seemed like
a very exciting idea.
500 01:24:21:26 01:24:24:08  NARRATOR:
The size and shape of the tank
501 01:24:24:10 01:24:26:29  not only gives people
a unique perspective,
502 01:24:27:01 01:24:30:10  but it's also beneficial
to the fish flourishing within.
503 01:24:30:12 01:24:34:13  CUTLER:
The circular structure is really
great for creating a current.

A current is important to the fish because as they swim against the current, it rushes water over their gills and maximizes their ability to obtain oxygen.

NARRATOR: The surface area within the tank is increased by the addition of a substrate, an authentic replica of a Caribbean coral reef.

WU: We have an artificial reef in the middle, and we can host a lot more animals, they have more hiding spaces, a lot more surface area, whereas that rectangular tank, all the substrates are in the back and the fish can only hide in the substrate.

They can't go around the substrate like we have in the giant ocean tank.

NARRATOR: While the giant ocean tank has inspired the development of similar--- and even grander displays--- all over the world, it's spiraling vistas and exotic seascapes will always provide a fascinating window into the mysteries of marine life for visitors to the New England Aquarium.