In the final session of the course, the class is given an opportunity to discuss how the ideas they have explored about measurement translate into classroom practice.

So, what I thought we'd do today is actually take one topic, the topic of area, and first, generate together some of the big ideas about area.

"Area is a measure of how much surface is covered."

It's one reason they confuse so often area, perimeter, and sometimes even refer to solid shapes as, "Oh, that's square," when they really are talking about a cube.

Namely, we can cover this floor with square tiles, but if we tried to cover this floor with circular tiles, we'd have to have a lot of grout or something in between to fill in the spaces.

The units associated with area measurement are square units.

Have you noticed how often in this course I would ask you... you would give me an answer, and I'd say, "What?"

Now, we as educators have to be very careful to use the correct units and not to just say the numbers.

"It's 36."

"It's 36 what?"

If we don't focus in this case, square units, we are missing opportunities to really help students identify that unit.
Getting back to "How do we measure?"

Well, we have to think about the attribute, and then what is the unit that best can be used to measure that attribute?

Well, here, because we want to cover-- and we want to cover it completely-- we want to use square units. "The smaller the square unit, the more square units are necessary to determine the area."

In our final class, what we did was, first, try to list some of the big ideas around the measurement of area. And then, once we had some of the ideas articulated, I sent everyone back into grade-level groups to discuss how do those ideas get translated into practice: What kinds of activities could one do at their grade level and what mathematics was being highlighted around, in this case, the topic of area?

When you talked about estimation, I think it's important for younger kids to have benchmarks for estimation. Many times we give them something to estimate, we're not giving them... we just say "Oh, use this to estimate something."

Instead of doing that is actually giving them a benchmark by putting two or three pieces of paper, or whatever it takes to cover one end of the table. And... giving them a benchmark.

In other words,
this would be three
and a little more.

"How many now do you think
would cover this area?"

This gives them
a visual to work with.

I think again, too,
going back
to what we were talking
about with area

on, um... shapes:

If we start to look
at how shapes fill...
a, um, a larger shape--
how many triangles,
how many trapezoids,
how many rhombuses
would fill a hexagon?

Starting
to encourage children
to think about the why of that.
"Why is that happening?"
And then that gets
to your units, to your, um...

and to the understanding
that the smaller the unit

units you need
a certain area.
The larger the...

But it also makes
them think
that some units have
to be broken down.
I mean, like, for instance,
with my kids,

I give them a shape,
and they have to fill it in
with different shapes

in order to see...
And I put it on acetate
and I slide it over
onto Plexiglas,

and they look underneath,
see the holes
and they realize that,
"Oh, there's a hole.

I need to fill it in
with something."

So they start breaking shapes
into smaller pieces;
it's no longer
an abstract thought.

We give them too many abstracts,
you know--
a number is abstract.

You know, area is abstract.

Perimeter, well, they can walk around a perimeter, but...

It's an abstract concept.

So we need to make sure we keep it tactile, keep it oral or keep it something that they can visually see.

I think it's essential for them to physically take one shape to cover another shape over and over again.

For them to get that understanding that area is the measure of a surface with no spaces.

And I love that idea of the acetate look up, because then they'll really get the idea of "Oh, look at all this light coming down and pouring through."

WOMAN: You had mentioned an umbrella as another way to do that...

We talk about building an umbrella, and that seems to be a good metaphor for them.

Because then they understand, "Oh, I really can't have any holes in there to get wet."

Conservation, I think, is a fundamental concept that they need to be able to have in order to build on other concepts.

WOMAN: And I guess the last one would be standardization, that they understand that you need to have
a standard measurement at any given time that you could... The measurement might change depending on your... setting, but when you're measuring one thing at one time you need to have a consistent standard or unit of measure.

Later in the school year, we visited one of the participants of the summer session, Mary Guerino, a second-grade teacher at the Mabelle M. Burrell Elementary School in Foxboro, Massachusetts.

Good morning.

Good morning, Mrs. Guerino.

