1	01:00:47:11	01:00:48:21	WOMAN: Measurement is the process
2	01:00:48:23	01:00:51:24	of quantifying properties of objects.
3	01:00:51:26	01:00:54:21	And to do that, we have set procedures
4	01:00:54:23	01:00:56:22	that enable us to measure.
5	01:00:56:24	01:00:58:14	Oh.
6	01:00:58:16	01:01:00:28	Measuring helps you to understand
7	01:01:01:00	01:01:03:09	how things relate to each other.
8	01:01:03:11	01:01:07:07	Our volume of a sphere actually has a formula
9	01:01:07:09	01:01:09:18	of four-thirds pi r-cubed.
10	01:01:09:20	01:01:13:04	This course really made me think about how I approach measurement
11	01:01:13:06	01:01:16:07	and how I can use measurement every day in the classroom.
12	01:01:22:00	01:01:23:00	Good morning.
13	01:01:23:02	01:01:24:10	In this session,
14	01:01:24:12	01:01:27:10	we are going to be exploring indirect measurement.
15	01:01:27:12	01:01:31:24	Now, an indirect measure is one that is not done directly.
16	01:01:31:26	01:01:34:18	Namely, we don't take a tool
17	01:01:34:20	01:01:39:12	and compare the length or the mass or the volume
18	01:01:39:14	01:01:40:28	to a set of units.
19	01:01:41:00	01:01:44:07	Instead, we still take some measurements,
20	01:01:44:09	01:01:46:13	but we use other mathematics
21	01:01:46:15	01:01:49:20	to derive, or come up with, the measure, um,
22	01:01:49:22	01:01:53:01	of the object that we're interested in.
23	01:01:53:03	01:01:54:27	Now, there are a number of ways
24	01:01:54:29	01:01:56:17	to do indirect measurement.
25	01:01:56:19	01:01:58:19	One uses similar triangles
26	01:01:58:21	01:02:02:10	and connects geometry, measurement and number,
27	01:02:02:12	01:02:05:26	because it also uses proportional reasoning.
28	01:02:05:28	01:02:08:06	Let's do a quick review
29	01:02:08:08	01:02:10:28	of what does it mean to have a similar figure
30	01:02:11:00	01:02:12:08	or similar triangles.
31	01:02:12:10	01:02:14:17	If you look up here, we see
32	01:02:14:19	01:02:18:16	that these two triangles are similar
33	01:02:18:18	01:02:22:29	and these two squares are similar.
34	01:02:23:01	01:02:27:16	Likewise, these two triangles are not similar

35	01:02:27:18	01:02:31:04	and these two are not similar.
36	01:02:31:06	01:02:33:26	Anybody, by looking at those examples,
37	01:02:33:28	01:02:37:04	can they come up with what seems
38	01:02:37:06	01:02:41:03	to be a characteristic of similar figures?
39	01:02:41:05	01:02:42:24	They would have to have the same angles.
40	01:02:42:26	01:02:44:16	They have to have the same angles.
41	01:02:44:18	01:02:45:23	Let's just double-check.
42	01:02:45:25	01:02:48:26	I could put that there, here and here.
43	01:02:48:28	01:02:50:17	Yep, that one works.
44	01:02:50:19	01:02:52:17	And they would have to have proportionate sides.
45	01:02:52:19	01:02:55:14	Sometimes we say that similar figures
46	01:02:55:16	01:02:58:04	have same shape, different size.
47	01:02:58:06	01:03:00:28	Now, we're going to use this idea
48	01:03:01:00	01:03:04:18	to help us make some measurements.
49	01:03:04:20	01:03:06:14	And there are two different things
50	01:03:06:16	01:03:08:11	that we are going to be exploring.
51	01:03:08:13	01:03:13:01	One is to look and see how far away something is.
52	01:03:13:03	01:03:15:21	For example, imagine
53	01:03:15:23	01:03:18:21	that we have a tree
54	01:03:18:23	01:03:24:16	and there's a river and here we are, all right?
55	01:03:24:18	01:03:32:08	What we want to do is we want to figure out that distance, okay?
56	01:03:32:10	01:03:37:04	Well, one way to do that is to make a triangle,
57	01:03:37:06	01:03:40:07	and we're then going to use a similar triangle
58	01:03:40:09	01:03:43:07	to actually do some measurements, all right?
59	01:03:43:09	01:03:46:07	Well, this triangle actually has to be somewhat in our minds,
60	01:03:46:09	01:03:48:20	because we aren't actually able to be out there,
61	01:03:48:22	01:03:51:15	running across that river.
62	01:03:51:17	01:03:55:17	But what we can do is we can stay at this point
63	01:03:55:19	01:04:00:11	and, thinking about a base line of a horizontal distance,
64	01:04:00:13	01:04:02:14	we can use an instrument
65	01:04:02:16	01:04:06:08	and make a 90-degree angle there, all right?

