1 01:00:47:09 01:00:48:19 WOMAN:
Measurement is the process
2 01:00:48:21 01:00:51:22 of quantifying properties
of objects.
3 01:00:51:24 01:00:54:19 And to do that,
we have set procedures
4 01:00:54:21 01:00:56:20 that enable us to measure.
5 01:00:56:22 01:00:58:12 Oh.
6 01:00:58:14 01:01:00:26 Measuring helps you
to understand
7 01:01:00:28 01:01:03:07 how things relate to each other.
8 01:01:03:09 01:01:07:05 Our volume of a sphere
actually has a formula
9 01:01:07:07 01:01:09:16 of 4/3 pi r-cubed.
10 01:01:09:18 01:01:13:02 This course really made me think
about how I approach measurement
11 01:01:13:04 01:01:16:05 and how I can use measurement
every day in the classroom.
12 01:01:21:22 01:01:23:18 CHAPIN:
In session four
on angle measurement,
13 01:01:23:20 01:01:26:04 I had a number of objectives.
14 01:01:26:06 01:01:28:10 One was, first, to review
15 01:01:28:12 01:01:31:15 what many participants already
knew about angle measure--
16 01:01:31:17 01:01:33:27 how to write angles,
what the symbolism was
17 01:01:33:29 01:01:36:18 and what different
angle measures mean.
18 01:01:36:20 01:01:39:05 But in addition,
we wanted to go beyond that
19 01:01:39:07 01:01:41:04 and look at angle relationships
20 01:01:41:06 01:01:46:14 and to get to the point of some
informal justification or proof
21 01:01:46:16 01:01:49:27 of what they supposedly
always take for granted.
22 01:01:49:29 01:01:53:16 An angle is a union of two rays
at a common point
23 01:01:53:18 01:01:55:22 which is known as the vertex.
24 01:01:55:24 01:01:57:27 And if we look up here...
25 01:02:06:24 01:02:13:23 we can name that angle
by ABC or CBA...
26 01:02:18:10 01:02:22:12 or even angle B,
in terms of the vertex.
27 01:02:22:14 01:02:26:27 When we want to talk
about measuring that angle,
28 01:02:26:29 01:02:28:28 we often put an M
29 01:02:29:00 01:02:32:11 to say that we're interested
in the measure of angle ABC,
30 01:02:32:13 01:02:35:03 and that's when we are
then going to consider
31 01:02:35:05 01:02:38:27 how many degrees
are involved in that angle.
32 01:02:38:29 01:02:42:13 Along with naming angles
and the symbolism,
33 01:02:42:15 01:02:46:20 we have some tools that are used

to measure angles.

34 01:02:46:22 01:02:51:08 One tool that you have
in front of you is a protractor.

35 01:02:51:10 01:02:56:01 Another tool that is also used
is called an angle ruler.

36 01:02:56:03 01:03:01:19 We're going to look at how this
works as well to measure angles.

37 01:03:01:21 01:03:03:00 All right?

38 01:03:03:02 01:03:04:11 Kind of interesting.

39 01:03:04:13 01:03:06:21 This one is what I call
a more dynamic instrument,

40 01:03:06:23 01:03:11:29 in that it shows the movement
as I open up the arc

41 01:03:12:01 01:03:15:19 and have a larger
and larger angle.

42 01:03:15:21 01:03:20:25 You also have
a very simple device

43 01:03:20:27 01:03:24:11 which is another way
to measure angles.

44 01:03:24:13 01:03:27:03 It's a simple straw protractor.

45 01:03:27:05 01:03:30:05 Now, would it matter if I...

46 01:03:30:07 01:03:33:12 Say, here's a 90-degree angle.

47 01:03:33:14 01:03:36:11 What if I cut the rays?

48 01:03:36:13 01:03:40:17 Would it affect the angle size?

49 01:03:40:19 01:03:45:27 The angle does not depend
on the length of the rays.

50 01:03:45:29 01:03:49:07 It's the region
between the rays.

51 01:03:49:09 01:03:51:13 Right,
absolutely.

52 01:03:51:15 01:03:58:01 This is a common misconception,
and one where many people think

53 01:03:58:03 01:04:00:13 that this angle
would be different

54 01:04:00:15 01:04:02:07 than if I cut it off,

55 01:04:02:09 01:04:04:09 so that they get easily swayed

56 01:04:04:11 01:04:07:10 by, again, looking at the sides
of the angle

57 01:04:07:12 01:04:10:20 and think that that's going
to make a difference

58 01:04:10:22 01:04:12:20 from the actual measure.

59 01:04:16:03 01:04:20:11 We can classify angles
in a number of different ways,

60 01:04:20:13 01:04:25:13 and one way is according
to their measure.

61 01:04:25:15 01:04:29:03 And listed up here
on the yellow pad

62 01:04:29:05 01:04:31:22 are a number of different ways
that we classify angles.

63 01:04:31:24 01:04:35:21 Anyone want to give us a quick
definition of an acute angle?

