

1 01:00:46:28 01:00:48:06 If it has the largest area,
2 01:00:48:08 01:00:49:17 it doesn't
necessarily mean
3 01:00:49:19 01:00:51:09 it's the largest
perimeter.
4 01:00:51:11 01:00:53:24 NARRATOR:
In an earlier session
of the measurement course,
5 01:00:53:26 01:00:56:05 the class explored the concept
of area and perimeter
6 01:00:56:07 01:00:58:15 using tangrams.
7 01:00:58:17 01:01:02:09 During the activity, John Belber
and his partner Rosalie Paillard
8 01:01:02:11 01:01:04:27 shared their thinking
with Professor Chapin.
9 01:01:04:29 01:01:09:25 Now, what is the shape with the
greatest or the largest area?
10 01:01:11:04 01:01:12:29 PILLARD:
See, I'm thinking
they're all the...
11 01:01:13:01 01:01:14:24 I have this thought
that they're all the same
12 01:01:14:26 01:01:16:14 because we use
the same exact pieces.
13 01:01:16:16 01:01:17:13 Mm-hmm.
14 01:01:17:15 01:01:18:20 But you say areas...
15 01:01:18:22 01:01:21:02 They all take up
the same amount of space.
16 01:01:21:04 01:01:24:15 The area of a triangle
is base times height
divided by two.
17 01:01:24:17 01:01:29:00 Is that going to be
the same as... as
the area of a square,
18 01:01:29:02 01:01:31:06 which is length
times width,
19 01:01:31:08 01:01:33:02 or a rectangle--
length times width?
20 01:01:35:00 01:01:36:21 CHAPIN:
This one looks bigger.
21 01:01:36:23 01:01:38:11 It does look
bigger, but...
22 01:01:38:13 01:01:39:14 PILLARD:
How can it be bigger?
23 01:01:39:16 01:01:40:19 We used the same pieces.
24 01:01:40:21 01:01:43:17 Is there overlap
of any of the pieces?
25 01:01:43:19 01:01:46:07 Are all the pieces
used completely
26 01:01:46:09 01:01:48:02 in each of your shapes?
27 01:01:48:04 01:01:49:17 Completely in each
of our shapes.
28 01:01:49:19 01:01:51:00 Yeah, they are
used completely.
29 01:01:51:02 01:01:53:04 So why wouldn't the area
all be the same?

30 01:01:53:06 01:01:54:10 I think
you're right.

31 01:01:54:12 01:01:56:04 I mean, I'm doing
the seeing-is-believing--

32 01:01:56:06 01:01:57:23 like, if we use
three pieces, then...

33 01:01:57:25 01:01:58:18 Right.

34 01:01:58:20 01:01:59:22 Each should be the same.

35 01:01:59:24 01:02:01:04 The perimeter
would be different,

36 01:02:01:06 01:02:03:08 because that's the part
that you're talking about--

37 01:02:03:10 01:02:04:27 the longer side
versus the shorter side

38 01:02:04:29 01:02:06:03 of the medium-size triangle.

39 01:02:06:05 01:02:07:29 Right, longer side
around the outside, yeah.

40 01:02:08:01 01:02:11:12 NARRATOR:
Later, in the final session
of the course,

41 01:02:11:14 01:02:14:12 Mr. Belber and other fourth-
and fifth-grade teachers

42 01:02:14:14 01:02:16:21 discuss how they would
apply ideas about area

43 01:02:16:23 01:02:18:16 to lessons
in their own classrooms.

44 01:02:18:18 01:02:21:10 The idea of length times
width takes some time--

45 01:02:21:12 01:02:23:23 I don't know,
developmentally
or whatever.

46 01:02:23:25 01:02:25:15 So many of them
can tell you,

47 01:02:25:17 01:02:27:19 "Well, this is
a 3 x 4 rectangle."

48 01:02:27:21 01:02:29:28 But if you ask them
then to build it,

49 01:02:30:00 01:02:33:15 they would create
the three and the four
along the edges,

50 01:02:33:17 01:02:35:01 but wouldn't fill it in.

51 01:02:35:03 01:02:38:02 And it's very hard
for them to get
that sense of it,

52 01:02:38:04 01:02:39:11 sort of expanding outward.

53 01:02:39:13 01:02:41:09 That it takes up
that amount of space...

54 01:02:41:11 01:02:42:28 And that it's
covering the space.

55 01:02:43:00 01:02:44:12 Covers the whole
space, right.