66	01:04:06:10	01:04:09:00	Now, the instrument that allows us
67	01:04:09:02	01:04:11:20	to measure, um, angles like that
68	01:04:11:22	01:04:16:25	that is used in surveying is something called a "transit."
69	01:04:16:27	01:04:21:16	Now, this is a very informal transit, all right?
70	01:04:21:18	01:04:26:27	And if you notice, it is a ruler with a protractor on it
71	01:04:26:29	01:04:32:29	and then a straw that allows us to move it along the protractor
72	01:04:33:01	01:04:35:08	and determine an angle.
73	01:04:35:10	01:04:39:17	If we can hold this up to our eye
74	01:04:39:19	01:04:44:08	and take a bead on our straw, I can say, you know, what angle
75	01:04:44:10	01:04:47:25	is going off in this direction from this point
76	01:04:47:27	01:04:52:00	in terms of... of how I've oriented myself, all right?
77	01:04:52:02	01:04:55:17	So we're going to use our transit
78	01:04:55:19	01:05:03:00	to determine a 90-degree angle with a line on the ground.
79	01:05:03:02	01:05:05:22	NARRATOR: Once a 90-degree angle has been created,
80	01:05:05:24	01:05:08:14	the length of the line on the ground is measured.
81	01:05:08:16	01:05:11:03	At the far end, the transit is used again
82	01:05:11:05	01:05:13:11	to determine the angle to the tree.
83	01:05:13:13	01:05:15:14	This forms a triangle.
84	01:05:15:16	01:05:17:17	Using a ratio or a scale factor
85	01:05:17:19	01:05:20:05	such as four meters to one centimeter,
86	01:05:20:07	01:05:23:00	a similar triangle can then be drawn on paper.
87	01:05:23:02	01:05:26:14	By measuring the distance between A and C on the drawing
88	01:05:26:16	01:05:28:22	and multiplying by the scale factor,
89	01:05:28:24	01:05:32:26	the distance to the tree on the original triangle can be found.
90	01:05:32:28	01:05:35:05	Another way to approach this problem
91	01:05:35:07	01:05:36:27	is setting up a proportion
92	01:05:36:29	01:05:40:00	and solving for X using the cross-product method,
93	01:05:40:02	01:05:42:10	where X is the distance to the tree.
94	01:05:42:12	01:05:45:16	To investigate this further, the class goes outside
95	01:05:45:18	01:05:47:21	with transits and trundle wheels
96	01:05:47:23	01:05:51:00	to measure the distance

			of a tree across the field
97	01:05:51:02	01:05:52:27	using this indirect method.
98	01:05:52:29	01:05:55:12	We've picked this large, leafy one directly across
99	01:05:55:14	01:05:57:16	so this will be our point
100	01:05:57:18	01:05:58:28	to make a 90-degree angle.
101	01:05:59:00	01:06:00:29	So let's put a stake in where we're standing
102	01:06:01:01	01:06:02:19	and then we'll do the 90-degree angle.
103	01:06:04:03	01:06:05:02	Whoops.
104	01:06:06:26	01:06:08:16	Okay, good.
105	01:06:08:18	01:06:11:16	CHAPIN: We were trying to find the distance to a tree
106	01:06:11:18	01:06:12:26	far across the field.
107	01:06:12:28	01:06:15:07	In this case, we can make a right angle
108	01:06:15:09	01:06:17:03	or set ourselves perpendicular
109	01:06:17:05	01:06:20:18	to a base line to the tree of interest.
110	01:06:20:20	01:06:24:21	From there, we measure out a set distance along the ground
111	01:06:24:23	01:06:26:28	until we come to a second point,
112	01:06:27:00	01:06:29:20	which we have now indicated with a stake,
113	01:06:29:22	01:06:32:21	and at that point we take a second measure.
114	01:06:32:23	01:06:34:03	And I'll bring the transit
115	01:06:34:05	01:06:36:09	so we can measure that angle down there.
116	01:06:36:11	01:06:41:14	One... two... three... four... five...
117	01:06:43:14	01:06:47:21	CHAPIN: The transit enables us to measure the size of the angle
118	01:06:47:23	01:06:50:24	between our point on our line and the tree.
119	01:06:50:26	01:06:53:15	We're going to draw the same type of a triangle.
120	01:06:53:17	01:06:54:25	We're going to make sure
121	01:06:54:27	01:06:57:18	we have, um, a 90-degree angle here,
122	01:06:57:20	01:06:58:25	a 70 one here
123	01:06:58:27	01:07:02:22	and then we have to have something proportional
124	01:07:02:24	01:07:04:05	to the 20 meters.
125	01:07:04:07	01:07:06:12	Exactly, because we certainly can't draw a line
126	01:07:06:14	01:07:07:16	20 meters