Building upon knowledge learned in the measurement course, Ms. Guerino is teaching a series of mathematics lessons on the concept of area.

My goal for today's lesson is to move children from nonstandard measurements to standard measurement. In the past few days, children have been using paper clips, links, buttons to cover different objects. Children need to be exposed to using different, nonstandard measurements to get the use of counting it and how to lay them out on top of objects. Before they can be introduced to the terminology of area, in that it is covering a unit with a specific, um, measurement tool or unit. Each one of these, we measured the same book, but we got different quantities. Would that help me if I was going to cover that book? Haley, would it help me?
Um, no.

No, why not?

Because objects might have holes in them and you...

and you couldn't cover the whole thing.

Okay, so none of these...

all right, were able to cover the entire object.

We used...

we used paper clips...

we used a playing card and we used buttons.

I want to go over this.

Why wouldn't buttons cover it?

When you put them on next to each other,

and there's a little space.

There's a little space, any circular object.

Would it cover the entire thing-- anybody know, yes or no?

If you put them like this and one on top,

there's a little hole in the middle.

There's still a little space.

So this isn't an object that always would cover an entire object,

like the book that we measured, right, okay?

How about the paper clip?

Chris.

They wouldn't cover it because they have holes in the middle.

It has holes in the middle.

Then we had rods.

Okay?

What happened with the rods?

Jessica.

Some went off, because one was in the middle and one was at the top that was sticking out.

Okay, can you use measurement words for them?

Was it too long?
Was it too short?

We are going to be doing a lesson this morning that talks about going from nonstandard measurement, okay?

We all got different answers. Didn't we all get different answers?

Because some of us were using different things, like some of us were using buttons.

So we were all using different objects?

Okay, so that's why we got different answers.

No.

Okay, so we got different answers.

Today we're going to use the same size objects to move from nonstandard measurement to what they call standard measurements--measurements that we use all the time... in the world.

Area is a very difficult concept.

We have to first make sure that children understand that it is a covering of a surface and that covering must be completed with units that do not leave any holes or gaps between the units.

This may seem rather straightforward, but surprisingly, that is not an easy idea for young children.

In our exploratory measurement today, we're going to use square-inch tiles... and centimeter cubes.

We're not concerned
about the cube today,
we're concerned about
the surface on the bottom,
which is a square... centimeter.
You have two papers
in front of you.
Would you turn them over now
and find the one that has
on the top of it...
a square tile.
I want you to take
a minute or two--
without using anything--
And then take a pencil--
you have to agree
between the two of you--
all right, and circle
the one where it says
number one
or number two.
A square is, like, bigger
on the other side.
Four, four equal sides
makes a square, remember?
And then four right angles
also makes a square.
Mm-hmm.
But they're pretty much
both squares, right?
Because, remember...
I would say it's this one,
wouldn't you?
Yeah.
All right.
This part is...
Hold on...
make a line through
half of it.
Put your
fingers on there.
And put your fingers
on here.
They're the same.
Mm-hmm, that way;
but this way...
This is.
Mm-hmm... so that
one would be...
Bigger.
Yup.
These activities were very
appropriate for second graders
because it involved them
actually covering surfaces,