56 01:02:44:14 01:02:47:16 So, using the color tiles
to actually do that--

57 01:02:47:18 01:02:49:10 it helps,
but like you said,

58 01:02:49:12 01:02:51:27 if you just ask them

59 01:02:51:29 later on to do it,
 01:02:54:21 they may not come up
 with that rectangle.
 60 01:02:54:23 01:02:56:01 Right.
 61 01:02:56:03 01:02:58:14 WOMAN:
 Right, they need to fill
 in the whole space
 62 01:02:58:16 01:02:59:23 to cover the entire surface,
 63 01:02:59:25 01:03:01:16 to understand
 that the surface area
 64 01:03:01:18 01:03:03:11 is covering the surface
 of the paper.
 65 01:03:03:13 01:03:05:01 It's like a TERC activity
 that asks them
 66 01:03:05:03 01:03:06:21 to practice their
 multiplication facts,
 67 01:03:06:23 01:03:09:25 and you have to fill in
 all the dimensions.
 68 01:03:09:27 01:03:13:16 And if you're counting
 six cubes across
 and... a 6 x 8,
 69 01:03:13:18 01:03:15:16 you don't want
 to do a 6 x 9...
 70 01:03:15:18 01:03:16:16 Right.
 71 01:03:16:18 01:03:18:07 Like you had mentioned,
 72 01:03:18:09 01:03:20:13 and that you want
 to make sure
 73 01:03:20:15 01:03:24:13 that you count
 the perimeter around
 it the same amount,
 74 01:03:24:15 01:03:26:02 so it is a 6 x 8
 in each area.
 75 01:03:26:04 01:03:28:26 But that again would
 be a great example
 to go over--
 76 01:03:28:28 01:03:31:15 perimeter versus... versus
 area, sort of around.
 77 01:03:31:17 01:03:33:01 WOMAN:
 And I think that...
 78 01:03:33:03 01:03:36:14 that activity
 of creating arrays,
 79 01:03:36:16 01:03:40:27 or creating, you know,
 areas, rectangles,
 80 01:03:40:29 01:03:43:27 gives them so many experiences
 81 01:03:43:29 01:03:48:15 with sort of seeing
 the squareness of the unit,
 82 01:03:48:17 01:03:51:09 and also seeing that it's...
 83 01:03:51:11 01:03:54:26 A 6 x 8 would be eight rows
 of six or six rows of eight,
 84 01:03:54:28 01:03:56:26 because when they're making it,
 85 01:03:56:28 01:03:58:28 they actually
 have to make those,
 86 01:03:59:00 01:04:01:03 and they see that it's,
 you know,
 87 01:04:01:05 01:04:03:18 a multiplication
 kind of a relationship,

88 01:04:03:20 01:04:05:21 rather than just six and eight.
89 01:04:05:23 01:04:08:00 WOMAN:
It leads into the formula
very well, too.
90 01:04:08:02 01:04:09:21 It allows them
to see the formula
91 01:04:09:23 01:04:13:19 for area of a rectangle,
and... and they can
come up with it.
92 01:04:13:21 01:04:17:04 They might even
come up with it on
their own that way.
93 01:04:17:06 01:04:19:24 WOMAN:
Well, and it helps them
with their multiplication,
94 01:04:19:26 01:04:22:23 because they understand
 $6 \times 8 = 48$.
95 01:04:22:25 01:04:25:20 They're visualizing
now what 48 is
96 01:04:25:22 01:04:29:19 because they've made the area
of 48 square units.
97 01:04:29:21 01:04:32:24 And incorporating and
emphasizing that vocabulary too,
98 01:04:32:26 01:04:35:09 with six rows of eight
or eight rows of six
99 01:04:35:11 01:04:37:18 really reinforces
what they're creating,
100 01:04:37:20 01:04:40:27 either on paper or out of tiles
or the fastening cubes.
101 01:04:40:29 01:04:42:29 And I've seen
a few kids,
102 01:04:43:01 01:04:45:20 and I think it's
sort of a stretch,
103 01:04:45:22 01:04:47:20 but a few kids who are able
104 01:04:47:22 01:04:50:13 to create that
conservation of area,
105 01:04:50:15 01:04:51:27 where when
they're doing
106 01:04:51:29 01:04:54:01 that multiplication
array activity
107 01:04:54:03 01:04:55:25 they can take
a 6×8 array
108 01:04:55:27 01:04:59:06 and they shift it... I don't
know, it's hard to tell,
109 01:04:59:08 01:05:02:08 but so if this is the six
and this is the eight,
110 01:05:02:10 01:05:03:21 they can shift it like this...
111 01:05:03:23 01:05:04:20 (*all agreeing*)
112 01:05:04:22 01:05:06:18 And now it's a 12×4 ...
113 01:05:06:20 01:05:07:28 (*all agreeing*)
114 01:05:08:00 01:05:11:04 And so they know that this
was 48 and this is 48...
115 01:05:11:06 01:05:13:00 That the area
hasn't changed.
116 01:05:13:02 01:05:15:09 And so 12×4 is
the same as 6×8 --

117 01:05:15:11 01:05:17:08 same area, but
it's a different shape.