			on our paper.
127	01:07:07:18	01:07:09:19	What might be a distance that we want to...
128	01:07:09:21	01:07:11:15	that we could easily draw on our paper?
129	01:07:11:17	01:07:13:03	So we could pick, like, 20 centimeters
130	01:07:13:05	01:07:14:16	or 20 millimeters.
131	01:07:14:18	01:07:17:25	CHAPIN: Or even five centimeters,
132	01:07:17:27	01:07:22:27	and we could have our ratio in terms of our scale
133	01:07:22:29	01:07:26:09	as 20 meters to five centimeters,
134	01:07:26:11	01:07:28:02	and then we could say that's...
135	01:07:28:04	01:07:31:04	This is in our one that we're actually going to draw
136	01:07:31:06	01:07:33:10	is going to be about five centimeters
137	01:07:33:12	01:07:35:21	and then we have to measure this angle--
138	01:07:35:23	01:07:36:23	70 degrees, still--
139	01:07:36:25	01:07:40:27	and we have to measure that.
140	01:07:40:29	01:07:42:20	And then we can set up a proportion.
141	01:07:42:22	01:07:45:09	And what do you think the other proportion would be?
142	01:07:45:11	01:07:47:00	MAN: X for the... this leg.
143	01:07:47:02	01:07:48:21	Right.
144	01:07:48:23	01:07:50:27	And then whatever this leg is.
145	01:07:50:29	01:07:54:04	So we actually have to build that triangle.
146	01:07:54:06	01:07:55:13	We have to actually measure it
147	01:07:55:15	01:07:57:13	and whatever that measure is is going to go here
148	01:07:57:15	01:07:59:02	and then we're only missing one value.
149	01:07:59:04	01:08:00:12	So if we have the three angles
150	01:08:00:14	01:08:02:13	and then the one proportional side,
151	01:08:02:15	01:08:03:18	then we're all set.
152	01:08:03:20	01:08:05:24	Yep. Excellent.
153	01:08:09:18	01:08:10:29	Okay.
154	01:08:11:01	01:08:15:04	Thank you and you hold it over there.
155	01:08:15:06	01:08:17:27	I still give you a 90-degree angle here.
156	01:08:17:29	01:08:19:01	Very good.
157	01:08:19:03	01:08:21:02	Now go as far as you can.
158	01:08:21:04	01:08:23:14	I'm trying to go

			along the base...
159	01:08:23:16	01:08:24:16	There you go.
160	01:08:24:18	01:08:25:16	Yeah, very good.
161	01:08:25:18	01:08:27:01	Ooh, excellent.
162	01:08:28:15	01:08:30:09	NARRATOR: In a second activity,
163	01:08:30:11	01:08:32:29	participants determine the height of a tree
164	01:08:33:01	01:08:34:14	by measuring its shadow
165	01:08:34:16	01:08:37:02	and then creating a similar right triangle
166	01:08:37:04	01:08:38:11	using a meter stick.
167	01:08:38:13	01:08:40:15	So here's the tree.
168	01:08:40:17	01:08:41:15	Yep.
169	01:08:41:17	01:08:43:01	And then we came out this far
170	01:08:43:03	01:08:44:28	and this was our shadow.
171	01:08:45:00	01:08:46:27	MAN: That is correct, yep.
172	01:08:46:29	01:08:49:18	Okay, and that was 500 and...
173	01:08:49:20	01:08:51:02	40 centimeters.
174	01:08:51:04	01:08:53:20	40 centimeters, or 5.4 meters
175	01:08:53:22	01:08:56:09	if we're doing it in meters.
176	01:08:56:11	01:08:57:14	All right.
177	01:08:57:16	01:08:58:14	Okay.
178	01:08:58:16	01:09:00:08	Um, there's our sun.
179	01:09:00:10	01:09:01:23	That's right.
180	01:09:01:25	01:09:04:19	CHAPIN: One very common application
181	01:09:04:21	01:09:06:28	of indirect measurement involves shadows.
182	01:09:07:00	01:09:10:05	So in this case what we did was we went out
183	01:09:10:07	01:09:13:02	and we were trying to figure out the heights of trees.
184	01:09:13:04	01:09:15:09	Instead of measuring directly that height,
185	01:09:15:11	01:09:18:05	we actually went on the ground and measured the shadow.
186	01:09:18:07	01:09:20:29	At that point now, we have a triangle
187	01:09:21:01	01:09:25:08	that has been formed by both the height and the shadow
188	01:09:25:10	01:09:28:02	of the... of the object forming a 90-degree angle
189	01:09:28:04	01:09:30:27	as well as with the angle that comes from the sun.
190	01:09:30:29	01:09:32:15	We need a similar triangle,
191	01:09:32:17	01:09:35:24	so we took a meter stick and placed that on the ground