263 01:12:12:04 01:12:15:20 counting the actual number
of units that were needed
264 01:12:15:22 01:12:19:05 and reflecting on the shape of
the unit that was used to cover
265 01:12:19:07 01:12:22:05 as well as the number of units
that were necessary
266 01:12:22:07 01:12:25:10 to determine an area.
267 01:12:34:19 01:12:36:23 Jillian, I noticed
you were using your hand.
268 01:12:36:25 01:12:39:04 Can you show me
what you did at the beginning?
269 01:12:39:06 01:12:42:14 I was looking
at the square
270 01:12:42:16 01:12:44:25 to see the size,
271 01:12:44:27 01:12:48:14 and then I kept on
moving my hand
right here
272 01:12:48:16 01:12:51:10 because I thought
it wasn't...
273 01:12:51:12 01:12:54:24 I thought I wasn't
moving my hand,
274 01:12:54:24 01:12:57:13 so then I measured it
this way.
275 01:12:57:15 01:13:00:26 And so which one
did you predict
would be bigger?
276 01:13:00:28 01:13:02:17 Um...
277 01:13:02:19 01:13:04:10 shape one.
278 01:13:04:12 01:13:06:02 But then, shape...
279 01:13:06:04 01:13:08:14 When we put on
the tiles,
280 01:13:08:16 01:13:10:22 shape two was
the bigger one.
281 01:13:10:24 01:13:14:01 Okay, so you used
your hand span, like this,
282 01:13:14:03 01:13:15:29 to make
a benchmark, okay,
283 01:13:16:01 01:13:18:27 to see whether this
was as big as this.
that didn't work?
285 01:13:21:19 01:13:25:03 Um... because
I kept on
286 01:13:25:05 01:13:28:04 moving my hand,
my fingers
287 01:13:28:06 01:13:31:04 and I didn't
really think I was.
288 01:13:31:06 01:13:33:12 Okay, so it wasn't
staying steady.
289 01:13:33:14 01:13:35:28 It wasn't a steady
measurement, okay.
290 01:13:36:00 01:13:38:01 Now, why do you
think it works
291 01:13:38:03 01:13:40:23 when we use
square tiles?
Because when we count them, they would be able to find out the number. And this one would have eight and two... This one has four in each row. So if we go like this, that would be eight, this would be eight, and this equals 16. And there's five going this way, and five plus five equals ten and ten plus five equals 15. Okay, so why isn't this one bigger where it has five in a row? Because when it goes down, it's not an even number... it's not the same number, and that because it would be right here and that's how we got 15 out of this and we got 16 out of this. Okay, very good. So would you use your hand as a measurement next time, or do you think tiles might be more accurate? Tiles. Okay, and that's... They were asked to cover up with square-inch tiles first, and they found that the second shape on the bottom was bigger. Then I asked them to do it again with centimeters to ask them whether they could conserve area.
In other words, if something is bigger in one spot, will it always be bigger or does it change with the size tile we use?

Devon, how many do you think you'll have?

More than 50.

I think I'll have more than 100.

Which one do you think will have more--

this one or this one?

This one.

Why?

I don't know.

I think this one will have more because this one has more at the top than that one does.

and on this side, this one has more than this one,

but this one at the top has more.

Why do you think this one will have more?

Do you think why, um... maybe because this one is shorter than that one, like the tiles did?

I don't know, but I still think this one will be bigger.

I think this one will be bigger, Devon,

because the last one when we used the tiles,

this one was less than that one,

but this one could be different,

because these little square centimeter cubes are smaller than the tiles are.

I noticed that you covered half of this, okay.

Can you predict which one will be bigger
covering half of it, do you think?

I think that it might be number two.

It might be number two.

Why do you think maybe two...

Well, before you thought it was number one that would be bigger,

before you started the activity.

Okay, how many...

Now you've filled half, okay.

You're setting a benchmark for yourself.

Forty-eight.

Forty-eight.

How many do you think would be twice as many as 48?

How much is 40 and 40?

Eighty.

Eighty.

And how much is eight and eight?

Sixteen.

Sixteen,

so 80 and 16 ninety-six.

So this would be your prediction?

There'd be 96

Okay, how many did you cover on here?

Fifty.

Fifty, okay.

How much is 50 and 50?

A hundred.

So you think this will have more in it because you've
covered half, okay?

So if we've covered half of each of these and if you're correct in your prediction, then the bottom one would be more.

Would you go ahead and try and see what the rest of it would be?

Thank you.

Okay, we've just done an activity.

What were your findings, okay?

Shannon and Devon, which did you find were bigger?

Um, the square tiles, shape two was bigger.

Okay.

And when you did the centimeter cubes, which one was bigger?

Shape two was bigger.

Shape two also.

When we find area, if something is bigger, do you think it would change no matter what we used?