118 01:05:17:10 01:05:18:16 (*all agreeing*)

119 01:05:18:18 01:05:20:23 Which is really powerful
when it can happen.

120 01:05:20:25 01:05:23:03 Because the number of tiles
does not change--

121 01:05:23:05 01:05:25:03 it stayed the same,
so they should know

122 01:05:25:05 01:05:26:27 that the area's
not going to change.

123 01:05:26:29 01:05:30:10 NARRATOR:
The following spring,
we visited Mr. Belber's class

124 01:05:30:12 01:05:33:06 at the Milton Academy
near Boston, Massachusetts.

125 01:05:33:08 01:05:36:13 He and his students were
investigating the relationship

126 01:05:36:15 01:05:39:18 between area and perimeter,
using pentominoes.

127 01:05:39:20 01:05:41:21 CHAPIN:
It's exciting to see ideas

128 01:05:41:23 01:05:44:04 that we as adults
explored this summer

129 01:05:44:06 01:05:47:08 become translated into fine
instructional practice

130 01:05:47:10 01:05:48:24 for students.

131 01:05:48:26 01:05:52:05 We looked at the relationships
between perimeter and area

132 01:05:52:07 01:05:55:21 among many different kinds of
shapes in our summer institute.

133 01:05:55:23 01:05:57:25 And here that
has been translated

134 01:05:57:27 01:05:59:22 into a very concrete approach

135 01:05:59:24 01:06:03:00 for... that is very appropriate
for fourth graders

136 01:06:03:02 01:06:04:24 with manipulative materials

137 01:06:04:26 01:06:08:22 and a limited number
of possibilities.

138 01:06:08:24 01:06:14:20 Does anybody remember the area,
the amount of space,

139 01:06:14:22 01:06:15:22 one pentomino takes up?

140 01:06:15:24 01:06:16:22 Sophie?

141 01:06:16:24 01:06:18:06 Five square inches.

142 01:06:18:08 01:06:20:05 BELBER:
So one pentomino

143 01:06:20:07 01:06:21:23 is five square inches, okay?

144 01:06:21:25 01:06:26:12 Do you remember how to find
the perimeter of one pentomino?

145 01:06:26:14 01:06:30:00 The perimeter, like we talked
about last week,

146 01:06:30:02 01:06:33:18 is around the outside
of a shape.

147 01:06:33:20 01:06:37:08 You can find out the perimeter
of a pentomino

148 01:06:37:10 01:06:41:07 by measuring the length

of the outside in inches,
 149 01:06:41:09 01:06:45:14 and we figured out that,
 last Friday,
 150 01:06:45:16 01:06:47:28 that there's only one pentomino
 151 01:06:48:00 01:06:50:17 that's perimeter
 is not 12 inches.
 152 01:06:50:19 01:06:52:17 BELBER:
 And we had a name
 for that--
 153 01:06:52:19 01:06:54:19 it was sort of
 a squished one, right?
 154 01:06:54:21 01:06:56:10 And so here it is right here.
 155 01:06:56:12 01:06:58:03 It has several interior lines,
 156 01:06:58:05 01:07:00:11 and if you count
 around the outside...
 157 01:07:00:13 01:07:04:14 I can put my marker
 here to start--
 158 01:07:04:16 01:07:07:27 one, two, three...
 159 01:07:07:29 01:07:12:10 four, five, six, seven,
 eight, nine, ten.
 160 01:07:12:12 01:07:13:19 Here's my starting point.
 161 01:07:13:21 01:07:15:08 Ten inches around the outside.
 162 01:07:15:10 01:07:17:07 You're right, Tim.
 163 01:07:17:09 01:07:21:22 Today's task is now that we know
 the area of a pentomino,
 164 01:07:21:24 01:07:24:07 and that all the pentominoes
 are 12 inches
 165 01:07:24:09 01:07:25:29 except for the squished one,
 166 01:07:26:01 01:07:29:25 we're going to put
 two pentominoes together.
 167 01:07:29:27 01:07:33:00 BELBER:
 The goals for today's lesson
 168 01:07:33:02 01:07:36:06 are to study area and perimeter
 of pentominoes.
 169 01:07:36:08 01:07:38:26 And in putting two pentominoes
 together
 170 01:07:38:28 01:07:40:15 and making a new design,
 171 01:07:40:17 01:07:42:28 I want the kids
 to discover the area
 172 01:07:43:00 01:07:46:08 of those two pentominoes
 and the perimeters.
 173 01:07:46:10 01:07:49:25 Get to the understanding
 of the range of perimeters,
 174 01:07:49:27 01:07:52:07 from smallest to largest.
 175 01:07:52:09 01:07:54:13 Each time you put
 a pair together,
 176 01:07:54:15 01:07:58:14 you put a pair together on one
 of your pieces of graph paper,
 177 01:07:58:16 01:08:02:24 and in the corner, you'll see
 "area" and "perimeter."
 178 01:08:02:26 01:08:04:03 So in trying to find
 179 01:08:04:05 01:08:06:19 the greatest area...
 greatest perimeter
 180 01:08:06:21 01:08:10:07 and smallest perimeter,
 you can record that information
 181 01:08:10:09 01:08:12:05 in the corner of your sheet.