192	01:09:35:26	01:09:37:12	in the exact, same location
193	01:09:37:14	01:09:40:07	so that the sun was hitting it in the same manner,
194	01:09:40:09	01:09:43:21	measured its shadow and again now we have a smaller triangle,
195	01:09:43:23	01:09:47:10	where we actually have some measures that we can use.
196	01:09:47:12	01:09:49:03	From there we could set up proportions
197	01:09:49:05	01:09:50:14	between the smaller triangle
198	01:09:50:16	01:09:54:19	to the larger one that has the length of interest
199	01:09:54:21	01:09:57:03	and calculate the height of our trees.
200	01:09:57:05	01:10:00:24	If we take the height of the meter stick,
201	01:10:00:26	01:10:02:29	which is 100 centimeters,
202	01:10:03:01	01:10:05:16	is to the height of the tree--
203	01:10:05:18	01:10:08:22	which we don't know, we call it X--
204	01:10:08:24	01:10:12:23	as, uh, the length of the shadow
205	01:10:12:25	01:10:18:24	of the meter stick, which is, uh, 90 centimeters
206	01:10:18:26	01:10:20:27	is to its length,
207	01:10:20:29	01:10:26:12	and solving for X, doing the cross product,
208	01:10:26:14	01:10:29:10	will give us, uh...
209	01:10:29:12	01:10:32:11	We'll get, uh, X times 90
210	01:10:32:13	01:10:37:08	will give us 90X equals 540 times 100 here,
211	01:10:37:10	01:10:39:22	and solving for X
212	01:10:39:24	01:10:42:19	will give us the height of the tree itself,
213	01:10:42:21	01:10:44:19	which is what you came up with.
214	01:10:44:21	01:10:46:27	Right, but what I would do in this
215	01:10:46:29	01:10:48:14	is reduce this fraction.
216	01:10:48:16	01:10:49:28	You can do that, too.
217	01:10:50:00	01:10:54:10	So that I know that nine is one-sixth of 54.
218	01:10:54:12	01:10:58:05	That's right, and 100 will be 600.
219	01:11:06:05	01:11:09:17	Okay, I'd like to move on to our next activity,
220	01:11:09:19	01:11:14:03	which is to think about indirect measurement in another way.
221	01:11:14:05	01:11:17:09	And there are other areas of mathematics
222	01:11:17:11	01:11:21:28	that have been developed to address some indirect measure,
223	01:11:22:00	01:11:25:24	and one

is trigonometry.
224 01:11:25:26 01:11:28:15 Now, the trigonometry
of right triangles,
225 01:11:28:17 01:11:32:22 which is what we are going to be
exploring, um, provides us a way
226 01:11:32:24 01:11:35:23 to again find some measures
indirectly
227 01:11:35:25 01:11:39:05 without actually physically
having to make
228 01:11:39:07 01:11:41:24 those measurements.
229 01:11:41:26 01:11:45:07 NARRATOR:
There are three ratios
in right-triangle trigonometry
230 01:11:45:09 01:11:47:26 that are often used
for indirect measurement:
231 01:11:47:28 01:11:51:12 the sine, cosine and tangent.
232 01:11:51:14 01:11:52:26 The sine of an angle
233 01:11:52:28 01:11:55:21 is the ratio of the height
to hypotenuse.
234 01:11:55:23 01:11:59:23 The cosine is the ratio
of the distance to hypotenuse.
235 01:11:59:25 01:12:01:05 And the tangent
236 01:12:01:07 01:12:04:02 is the ratio of the height
to distance of the legs.
237 01:12:04:04 01:12:05:21 In the next activity,
238 01:12:05:23 01:12:09:15 the class will begin
to explore the tangent ratio.
239 01:12:09:17 01:12:10:28 Making the assumption
240 01:12:11:00 01:12:13:29 that we're starting first
from a right triangle,
241 01:12:14:01 01:12:19:26 what is the ratio of height
to distance, all right?
242 01:12:19:28 01:12:22:16 Now, how are we going
to figure that out?
243 01:12:22:18 01:12:25:13 Well, one way is for you
to draw a triangle
244 01:12:25:15 01:12:27:11 where you have a right triangle
245 01:12:27:13 01:12:29:11 and that angle here
is 45 degrees
246 01:12:29:13 01:12:33:04 and then find out what is the
ratio of the height to distance
247 01:12:33:06 01:12:34:23 by measuring.
248 01:12:34:25 01:12:40:24 Okay, gather some data, make a
little sketch, record your data.
249 01:12:43:00 01:12:45:17 45 degrees.
250 01:12:50:04 01:12:55:10 (*students conversing quietly*)
251 01:13:02:00 01:13:04:07 Forty-five.
252 01:13:04:09 01:13:06:25 CHAPIN:
To help them make sense
of the tangent ratio,
253 01:13:06:27 01:13:07:29 I gave them information
254 01:13:08:01 01:13:11:07 and asked them
to construct triangles.
255 01:13:11:09 01:13:13:00 One of the main purposes
of this activity
256 01:13:13:02 01:13:14:22 was to help them understand