64 01:04:35:23 01:04:38:17 That's an angle that measures
less than 90 degrees.

65 01:04:38:19 01:04:39:24 All right.

66 01:04:39:26 01:04:41:05 How about
obtuse angle?

67 01:04:41:07 01:04:45:22 An obtuse angle is greater than 90, but less than 180.
68 01:04:45:24 01:04:48:00 Now, I think we all know a right angle.
69 01:04:48:02 01:04:56:08 Straight angle is simply, by definition, a straight line.
70 01:04:56:10 01:04:59:05 So ABC there is a straight angle.
71 01:04:59:07 01:05:01:22 CHAPIN: We reviewed some of the terms
72 01:05:01:24 01:05:05:03 that are associated with angle and angle measure.
73 01:05:05:05 01:05:08:26 We looked at how we classified angles by their measure
74 01:05:08:28 01:05:11:04 such as acute and obtuse angles.
75 01:05:11:06 01:05:14:15 We looked at also other ways of classifying them
76 01:05:14:17 01:05:16:17 by their relationship with other angles.
77 01:05:16:19 01:05:19:15 We looked at the supplementary angles, complementary.
78 01:05:19:17 01:05:22:28 What's the difference?
79 01:05:23:00 01:05:24:24 Two angles are supplementary
80 01:05:24:26 01:05:28:14 when the sum of their measures is equal to 180 degrees;
81 01:05:28:16 01:05:30:29 and whereas complementary--
82 01:05:31:01 01:05:34:25 their sum will be equal to 90 degrees.
83 01:05:34:27 01:05:37:03 We also have congruent angles--
84 01:05:37:05 01:05:40:00 angles that are exactly the same size.
85 01:05:40:02 01:05:44:15 Adjacent angles are those that are right next to each other,
86 01:05:44:17 01:05:47:07 that share a common side, all right?
87 01:05:47:09 01:05:49:15 And then we have four relationships
88 01:05:49:17 01:05:52:18 that we may not have thought about in a while--
89 01:05:52:20 01:05:55:11 vertical angles, alternate interior angles,
90 01:05:55:13 01:05:57:03 alternate exterior angles,
91 01:05:57:05 01:06:00:27 and we're going to hold off on corresponding angles first
92 01:06:00:29 01:06:02:26 for a minute.
93 01:06:02:28 01:06:07:06 Now, these relationships are ones that are true
94 01:06:07:08 01:06:09:27 when we have two parallel lines.
95 01:06:09:29 01:06:12:26 Now, bear with my drawing, all right?
96 01:06:12:28 01:06:20:10 Imagine that line one here and line two are perfectly parallel,
97 01:06:20:12 01:06:23:27 equal distance apart,
98 01:06:23:29 01:06:26:11 and they are cut by another line

99 01:06:26:13 01:06:29:14 which we call the transversal, all right?
100 01:06:29:16 01:06:33:24 Now, there's some interesting relationships that occur here.
101 01:06:33:26 01:06:37:06 Does anyone want to share with us any of those?
102 01:06:37:08 01:06:40:13 Can you letter them, like, A through G?
103 01:06:40:15 01:06:41:21 Is that the eighth letter?
104 01:06:41:23 01:06:42:26 How's that?
105 01:06:42:28 01:06:44:00 For example,
106 01:06:44:02 01:06:46:18 angles A and D are vertical angles.
107 01:06:46:20 01:06:49:04 They're across a V from each other.
108 01:06:49:06 01:06:51:04 That's kind of where the word "vertical"
109 01:06:51:06 01:06:52:15 can be a nice little hint.
110 01:06:52:17 01:06:54:01 And they're the same measurement.
111 01:06:54:03 01:06:55:07 Okay.
112 01:06:55:09 01:06:58:00 Similarly, B and C would be the same.
113 01:06:58:02 01:07:01:06 Okay, now can I stop for a minute and say
114 01:07:01:08 01:07:05:10 how do we know that vertical angles are equal to each other
115 01:07:05:12 01:07:07:10 and their measure is the same?
116 01:07:07:12 01:07:09:09 How... you know.
117 01:07:09:11 01:07:14:21 A and B are supplementary, so we know that A and B add up to 180.
118 01:07:14:23 01:07:18:03 And similarly, B and D are supplementary.
119 01:07:18:05 01:07:20:14 They add up to 180.
120 01:07:20:16 01:07:22:21 So if A plus B equals 180
121 01:07:22:23 01:07:24:23 and B plus D equals 180
122 01:07:24:25 01:07:26:24 because they're both...
123 01:07:26:26 01:07:29:21 A and D both have the same thing
124 01:07:29:23 01:07:32:15 being added to them, and you get the same result,
125 01:07:32:17 01:07:33:24 they have to be equal.
126 01:07:33:26 01:07:35:10 Okay, so we've got it
127 01:07:35:12 01:07:39:12 that vertical angles are equal measures
128 01:07:39:14 01:07:41:08 which is a handy thing to have.
129 01:07:41:10 01:07:44:08 Now, what do you notice

about B and F?