182 01:08:12:07 01:08:13:17 Okay?

183 01:08:13:19 01:08:17:16 You have a thick magic marker
at your desk, also.

184 01:08:17:18 01:08:20:17 That is to trace
around the perimeter.

185 01:08:20:19 01:08:23:06 So, just to be clear,

186 01:08:23:08 01:08:26:21 I'd love for someone just
to repeat back the directions
that you're doing now.

187 01:08:26:23 01:08:28:05

188 01:08:28:07 01:08:29:14 Okay.

189 01:08:29:16 01:08:30:28 Alex.

190 01:08:31:00 01:08:33:29 We're going to put
two pentominoes together
and make a new...

191 01:08:34:01 01:08:36:24 and make a new shape.

192 01:08:36:26 01:08:38:09 BELBER:
Right.

193 01:08:38:11 01:08:41:02 And then we have to find the
area and the perimeter of them.

194 01:08:41:04 01:08:42:16 Right, and as you
find the area...

195 01:08:42:18 01:08:44:02 That's great, Alex.

196 01:08:44:04 01:08:46:16 As you find the area and the
perimeter of each new shape,
you're trying to find
the largest perimeter
around the outside
and the smallest perimeter
around the outside.

197 01:08:46:18 01:08:47:26

198 01:08:47:28 01:08:50:10

199 01:08:50:12 01:08:53:04

200 01:08:53:06 01:08:54:04 Yeah.

201 01:08:56:04 01:09:00:22 (*children and Belber
talking quietly*)

202 01:09:06:23 01:09:09:02 GIRL:
Okay, so, let's start here.

203 01:09:09:04 01:09:11:27 So, one, two, three,

204 01:09:11:29 01:09:17:21 four, five, six, seven,
eight, nine, ten,

205 01:09:17:23 01:09:19:00 11, 12, 13.

206 01:09:19:02 01:09:20:14 It's 13.

207 01:09:20:16 01:09:22:27 CHAPIN:
There were a number of purposes
of this lesson.

208 01:09:22:29 01:09:25:04 One was to look
at the relationship
between perimeter and area.

209 01:09:25:06 01:09:27:19

210 01:09:27:21 01:09:30:22 And when you have a fixed area--

211 01:09:30:24 01:09:34:00 in this case,
ten square inches--
there are a variety of
perimeters that are possible.

212 01:09:34:02 01:09:38:20

213 01:09:38:22 01:09:40:29 We want young students
to realize
that perimeter and area change,
and that you can get a variety
of perimeters with a fixed area.

214 01:09:41:01 01:09:43:07

215 01:09:43:09 01:09:47:23

216 01:09:47:25 01:09:50:20 There were other purposes

as well, however.

217 01:09:50:22 01:09:52:28 One was a problem-solving purpose,

218 01:09:53:00 01:09:54:28 where the children were asked

219 01:09:55:00 01:09:58:09 to find what are the various perimeters possible.

220 01:09:58:11 01:10:02:05 They had to really investigate, look for patterns,

221 01:10:02:07 01:10:05:08 try to reason through what was possible

222 01:10:05:10 01:10:07:06 and what was impossible.

223 01:10:07:08 01:10:10:09 How many squares do you join together?

224 01:10:11:13 01:10:12:26 Ten.

225 01:10:12:28 01:10:14:11 Yeah, one, two, right?

226 01:10:14:13 01:10:16:10 One, two, three, four, five,

227 01:10:16:12 01:10:17:19 six, seven, eight.

228 01:10:17:21 01:10:18:26 Nine, ten.

229 01:10:20:10 01:10:21:18 Oh, wait.