that there is
257 01:13:14:24 01:13:18:10 a consistent relationship
between the ratio and the angle.
258 01:13:18:12 01:13:23:14 STUDENT:
Um, I don't think
I am really...
259 01:13:23:16 01:13:25:04 Hi, guys,
what are you up to?
260 01:13:25:06 01:13:26:04 We are... confused.
261 01:13:26:06 01:13:27:18 Having a little
discussion here.
262 01:13:27:20 01:13:29:03 Okay.
263 01:13:29:05 01:13:31:21 Because we were
expecting something,
264 01:13:31:23 01:13:33:23 and we found
the other thing...
265 01:13:33:25 01:13:35:07 you know, experiment.
266 01:13:35:09 01:13:36:09 Ah, all right.
267 01:13:36:11 01:13:39:22 So explain to me
where you are and...
268 01:13:39:24 01:13:41:10 All right,
my confusion is,
269 01:13:41:12 01:13:45:12 if, um, we had to make
alpha equal 30 degrees...
270 01:13:45:14 01:13:47:19 This is 60 and 90.
271 01:13:47:21 01:13:51:13 So I assume, since this angle--
60 degrees--
272 01:13:51:15 01:13:53:20 is twice the 30 degrees,
273 01:13:53:22 01:13:56:17 that this opposite length
would be exactly twice
274 01:13:56:19 01:13:58:25 that opposite of the 30,
but it's not.
275 01:13:58:27 01:14:01:27 Oh... okay.
276 01:14:01:29 01:14:04:15 CHAPIN:
One group started to realize
that there seemed to be
277 01:14:04:17 01:14:08:02 a relationship between
angle measure and the ratio.
278 01:14:08:04 01:14:10:28 They then made a quick leap
and felt that,
279 01:14:11:00 01:14:15:29 well, if the ratio was 2:1,
then they were going to have
280 01:14:16:01 01:14:17:28 a 2:1 relationship
in terms of the angles,
281 01:14:18:00 01:14:19:28 so they thought,
well, at first it was 30,
282 01:14:20:00 01:14:21:20 so maybe it's going to be 60.
283 01:14:21:22 01:14:24:29 Likewise, they then went
and built the other 1:2
284 01:14:25:01 01:14:27:16 and found... hmm,
this is not working.
285 01:14:27:18 01:14:29:19 This is a common misconception,
286 01:14:29:21 01:14:32:12 because sometimes
learners overgeneralize,
287 01:14:32:14 01:14:35:05 and rather than gathering
enough data to see

288	01:14:35:07	01:14:38:04	how there are differences between relationships,
289	01:14:38:06	01:14:40:16	they immediately jump to conclusions.
290	01:14:40:18	01:14:43:21	I'd like to first fill in, briefly, this chart
291	01:14:43:23	01:14:48:02	that has the angle measure and the ratio of height to distance
292	01:14:48:04	01:14:52:15	that we were exploring in this situation.
293	01:14:52:17	01:14:56:23	When we have an angle measure of 45 degrees,
294	01:14:56:25	01:15:00:23	our ratio, I think everybody found, was 1:1,
295	01:15:00:25	01:15:06:05	yup, and kind of reminds us about the old isosceles triangle
296	01:15:06:07	01:15:11:12	always having equal angle measure and congruent two sides.
297	01:15:11:14	01:15:14:15	What about, though, when we built a triangle
298	01:15:14:17	01:15:17:14	with a ratio of height to distance of 2:1?
299	01:15:17:16	01:15:20:26	What did we find as our alpha measure?
300	01:15:20:28	01:15:24:26	We found alpha to be 63.5 degrees.
301	01:15:24:28	01:15:28:05	Okay, so, um, were you just rounding there, based...
302	01:15:28:07	01:15:31:11	or did your instrument actually go to half degrees?
303	01:15:31:13	01:15:33:07	It seemed to be in between 63 and 64.
304	01:15:33:09	01:15:34:28	CHAPIN: Okay, so I'm going to say
305	01:15:35:00	01:15:37:19	that we're going to say between 63, 64,
306	01:15:37:21	01:15:40:15	or, as you said, 63.5, right around there.
307	01:15:40:17	01:15:42:03	All right?
308	01:15:42:05	01:15:44:20	NARRATOR: As the class continues to put up their results
309	01:15:44:22	01:15:47:19	for alpha angles and tangent ratios,
310	01:15:47:21	01:15:49:07	a pattern emerges.
311	01:15:49:09	01:15:50:21	We got 27 degrees.
312	01:15:50:23	01:15:52:09	It was close to 30,
313	01:15:52:11	01:15:56:17	but it was, it was almost the complement of the 63.
314	01:15:56:19	01:15:57:28	Ah, now, say that again.
315	01:15:58:00	01:16:01:18	So, 27 degrees-- what do you mean by complement?
316	01:16:01:20	01:16:03:02	Um, when we had the 90-degree angle
317	01:16:03:04	01:16:08:11	before, for the 2:1, the angle was 63