130 01:07:44:10 01:07:48:07 If line one and line two are parallel,

131 01:07:48:09 01:07:50:03 and the transversal line

132 01:07:50:05 01:07:51:19 is moving across them

133 01:07:51:21 01:07:53:11 at the same angle...

134 01:07:53:13 01:07:55:03 in the same direction--

135 01:07:55:05 01:07:56:12 it isn't moving--

136 01:07:56:14 01:07:59:06 then those angles would have to remain the...

137 01:07:59:08 01:08:02:04 would have to be the same because of the cross.

138 01:08:02:06 01:08:03:22 Right, and those are known

139 01:08:03:24 01:08:05:06 as corresponding angles,

140 01:08:05:08 01:08:06:11 that they correspond

141 01:08:06:13 01:08:09:17 in regards to the parallel lines

142 01:08:09:19 01:08:15:21 and so that we can say that B and F are also equal in measure,

143 01:08:15:23 01:08:17:21 C and E, D and H

144 01:08:17:23 01:08:22:10 because they are getting cut, again, by the transversal...

145 01:08:22:12 01:08:27:00 the parallel lines at the same angle by that transversal.

146 01:08:27:02 01:08:28:27 So we've got corresponding.

147 01:08:28:29 01:08:30:25 We've done vertical.

148 01:08:30:27 01:08:34:27 Well, that leaves alternate interior and alternate exterior.

149 01:08:34:29 01:08:37:28 Well, alternate interior have to do in terms

150 01:08:38:00 01:08:41:14 of the interior angles formed by the parallel lines.

151 01:08:41:16 01:08:44:28 Finally, we have alternate exterior angles.

152 01:08:45:00 01:08:50:17 Well, here the exterior, in terms of the parallel lines,

153 01:08:50:19 01:08:54:21 alternate exterior is B and G and A and H

154 01:08:54:23 01:08:56:21 because they're on the exterior.

155 01:08:56:23 01:08:58:04 They're alternating.

156 01:09:01:02 01:09:03:26 NARRATOR: The class is now given the opportunity

157 01:09:03:28 01:09:07:18 to apply some of the concepts they have been examining.

158 01:09:07:20 01:09:10:12 Using power polygons, they begin the task

159 01:09:10:14 01:09:13:04 of finding the measurement of each angle

160 01:09:13:06 01:09:15:00 found in different shapes.

161 01:09:15:02 01:09:17:01 So we've got
a parallelogram,
162 01:09:17:03 01:09:19:03 and the two sides
are equal.
163 01:09:19:05 01:09:21:01 All the angles have
to equal 360.
164 01:09:21:03 01:09:24:14 WOMAN:
Okay, and we have two acute
angles and two obtuse.
165 01:09:24:16 01:09:26:04 Right, and
they're opposite.
166 01:09:26:06 01:09:27:25 And the opposites
are equal.
167 01:09:27:27 01:09:31:22 So we need to figure out
what they measure.
168 01:09:31:24 01:09:35:07 I'm taking
an equilateral
triangle here.
169 01:09:35:09 01:09:37:17 And we know that
all three angles
170 01:09:37:19 01:09:39:03 in this one
measure 60,
171 01:09:39:05 01:09:40:28 so this angle
measures 60.
172 01:09:41:00 01:09:42:20 And that one
measures 60.
173 01:09:42:22 01:09:47:05 So if these measure 60,
together that's 120.
174 01:09:47:07 01:09:48:21 Right.
175 01:09:48:23 01:09:50:06 And angle one was 60;
176 01:09:50:08 01:09:52:19 angle two was 60
and then 120, 120.
177 01:09:52:21 01:09:54:10 And that equals 360.
178 01:09:54:12 01:09:56:01 Yep.
179 01:09:56:03 01:09:58:20 CHAPIN:
In the power polygon activity,
180 01:09:58:22 01:10:01:15 participants were asked
to determine the measure
181 01:10:01:17 01:10:03:25 of the angles
in the different polygons.
182 01:10:03:27 01:10:07:11 And one of the interesting
things that occurred was
183 01:10:07:13 01:10:09:14 they started to use information
184 01:10:09:16 01:10:13:16 that they had gleaned from some
of the other power polygons.
185 01:10:13:18 01:10:15:05 So, for example,
186 01:10:15:07 01:10:18:18 they used the idea that
in an equilateral triangle,
187 01:10:18:20 01:10:20:27 the angle measures are
60 degrees,
188 01:10:20:29 01:10:23:14 and then they could place
that triangle
189 01:10:23:16 01:10:27:03 on top of another polygon
to determine a new measure.
190 01:10:27:05 01:10:30:24 They also started to build
and put together power polygons