You're not switching the shapes or the sizes of the shapes.

Okay.

Now we're going to do an activity, okay, that requires us to use the tiles only.

We're not going to use the centimeters anymore.

And I am going to ask you to make something with the area--

and this is a little different than what we did before, all right--

that has an area of 24 square inches

and we are going to make rectangles only.

In the last activity,

many students made rectangles that were hollow inside.

They did not realize that when they were trying to make the rectangle, it had to be a solid rectangle

in order to be able to talk about its area.
The teacher went and asked the students questions to try to probe their understanding of whether or not a surface could be covered with a hole in the middle. Children, when they were actually confronted with their error, then began to reconsider. Are we sure we have 24 tiles out? Let's count. BOTH: One, two, three, four, five, six, seven, eight, nine, ten, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24. So what do you think we need to do? Hmm... Are we allowed to leave them out? Nope. It has to cover the entire thing and it has to be a rectangle. So can we have that shape, do you think? KIDS: No. Can you try it a different way? Oh, oh, maybe we could make it half skinnier maybe. Like that. But then we can also make it a little longer. No, we can make it like that. Why are you always leaving this opening? Because then we have to fill in that. Well, why don't you fill it in as you go? See, maybe that will help you, okay. to make a rectangle shape.
Yeah.
Oh, wait, I don't...
yep, we have enough.
GUERINO: Does it fit?
Yeah!
Okay, now let's see if you can make some more.
GUERINO: Does it fit?
Yeah!
Okay, now let's see if you can make some more.
Do each of these rectangles have an area of 24?
Yup.
Can you show me which ones do?
This one does.
How do we know that it has an area of 24?
We could count. Amanda, would you count yours?
We could count by two's. Two's, okay.
Two, four, six, eight, ten, 12, 14, 16, 18, 20, 22, 24.
Okay, what's the definition of area, do you think?
How many it has all together? How many it has all together.
If I filled a cup, is that area?
When we talked before about...
what did I ask you to do with the two shapes, when we did it before with tiles?
You... asked us to... fill in the shapes.
To fill in the shapes or to... cover them, okay.
So an area is
When you use 24 squares, there are exactly four distinct rectangles that can be made--
a 1 x 24, a 2 x 12,
a 3 x 8 and a 4 x 6.
It was very difficult for students because in many cases they did not see that two rectangles such as a 3 x 8 and an 8 x 3 were identical.
They thought that as soon as you changed the orientation of the rectangle it actually had changed in some way.
Okay, is this a different rectangle than this?
Yup.
Yes.
Why, Amanda?
Because this rectangle is facing, um... across
and this rectangle is facing, um... up.
All right, if I flipped them,
would they be the same rectangle?
Yes.
Yes, okay.
So they're not a different rectangle.
They're just placed differently.
Okay, we're going to get more into that, okay?
How many made this rectangle?
Okay, how many shapes, Jonathan, are on... how many are on the top of this?
Twelve.
And twelve on the bottom.
How many are on the side?
Two.
And how many are on the other side?
Two.
How many made this one?
Is it the same rectangle?

KIDS:
Yes.

Okay?

Whether I move up or down, it's the same rectangle, okay?

How many made this shape?

Shannon, how many are on the top of this?

Six.

How many are on the bottom?

Six.

How many are on the side?

Four.

How many are on the other side?

Four.

How many made this one?

I think in the future, the teacher will probably want to do more activities that involve covering with standard units, asking students to reflect on both the size of the unit and the number of units, as well as continuing to look at what happens to the area of a shape when it is rotated.

Are square tiles more efficient for covering something than buttons?

Or paper clips?

Or the other kinds of things that we used?

Yes.

Why?

Because squares have, usually have sharp edges and they're easier, and they don't leave much spaces.

Okay, in our measurement system uses square inches, square yards to measure different things.

So you've learned why we need to use a standard form of measurement to cover things.

And to cover things is what, Amanda?

Area.

Area--
everybody?

Area.

Area, when we cover something.

Okay, good job.

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