318	01:16:08:13	01:16:10:12	and if we measured the other angle,
319	01:16:10:14	01:16:11:21	it would have been 27,
320	01:16:11:23	01:16:13:24	and so this time, it was the reverse.
321	01:16:13:26	01:16:15:07	CHAPIN: Right.
322	01:16:15:09	01:16:18:17	So we have this interesting relationship occurring
323	01:16:18:19	01:16:20:08	that as the ratios flip--
324	01:16:20:10	01:16:22:18	in terms of what is being compared--
325	01:16:22:20	01:16:25:10	we actually are finding angles
326	01:16:25:12	01:16:28:14	that are complements to each other.
327	01:16:28:16	01:16:30:27	Notice that one angle in these triangles
328	01:16:30:29	01:16:33:05	is a right triangle, already set at 90,
329	01:16:33:07	01:16:36:16	so the other two angles are going to sum to 90 degrees.
330	01:16:36:18	01:16:38:03	It's kind of difficult for us to get
331	01:16:38:05	01:16:39:21	a sense of what's the relationship
332	01:16:39:23	01:16:42:03	between the angle size and the ratio,
333	01:16:42:05	01:16:45:11	but one way we might be able to get some insight
334	01:16:45:13	01:16:47:06	into that information
335	01:16:47:08	01:16:51:15	is to graph the ratios against the angle measure.
336	01:16:51:17	01:16:54:02	Um, would anybody be willing to come up
337	01:16:54:04	01:16:56:29	and help us plot these points on our graph?
338	01:16:57:01	01:16:59:13	Great, John, come on up.
339	01:16:59:15	01:17:02:02	CHAPIN: The steepness graph is one way to help illustrate
340	01:17:02:04	01:17:05:22	what happens in terms of the ratio
341	01:17:05:24	01:17:08:00	as the angle size gets larger.
342	01:17:08:02	01:17:10:21	The graph is not a straight line-- it's a curve--
343	01:17:10:23	01:17:15:05	and at 45 degrees, the ratio is equal to one.
344	01:17:15:07	01:17:20:07	Beyond 45 degrees, the ratios increase at a much faster rate.
345	01:17:20:09	01:17:22:07	So angles that are...
346	01:17:22:09	01:17:25:07	angle measures that are greater than 45
347	01:17:25:09	01:17:28:06	have ratios that are greater than one,
348	01:17:28:08	01:17:31:09	and angles that are less than 45 degrees

349	01:17:31:11	01:17:35:09	have height-distance ratios that are less than one.
350	01:17:35:11	01:17:37:21	CHAPIN: What seems to be the relationship, here,
351	01:17:37:23	01:17:43:20	between the angle measure in degrees and the ratio?
352	01:17:43:22	01:17:47:02	It appears the greater the angle measurement,
353	01:17:47:04	01:17:48:24	the greater the ratio.
354	01:17:48:26	01:17:50:21	And so that an interesting...
355	01:17:50:23	01:17:53:07	and it starts out pretty gradually here,
356	01:17:53:09	01:17:55:28	and then... slips right up.
357	01:17:56:00	01:18:01:18	Once after 45, it seems to get even steeper, so it seems to...
358	01:18:01:20	01:18:03:06	that that angle, that ratio,
359	01:18:03:08	01:18:06:05	is increasing even more past the 45-degree mark
360	01:18:06:07	01:18:09:06	than from before, but each time it is increasing.
361	01:18:09:08	01:18:13:06	Well, now, we've learned a lot about the tangent--
362	01:18:13:08	01:18:16:04	that it is... the tangent of an angle
363	01:18:16:06	01:18:20:04	is the height-to-distance ratio in a right triangle.
364	01:18:20:06	01:18:22:09	And we can actually use this now
365	01:18:22:11	01:18:25:25	to solve some of the indirect measurement problems.
366	01:18:25:27	01:18:27:14	I'd like us to revisit
367	01:18:27:16	01:18:30:12	the one about that tree across the field,
368	01:18:30:14	01:18:34:05	and here it is, up above, on the board.
369	01:18:34:07	01:18:39:01	We notice that this is where John and Susan were standing.
370	01:18:39:03	01:18:42:21	Okay? They wanted to find the distance to their tree.
371	01:18:42:23	01:18:45:21	They measured out 20 meters,
372	01:18:45:23	01:18:49:11	and then with their transit, they determined
373	01:18:49:13	01:18:51:17	that that was 70 degrees.
374	01:18:51:19	01:18:54:02	Now, if we know
375	01:18:54:04	01:18:58:18	that the height-to-distance ratio is the tangent,
376	01:18:58:20	01:19:05:22	we can say the tangent of 70 is equal to X over 20-- this ratio.
377	01:19:05:24	01:19:07:04	Right?
378	01:19:07:06	01:19:09:11	Now, let's just use a little mathematics
379	01:19:09:13	01:19:11:11	and see what we're going to do here.
380	01:19:11:13	01:19:14:27	Well, I want X by itself, right?
381	01:19:14:29	01:19:18:18	So let's multiply