191 01:10:30:26 01:10:32:26 to use some of the relationships
192 01:10:32:28 01:10:35:15 with parallel lines
cut by a transversal,
193 01:10:35:17 01:10:37:19 so they were going to make sense
194 01:10:37:21 01:10:41:12 of alternate interior angles
and corresponding angles and,
195 01:10:41:14 01:10:44:09 using what
they had discussed earlier,
196 01:10:44:11 01:10:46:23 and apply it to the solution
of these problems.
197 01:10:46:25 01:10:49:15 If you notice up here,
I have a very large triangle,
198 01:10:49:17 01:10:51:03 just any old triangle.
199 01:10:51:05 01:10:54:13 It can be scalene, isosceles
or equilateral.
200 01:10:54:15 01:10:58:07 And I've labeled each
of the angles on the inside.
201 01:10:58:09 01:11:03:10 And what I'd like you to do with
the paper that I've passed out--
202 01:11:03:12 01:11:08:09 that gray sheet-- is to cut out,
using your scissors, a triangle.
203 01:11:08:11 01:11:15:13 And I'd like you also to label
the angles inside your triangle.
204 01:11:15:15 01:11:17:27 And then what
we're going to do is,
205 01:11:17:29 01:11:22:02 we are going to actually cut
them or tear those angles off.
206 01:11:32:00 01:11:34:29 CHAPIN:
Everyone knows that the sum
of the angles in a triangle
207 01:11:35:01 01:11:36:20 is 180 degrees.
208 01:11:36:22 01:11:40:09 What's a little more difficult
is to explain why.
209 01:11:40:11 01:11:42:15 And so we grappled with that
210 01:11:42:17 01:11:47:04 and came up with two different
ways that we could justify...
211 01:11:47:06 01:11:50:13 One that is very accessible
to learners is,
212 01:11:50:15 01:11:53:08 we had triangles where
we labeled each angle.
213 01:11:53:10 01:11:55:02 We tore off the angles,
214 01:11:55:04 01:11:58:08 and then we realigned them
along a straight line,
215 01:11:58:10 01:12:01:25 finding that the three angles
formed a straight angle,
216 01:12:01:27 01:12:03:22 thus summed to 180.
217 01:12:03:24 01:12:08:06 A very informal proof
or justification,
218 01:12:08:08 01:12:10:22 but it allows us
to really make sense
219 01:12:10:24 01:12:13:07 of where does
that number come from.
220 01:12:13:09 01:12:16:20 Now, there's another way that
we can look at this same idea
221 01:12:16:22 01:12:18:17 that's a little bit more formal.
222 01:12:18:19 01:12:22:07 Take a look

at this diagram up here.

223 01:12:22:09 01:12:30:10 We have triangle ABC,
and I've drawn another line

224 01:12:30:12 01:12:35:03 that is parallel to the base
of triangle ABC.

225 01:12:35:05 01:12:41:12 Why can we say angle measures
in this triangle sum to 180?

226 01:12:41:14 01:12:46:00 Angles A and 1 are
alternate interior
angles.

227 01:12:46:02 01:12:48:26 They're equal.

228 01:12:48:28 01:12:51:26 And then also
angles 3 and B

229 01:12:51:28 01:12:53:17 would also be
the same,

230 01:12:53:19 01:12:56:20 and we see that
1, 2 and 3 add
up to 180,

231 01:12:56:22 01:12:59:26 so A, B and C should
also add up to 180.

232 01:12:59:28 01:13:01:29 CHAPIN:
So we can see

233 01:13:02:01 01:13:06:03 that there are a number of ways
that we can justify

234 01:13:06:05 01:13:10:16 why the measure of the angles
in a triangle add to 180.

235 01:13:10:18 01:13:14:08 What about the measure of
the angles in quadrilaterals?

236 01:13:14:10 01:13:16:15 There are a lot
of quadrilaterals

237 01:13:16:17 01:13:18:11 in your shape set.

238 01:13:18:13 01:13:20:10 What do those angles sum to?

239 01:13:20:12 01:13:22:00 They sum
to 360 degrees.

240 01:13:22:02 01:13:23:19 Why do you know
that, John?

241 01:13:23:21 01:13:24:23 Why do
I know that?

242 01:13:24:25 01:13:25:26 I don't know.

243 01:13:25:28 01:13:28:02 It's a rule
I learned in school.

244 01:13:28:04 01:13:29:24 (*laughter*)

245 01:13:29:26 01:13:31:09 Can anyone
help John?

246 01:13:31:11 01:13:32:23 Let's see.

247 01:13:32:25 01:13:34:05 Mary.