both sides by 20,
382 01:19:18:20 01:19:25:18 and so I get: 20 times
the tangent of 70 degrees
383 01:19:25:20 01:19:28:17 is going to equal X.
384 01:19:28:19 01:19:31:08 Could somebody use
their calculator
385 01:19:31:10 01:19:34:15 and find for us
what is the tangent of 70?
386 01:19:34:17 01:19:35:25 Think about it for a minute.
387 01:19:35:27 01:19:39:14 Is it going to be a big ratio,
small ratio?
388 01:19:39:16 01:19:40:26 Dave, what do you get?
389 01:19:40:28 01:19:45:24 2.75, rounded off.
390 01:19:45:26 01:19:49:08 Okay, so we have 20 times 2.75,
rounded, and when...
391 01:19:49:10 01:19:51:10 Somebody multiply that for us?
392 01:19:51:12 01:19:53:11 What do we end up with?
393 01:19:53:13 01:19:56:26 I get 54.95.
394 01:19:56:28 01:19:59:12 Okay, or...
and that is in meters,
395 01:19:59:14 01:20:01:17 and what did we get earlier?
396 01:20:01:19 01:20:04:01 Didn't we get just about this?
397 01:20:04:03 01:20:07:07 I think it was about 50... 58.
398 01:20:07:09 01:20:09:23 So again, we're going
to have a little bit off
399 01:20:09:25 01:20:13:08 in terms of accuracy and, again,
in terms of our own precision,
400 01:20:13:10 01:20:16:03 but we found
two different ways, now,
401 01:20:16:05 01:20:21:03 that we have been able to get a
good approximation of a distance
402 01:20:21:05 01:20:25:05 that would be impossible
for us to measure directly.
403 01:20:25:07 01:20:28:20 CHAPIN:
Indirect measurement is
a major area of measurement.
404 01:20:28:22 01:20:31:13 Many, many times, we cannot
measure things directly.
405 01:20:31:15 01:20:33:24 Architects,
astronomers, scientists
406 01:20:33:26 01:20:37:00 are using these techniques
on a regular basis,
407 01:20:37:02 01:20:39:17 so we want to be
comfortable with them,
408 01:20:39:19 01:20:42:12 plus we want to understand
the mathematics
409 01:20:42:14 01:20:44:27 behind how
these measures are derived.
410 01:20:50:23 01:20:54:14 (*sirens wailing*)
411 01:21:00:23 01:21:02:06 NARRATOR:
It's another busy morning
412 01:21:02:08 01:21:04:03 for the Emergency
Communications Department
413 01:21:04:05 01:21:07:18 in the city
of Cambridge, Massachusetts.
414 01:21:07:20 01:21:10:02 MAN:

We're the people
that answer the 9-1-1 phone
415 01:21:10:04 01:21:12:01 and get callers
the help they need--
416 01:21:12:03 01:21:14:07 whether it's an ambulance,
a fire truck,
417 01:21:14:09 01:21:16:14 or a police car
or all of those.
418 01:21:16:16 01:21:18:14 MAN 2:
9-1-1, this call is recorded.
419 01:21:18:16 01:21:20:08 What's the location
of your emergency?
420 01:21:20:10 01:21:23:14 MAN 1:
When somebody calls 9-1-1
to report a medical emergency...
421 01:21:23:16 01:21:27:09 MAN 2:
All right, is there
any serious bleeding?
422 01:21:27:11 01:21:29:08 MAN 1:
After we ask four,
five questions...
423 01:21:29:10 01:21:31:13 MAN 2:
All right, do you know
how far she fell?
424 01:21:31:15 01:21:33:27 MAN 1:
We're able to figure out
how many and what type
425 01:21:33:29 01:21:35:28 of ambulances,
fire trucks and police cars
426 01:21:36:00 01:21:37:18 they need sent
to their location.
427 01:21:37:20 01:21:41:09 We've set up a map display
in front of the dispatchers
428 01:21:41:11 01:21:44:05 that shows them
the location of the units
429 01:21:44:07 01:21:46:19 and the location
of the 9-1-1 calls.
430 01:21:46:21 01:21:50:08 The idea is to let them see
which units are closest.
431 01:21:50:10 01:21:53:23 MAN 3:
Attention, rescue company
from headquarters.
432 01:21:53:25 01:21:55:05 Respond to 489 Broadway.
433 01:21:55:07 01:21:57:24 This is for a ten-year-old
female who has fallen.
434 01:21:57:26 01:22:01:03 MAN 1:
G.P.S. is the device
that's in the police cars,
435 01:22:01:05 01:22:03:01 fire trucks and ambulances,
436 01:22:03:03 01:22:06:17 that lets the dispatcher
see their location.
437 01:22:08:23 01:22:12:03 NARRATOR:
G.P.S. stands for
"Global Positioning System."
438 01:22:12:05 01:22:15:09 First developed by the U.S.
Department of Defense