248 01:13:34:07 01:13:36:06 I didn't see
the correlation at first,

249 01:13:36:08 01:13:39:04 but Sue came up with, that
if you can take any vertex

250 01:13:39:06 01:13:44:20 and make a triangle
to another vertex,

251 01:13:44:22 01:13:46:00 how many triangles
in it

252 01:13:46:02 01:13:48:04 makes how many degrees
will be in it

253 01:13:48:06 01:13:49:20 because we know every triangle
254 01:13:49:22 01:13:52:04 has 180 degrees.
255 01:13:52:06 01:13:54:23 So would that work for all quadrilaterals?
256 01:13:54:25 01:13:56:02 Yes.
257 01:13:56:04 01:13:58:12 That we could divide it into two triangles
258 01:13:58:14 01:14:04:05 and come up with 180 plus 180 is 360?
259 01:14:04:07 01:14:06:21 NARRATOR: The class considers whether this method
260 01:14:06:23 01:14:09:16 for finding the sum of the angles in a quadrilateral
261 01:14:09:18 01:14:12:05 will also apply to other polygons
262 01:14:12:07 01:14:15:27 such as a pentagon or hexagon.
263 01:14:15:29 01:14:20:10 If you choose any vertex on your polygon
264 01:14:20:12 01:14:25:16 and draw it to another vertex,
265 01:14:25:18 01:14:28:04 you have three triangles right here,
266 01:14:28:06 01:14:31:01 and we know that each triangle has 180 degrees,
267 01:14:31:03 01:14:34:08 so we can say the sum of the interior angles of this polygon
268 01:14:34:10 01:14:39:23 is 3 times 180, which is... what is it?
269 01:14:39:25 01:14:40:23 MAN: 540.
270 01:14:40:25 01:14:42:05 540.
271 01:14:42:07 01:14:44:10 Okay, I know it, but I can't figure it out.
272 01:14:44:12 01:14:46:24 So I can do that with any polygon,
273 01:14:46:26 01:14:49:15 so if I choose this six-sided figure,
274 01:14:49:17 01:14:51:29 I just draw my diagonals.
275 01:14:53:20 01:14:57:13 Now I have four triangles.
276 01:14:57:15 01:15:03:20 4 times 180, which is 720.
277 01:15:04:28 01:15:07:26 And this works with any polygon.
278 01:15:07:28 01:15:10:23 You can choose a number... any number of sides you want,
279 01:15:10:25 01:15:15:26 draw in all the diagonals and multiply the triangles by 180.
280 01:15:15:28 01:15:19:06 Is there any pattern of how many triangles you get
281 01:15:19:08 01:15:23:08 based on the number of sides of the figure?
282 01:15:23:10 01:15:24:08 What did you notice?
283 01:15:24:10 01:15:26:23 Well, if there's, um...
284 01:15:26:25 01:15:28:18 I always start with a triangle first.

285 01:15:28:20 01:15:33:20 Triangle has three sides,
no diagonal, 180 degrees.
286 01:15:33:22 01:15:37:22 There's four sides,
two triangles.
287 01:15:37:24 01:15:40:27 This is five sides,
three triangles.
288 01:15:40:29 01:15:44:02 So it's the number of sides
less two.
289 01:15:44:04 01:15:47:26 That's my, uh... my idea.
290 01:15:47:28 01:15:50:25 If we called N the number
of sides, minus two
291 01:15:50:27 01:15:53:05 gives us the number
of triangles,
292 01:15:53:07 01:16:01:17 and then times 180
will give us the total sum
293 01:16:01:19 01:16:04:00 of the angles in that shape.
294 01:16:04:02 01:16:06:18 One thing we have
to be careful about, though,
295 01:16:06:20 01:16:09:19 is that unless the shape
is a regular shape
296 01:16:09:21 01:16:11:27 and all the angles
are congruent,
297 01:16:11:29 01:16:17:08 we still don't know what is the
measure of each of those angles.
298 01:16:17:10 01:16:22:07 We know all together, those five
add up to 540 degrees,
299 01:16:22:09 01:16:25:22 but we don't know what
any one is using this method
300 01:16:25:24 01:16:29:06 unless it happened
that all five were identical.
301 01:16:29:08 01:16:30:26 And in this case, they're not.
302 01:16:34:05 01:16:37:08 NARRATOR:
The class is now introduced
to GeoLogo,
303 01:16:37:10 01:16:38:29 a computer program
that lends itself
304 01:16:39:01 01:16:41:19 to the exploration
of angle and shape.
305 01:16:41:21 01:16:43:15 After a short review,
306 01:16:43:17 01:16:46:11 Professor Chapin lets
participants begin
307 01:16:46:13 01:16:47:27 by building their own polygon.
308 01:16:47:29 01:16:49:13 Jump ahead.
309 01:16:49:15 01:16:52:00 If you think automatically
"I know how to make a square,
310 01:16:52:02 01:16:53:24 and I understand
what's going on there,"
311 01:16:53:26 01:16:55:23 try to make a pentagon,
a regular pentagon.
312 01:16:55:25 01:16:59:06 Try to make a regular hexagon
or a regular octagon, okay?
313 01:16:59:08 01:17:00:25 So experiment a little bit
314 01:17:00:27 01:17:03:11 with how you are going
to make different shapes
315 01:17:03:13 01:17:05:25 using what
we have learned

about angles
316 01:17:05:27 01:17:08:17 so far in our
session today,
317 01:17:08:19 01:17:12:09 and likewise,
considering
what is the sum
318 01:17:12:11 01:17:16:06 of the exterior angles
when I make a shape
each time.
319 01:17:16:08 01:17:17:24 All right?
320 01:17:17:26 01:17:19:05 We've tried
a couple of things
321 01:17:19:07 01:17:20:17 to make this
regular pentagon,
322 01:17:20:19 01:17:22:20 but what do you think
with the 72-degree angles?
323 01:17:22:22 01:17:24:05 You think we'll have
some success?
324 01:17:24:07 01:17:28:02 Okay, I think if we look
at the exterior of the pentagon
325 01:17:28:04 01:17:32:09 and make sure we know all
the angles will be 180 degrees,
326 01:17:32:11 01:17:35:06 if we subtract that
from 180, get 72.
327 01:17:35:08 01:17:37:04 So if we do the "repeat" button
328 01:17:37:06 01:17:40:26 and do it five times,
forward 40,
329 01:17:40:28 01:17:42:14 70 right angle, 72 degrees.
330 01:17:42:16 01:17:44:01 Let's try that.
331 01:17:44:03 01:17:46:17 So we have to do
the brackets.
332 01:17:46:19 01:17:48:25 So now we're going to do
the repeat command.
333 01:17:48:27 01:17:50:03 Repeat.
334 01:17:50:05 01:17:51:19 That will make and allow us
335 01:17:51:21 01:17:53:17 to do the same command
five times.
336 01:17:53:19 01:17:57:06 Okay, and we want to repeat it
"five" space "five."
337 01:17:57:08 01:17:58:07 Bracket?
338 01:17:58:09 01:17:59:12 No, space.
339 01:17:59:14 01:18:01:01 Okay.
340 01:18:01:03 01:18:02:09 Bracket.
341 01:18:02:11 01:18:04:10 Okay, now, how far you
want to move it forward?
342 01:18:04:12 01:18:05:21 We want to do 40?
343 01:18:05:23 01:18:06:28 Yeah.
344 01:18:07:00 01:18:08:07 Or maybe we can make it...
345 01:18:08:09 01:18:09:14 Well...
346 01:18:09:16 01:18:10:24 Want to try 50?
347 01:18:10:26 01:18:16:21 50, and then we're going
to do right turn, 72, right?
348 01:18:16:23 01:18:18:04 And repeat.
349 01:18:18:06 01:18:19:14 And close.
350 01:18:19:16 01:18:21:08 And let's see.
351 01:18:21:10 01:18:22:11 Yes.

352 01:18:22:13 01:18:23:15 Yay!
353 01:18:23:17 01:18:24:20 (*both giggling*)
354 01:18:24:22 01:18:26:07 Finally!
355 01:18:26:09 01:18:27:23 Okay, that's great
356 01:18:27:25 01:18:29:26 because we had tried
compensating
357 01:18:29:28 01:18:31:18 and making
one angle 72
358 01:18:31:20 01:18:35:16 and then figured
we had to compensate
for the others
359 01:18:35:18 01:18:38:06 by making them 108,
and we didn't.
360 01:18:38:08 01:18:40:02 We had to keep
them all 72.
361 01:18:40:04 01:18:41:17 That's fabulous.
362 01:18:41:19 01:18:42:27 So this was
a little different
363 01:18:42:29 01:18:44:08 than some of
the other figures.
364 01:18:44:10 01:18:47:12 When we were, um,
building shapes
up there,
365 01:18:47:14 01:18:53:01 I asked you to look
for the sum of the
exterior angles.
366 01:18:53:03 01:18:55:21 Can anyone tell us
what they found out,
367 01:18:55:23 01:18:58:01 and perhaps justify
why they are...
368 01:18:58:03 01:19:01:13 what they say the sum
of the exterior angles was
369 01:19:01:15 01:19:04:07 in any of the shapes
that they looked at?
370 01:19:04:09 01:19:06:19 Supplementary angle
to the angle of the...
371 01:19:06:21 01:19:09:13 to the interior angle
of the shape we were drawing.
372 01:19:09:15 01:19:14:05 All right, so if
this interior angle
here is 60,
373 01:19:14:07 01:19:16:17 the exterior angle
out here is 120,
374 01:19:16:19 01:19:19:28 and then what is the sum
of the exterior angles
375 01:19:20:00 01:19:20:28 of this shape?
376 01:19:21:00 01:19:21:28 360.
377 01:19:22:00 01:19:23:03 360.
378 01:19:23:05 01:19:27:20 Now, is that true
for all shapes?
379 01:19:27:22 01:19:31:16 On the square that we made,
it had a sum of 360.
380 01:19:31:18 01:19:34:12 Okay, so it
also had 360.
381 01:19:34:14 01:19:38:25 Anybody else find one
on any other shapes?