439	01:22:15:11	01:22:16:27	in the '70s, it involves
440	01:22:16:29	01:22:19:08	a series of satellites and computers
441	01:22:19:10	01:22:22:24	designed to calculate positions anywhere in the world,
442	01:22:22:26	01:22:27:04	any time of day, to an accuracy of three to four meters.
443	01:22:27:06	01:22:31:25	The way G.P.S. works... if I'm located, say, here in Cambridge,
444	01:22:31:27	01:22:34:26	and I need to work out where I'm located,
445	01:22:34:28	01:22:36:08	what I can use
446	01:22:36:10	01:22:39:05	is G.P.S. satellites which are located up here,
447	01:22:39:07	01:22:41:07	well above the surface of the Earth.
448	01:22:41:09	01:22:44:19	Each of these satellites is transmitting a radio signal
449	01:22:44:21	01:22:47:21	that tells me the time the signal was transmitted,
450	01:22:47:23	01:22:50:17	and it propagates from the satellite
451	01:22:50:19	01:22:53:00	down to positions on the ground,
452	01:22:53:02	01:22:54:14	and at my ground receiver,
453	01:22:54:16	01:22:56:14	I measure the time the signal arrives.
454	01:22:56:16	01:22:58:26	By looking at the time difference
455	01:22:58:28	01:23:01:25	and the fact that the radio waves travel
456	01:23:01:27	01:23:05:13	at a known velocity, I can calculate the distance
457	01:23:05:15	01:23:08:06	from this satellite to me on the ground.
458	01:23:08:08	01:23:10:24	But that doesn't do me very good by itself,
459	01:23:10:26	01:23:12:16	because I could be anywhere
460	01:23:12:18	01:23:15:00	along a circle with that as its radius.
461	01:23:15:02	01:23:18:00	If I have a second satellite up here,
462	01:23:18:02	01:23:21:15	which I do the same type of measurement on,
463	01:23:21:17	01:23:23:18	then I can measure that distance.
464	01:23:23:20	01:23:26:16	Now, the signals that the G.P.S. satellites transmit
465	01:23:26:18	01:23:28:13	tell me where they are in space,
466	01:23:28:15	01:23:32:21	so I can calculate, from the positions of those satellites,
467	01:23:32:23	01:23:34:28	this side of the triangle.
468	01:23:35:00	01:23:38:04	Now I have a triangle with three known sides.
469	01:23:38:06	01:23:40:18	That allows me to uniquely work out
470	01:23:40:20	01:23:44:02	where I am on the ground

			if the Earth was flat.
471	01:23:44:04	01:23:48:11	The problem with this is that in three dimensions,
472	01:23:48:13	01:23:51:06	this triangle that I have defined
473	01:23:51:08	01:23:54:26	can rotate in and out of the board.
474	01:23:54:28	01:23:56:18	To solve that problem,
475	01:23:56:20	01:23:59:11	I need to use a third G.P.S. satellite,
476	01:23:59:13	01:24:02:25	which would itself be positioned in three dimensions,
477	01:24:02:27	01:24:06:25	and so now I have a pyramid, with me at the apex,
478	01:24:06:27	01:24:08:24	and in three dimensions,
479	01:24:08:26	01:24:13:12	I am able to work out where I am located.
480	01:24:13:14	01:24:15:11	(*sirens wailing*)
481	01:24:15:13	01:24:19:03	NARRATOR: The first emergency vehicles in Cambridge
482	01:24:19:05	01:24:20:17	to be installed with G.P.S.
483	01:24:20:19	01:24:22:18	were the city's fire rescue units.
484	01:24:22:20	01:24:26:28	MAN 1: We have a G.P.S. receiver, with an antenna on the roof,
485	01:24:27:00	01:24:29:25	that receives the position of the unit
486	01:24:29:27	01:24:32:24	and broadcasts that position over a cable
487	01:24:32:26	01:24:35:17	to a laptop that's in the fire rescue.
488	01:24:35:19	01:24:38:26	That laptop computer has a radio attached to it,
489	01:24:38:28	01:24:41:17	and it sends that position periodically
490	01:24:41:19	01:24:43:16	back to the dispatch center,
491	01:24:43:18	01:24:47:04	and that position of that unit is displayed on a map
492	01:24:47:06	01:24:48:28	in front of a dispatcher.
493	01:24:49:00	01:24:51:27	MAN 2: Okay, we'll have an officer respond out there to help you.
494	01:24:51:29	01:24:53:15	All right, thank you... bye.
495	01:24:53:17	01:24:55:19	NARRATOR: G.P.S. is not only helpful to dispatchers
496	01:24:55:21	01:24:58:14	but to responding units as well.
497	01:24:58:16	01:25:00:08	(*siren wails*)
498	01:25:00:10	01:25:04:02	MAN 1: In many large counties, the map display in the unit
499	01:25:04:04	01:25:07:19	can help a police car or an ambulance or a fire truck

500	01:25:07:21	01:25:11:26	find the location quickly and, in fact, in some places,
501	01:25:11:28	01:25:15:28	the map can show the optimal route to the location,
502	01:25:16:00	01:25:19:02	which might save five or ten minutes in a response
503	01:25:19:04	01:25:21:05	and help to save someone's life.
504	01:25:21:07	01:25:24:22	NARRATOR: In today's world, there is a growing application
505	01:25:24:24	01:25:26:26	for the Global Positioning System--
506	01:25:26:28	01:25:30:17	from mapping family trips to navigating boats,
507	01:25:30:19	01:25:32:24	from laying out construction sites
508	01:25:32:26	01:25:34:18	to studying earthquakes.
509	01:25:36:26	01:25:40:10	One of the most important, though, will always be
510	01:25:40:12	01:25:43:05	when you pick up the phone and call 9-1-1.
511	01:25:46:02	01:25:49:20	Captioned by Media Access Group at WGBH access.wgbh.org