382 01:19:38:27 01:19:41:05 It would have to have
an exterior sum of 360,
383 01:19:41:07 01:19:42:22 'cause if using the program,
384 01:19:42:24 01:19:45:19 if we're going to continue
having the turtle make turns
385 01:19:45:21 01:19:47:06 to get back to where we are,
386 01:19:47:08 01:19:49:28 we have to turn eventually
360 degrees to get back.
387 01:19:50:00 01:19:52:22 So the exterior angles being
the opposite of the interior,
388 01:19:52:24 01:19:54:15 we have to turn 360
for all of them.
389 01:19:54:17 01:19:56:12 For example, for a pentagon,
390 01:19:56:14 01:19:59:00 we want interior angles
of 108 degrees;
391 01:19:59:02 01:20:00:25 we want to turn five times.
392 01:20:00:27 01:20:03:16 So we're going to turn
72 degrees five times
393 01:20:03:18 01:20:05:17 for a total of 360
on the outside
394 01:20:05:19 01:20:07:22 and 540 on the inside.
395 01:20:07:24 01:20:11:11 CHAPIN:
When you are using
a computer program like this,
396 01:20:11:13 01:20:14:04 one of the first things
you have to realize
397 01:20:14:06 01:20:17:24 is that when you turn, that is
the actual exterior angle
398 01:20:17:26 01:20:20:05 and it is forming
an interior angle.
399 01:20:20:07 01:20:23:01 So if you wanted to have
a triangle
400 01:20:23:03 01:20:25:27 with 60-degrees
interior measures,
401 01:20:25:29 01:20:29:15 you actually have to make
a turn of 120 degrees.
402 01:20:29:17 01:20:30:26 This allows us then,
403 01:20:30:28 01:20:34:01 as we think about going
around the triangle--
404 01:20:34:03 01:20:36:21 we have to do that three times--
405 01:20:36:23 01:20:38:17 to start to investigate
406 01:20:38:19 01:20:43:09 what is the sum of the exterior
angles now of any polygon.
407 01:20:43:11 01:20:47:05 Exterior angles
in all polygons
sum to 360,
408 01:20:47:07 01:20:51:15 and we can show this very
clearly with the GeoLogo
409 01:20:51:17 01:20:55:21 because we can look at
the addition of the turns,
410 01:20:55:23 01:20:57:23 or the exterior angles.
411 01:20:57:25 01:20:58:29 CHAPIN:
Well, I hope that
412 01:20:59:01 01:21:01:18 as you reflect
on this session today,
413 01:21:01:20 01:21:05:09 that the various activities

and discussions we've had
414 01:21:05:11 01:21:06:25 around angle measure
415 01:21:06:27 01:21:10:02 have enabled you to expand your
understanding of this topic--
416 01:21:10:04 01:21:13:00 both in terms of how
we can classify angles
417 01:21:13:02 01:21:16:00 and how we can use
what we know about angles
418 01:21:16:02 01:21:19:10 to prove different relationships
in mathematics.
419 01:21:27:26 01:21:31:13 NARRATOR:
John Feldman,
a doctoral candidate from MIT,
420 01:21:31:15 01:21:33:10 is challenging Albert Chou,
421 01:21:33:12 01:21:36:19 an assistant professor
of mathematics at Harvard,
422 01:21:36:21 01:21:38:17 to a friendly game of pool.
423 01:21:38:19 01:21:42:11 They are playing eight ball,
a complex game of angles
424 01:21:42:13 01:21:46:06 that is as difficult to master
as it is fun to play.
425 01:21:46:08 01:21:49:12 The first step is learning how
to determine the correct angle
426 01:21:49:14 01:21:51:03 to strike the ball.
427 01:21:51:05 01:21:55:10 MAN:
What a lot of people do is stick
their cue behind the ball
428 01:21:55:12 01:21:57:04 as if they're going to hit it
straight.
429 01:21:57:06 01:21:59:11 And then you sort of look
at where the cue ball is
430 01:21:59:13 01:22:00:28 in relation
to that straight line
431 01:22:01:00 01:22:02:22 and you can really see exactly
432 01:22:02:24 01:22:05:12 what angle you have to hit
the target ball at.
433 01:22:08:11 01:22:12:05 But equally importantly, there's
an angle behind every shot
434 01:22:12:07 01:22:15:05 which goes mostly unrecognized
with beginners,
435 01:22:15:07 01:22:16:22 and that is the angle
436 01:22:16:24 01:22:20:25 that your cue ball comes off
the object ball after the shot,
437 01:22:20:27 01:22:23:14 And this angle is actually
always constant.
438 01:22:25:00 01:22:26:13 Consider this shot:
439 01:22:26:15 01:22:29:28 I want to put the object ball--
in this case, the 1 ball--
440 01:22:30:00 01:22:31:21 into the side pocket.
441 01:22:35:13 01:22:36:29 Now notice what happened.
442 01:22:37:01 01:22:42:15 I sent the object ball from a
position here in this direction.
443 01:22:42:17 01:22:44:22 And what happened
to the cue ball?
444 01:22:44:24 01:22:47:22 After hitting the object ball,
you would have noticed

445 01:22:47:24 01:22:51:00 the cue ball traveled more
or less in this direction here.
446 01:22:51:02 01:22:52:16 And notice
447 01:22:52:18 01:22:55:16 that this direction and this
make an angle of 90 degrees.
448 01:22:55:18 01:22:58:14 It turns out that this angle is
constant in every shot.
449 01:22:58:16 01:23:01:26 Now I'm going to make
the same shot I made before
450 01:23:01:28 01:23:03:19 but with a steeper angle.
451 01:23:06:05 01:23:09:16 The cue ball again went
at an angle of 90 degrees.
452 01:23:09:18 01:23:11:10 If I know now, initially,
453 01:23:11:12 01:23:13:09 what direction my cue ball's
going to go
454 01:23:13:11 01:23:14:21 after sinking this shot,
455 01:23:14:23 01:23:18:23 I could now work with it
and use the effects of spin.
456 01:23:18:25 01:23:22:13 So if I hit the cue ball
below the center...
457 01:23:24:00 01:23:25:16 notice it comes back.
458 01:23:25:18 01:23:27:15 So, this is really
the beginning,
459 01:23:27:17 01:23:30:07 the first two steps in playing
position pool,
460 01:23:30:09 01:23:33:07 I can now use this predictable
angle of 90 degrees
461 01:23:33:09 01:23:36:04 on a shot which is not straight,
and combine it
462 01:23:36:06 01:23:38:26 with these effects
of reverse and forward spin
463 01:23:38:28 01:23:40:09 to change the angle.
464 01:23:40:11 01:23:43:26 So, I'm going to make the same
shot I made before.
465 01:23:43:28 01:23:46:29 I'm going to predict the
cue ball to come off initially
466 01:23:47:01 01:23:49:13 at an angle of roughly 90
degrees, like this.
467 01:23:49:15 01:23:52:29 But if I put reverse spin on the
cue ball, I'm going to predict
468 01:23:53:01 01:23:56:09 that this 90-degree angle
will be perturbed
469 01:23:56:11 01:23:58:23 to an angle
slightly more than 90
470 01:23:58:25 01:24:01:24 with the direction
of the object ball.
471 01:24:07:08 01:24:10:08 Let me make the same shot now
with forward spin,
472 01:24:10:10 01:24:12:28 and again I'm going to predict,
as opposed to a 90-degree,
473 01:24:13:00 01:24:15:11 I'm... the forward spin is going
to change this angle
474 01:24:15:13 01:24:17:17 to something slightly less
than 90.
475 01:24:21:10 01:24:23:21 FELDMAN:
You really have to think

not only about one shot
476 01:24:23:23 01:24:25:12 but the next shot
you're going to make.
477 01:24:25:14 01:24:27:08 And if you can set up your shot
in such a way
478 01:24:27:10 01:24:29:20 so that the cue ball will come
off the ball that you hit
479 01:24:29:22 01:24:31:10 and end up in a place
that will be easy
480 01:24:31:12 01:24:32:12 to make the next shot...
481 01:24:32:14 01:24:34:23 14 in the far
corner.
482 01:24:34:25 01:24:37:22 FELDMAN:
Then you'll be able to make
two shots in a row and not one.
483 01:24:39:17 01:24:42:23 NARRATOR:
In order to play competitively,
it is important
484 01:24:42:25 01:24:45:22 to master one more angle in
pool, and that's the bank shot.
485 01:24:45:24 01:24:48:00 This is where the cue ball
bounces off the rail
486 01:24:48:02 01:24:50:03 before hitting a ball
into the pocket.
487 01:24:50:05 01:24:51:26 FELDMAN:
One principle we can use
488 01:24:51:28 01:24:54:17 is that the angle
that it comes in to the rail
489 01:24:54:19 01:24:58:00 is the same angle that it will
have as it goes off of the rail.
490 01:24:58:02 01:24:59:13 And so we'd like a point
491 01:24:59:15 01:25:01:21 such that that principle
will allow us
492 01:25:01:23 01:25:04:07 to have the ball go
in the right direction.
493 01:25:04:09 01:25:08:28 For instance, if we hit it
too far to the right...
494 01:25:09:00 01:25:11:23 that time our angle was too
steep, and so we miss the ball.
495 01:25:11:25 01:25:14:08 On the other hand, if we go
too far to the left,
496 01:25:14:10 01:25:16:01 then the angle
will be too small,
497 01:25:16:03 01:25:18:08 and we're going to miss
that way.
498 01:25:18:10 01:25:22:23 For instance, if I shoot it down
this way, the angle's too small
499 01:25:22:25 01:25:25:09 and I miss the ball
in the other direction.
500 01:25:25:11 01:25:27:14 So what we really want to do
is judge the point
501 01:25:27:16 01:25:29:24 where, if we draw a line
from the cue to that point
502 01:25:29:26 01:25:31:29 and a line from that point
to our target ball,
503 01:25:32:01 01:25:35:04 we want those two lines

to have the same angle

504 01:25:35:06 01:25:36:19 in relation to the rail.

505 01:25:36:21 01:25:40:20 Okay, so now we're going to make our shot.

506 01:25:45:01 01:25:46:05 And there we go.

507 01:25:48:08 01:25:54:18 Captioned by Media Access Group at WGBH access.wgbh.org