230 01:10:21:20 01:10:25:26 You're doing eight squares together or back?

231 01:10:25:28 01:10:27:20 Yeah, I wonder if you can get more squares

232 01:10:27:22 01:10:29:06 joined together with other pieces.

233 01:10:30:29 01:10:34:27 (*children talking*)

234 01:10:36:06 01:10:38:05 Oh, ho.

235 01:10:38:07 01:10:39:27 Four.

236 01:10:39:29 01:10:40:29 That's just seven.

237 01:10:41:01 01:10:41:28 Yeah.

238 01:10:42:00 01:10:43:04 No, wait, it's eight.

239 01:10:45:23 01:10:46:21 Yeah, it's eight.

240 01:10:46:23 01:10:48:05 But you're under a good notion--

241 01:10:48:07 01:10:51:01 that idea of joining together...

242 01:10:51:03 01:10:53:08 covers up your shapes,

243 01:10:53:10 01:10:55:17 and makes them smaller.

244 01:10:55:19 01:10:57:16 CHAPIN:
One of the interesting approaches

245 01:10:57:18 01:10:59:22 was where they used a guess-and-check method

246 01:10:59:24 01:11:04:25 to basically test what perimeters would be possible.

247 01:11:04:27 01:11:08:03 They placed two pentominoes next to each other

248 01:11:08:05 01:11:11:02 and then counted to discover the perimeter.

249 01:11:11:04 01:11:13:16 However, after they'd done that a few times,

250 01:11:13:18 01:11:15:14 they began to analyze the situation

251 01:11:15:16 01:11:18:20 and realize that they could

252 01:11:18:22 use mathematical computations
 01:11:23:16 to discern what
 were possible perimeters.
 253 01:11:23:18 01:11:25:23 I see the range here,
 "14 to 22."
 254 01:11:25:25 01:11:26:26 Yeah.
 255 01:11:26:28 01:11:28:22 What are some ones
 you have in between?
 256 01:11:28:24 01:11:30:29 Can you have a 17
 or a 19 in between,
 or a 15?
 257 01:11:31:01 01:11:32:23 No, because they have
 to be all even.
 258 01:11:32:25 01:11:34:09 Why?
 259 01:11:34:11 01:11:39:05 Because... well, none of them
 are odd numbers, are they?
 260 01:11:39:07 01:11:40:11 No.
 261 01:11:40:13 01:11:43:14 So... and when you add
 an even and an even,
 262 01:11:43:16 01:11:45:06 you always get an even.
 263 01:11:45:08 01:11:46:06 Excellent example.
 264 01:11:46:08 01:11:47:13 That's good.
 265 01:11:47:15 01:11:50:16 So, is 18 the only range
 in between 14 and 22?
 266 01:11:50:18 01:11:51:28 No, you could
 also get a 16.
 267 01:11:52:00 01:11:53:19 Can you try to put
 that together?
 268 01:11:53:21 01:11:55:04 Yeah.
 269 01:11:55:06 01:11:57:25 BELBER:
 Which ones do you have so far,
 what perimeters?
 270 01:11:57:27 01:12:02:02 Well, 18, 13...
 271 01:12:02:04 01:12:04:24 16, another 18,
 and that's it.
 272 01:12:04:26 01:12:05:23 Okay.
 273 01:12:05:25 01:12:08:02 Can I see your 13?
 274 01:12:08:04 01:12:09:08 That sounds small.
 275 01:12:11:00 01:12:12:22 GIRL:
 One, two, three, four...
 276 01:12:12:24 01:12:15:11 BELBER:
 Okay, right when you
 got to two there...
 277 01:12:15:13 01:12:16:11 Oh, it's 14.
 278 01:12:16:13 01:12:17:11 Right.
 279 01:12:17:13 01:12:18:24 Why is it 14?
 280 01:12:18:26 01:12:20:27 Because there's
 this one right...
 281 01:12:20:29 01:12:23:18 that goes up there, and
 then there's this one.
 282 01:12:23:20 01:12:26:13 It's easy, because you get into
 the pattern of counting squares.
 283 01:12:26:15 01:12:27:13 GIRL:
 Right.
 284 01:12:27:15 01:12:30:06 BELBER:
 How'd you catch that?

285 01:12:30:08 01:12:33:26 Well, because you said "two,"
and then I looked at it,

286 01:12:33:28 01:12:35:11 and since it was,
like, an angle...

287 01:12:35:13 01:12:37:24 Yep, in fact,
it's a right angle.

288 01:12:37:26 01:12:38:29 Yeah, we talked
about that.

289 01:12:39:01 01:12:39:29 Right, so, yeah.

290 01:12:40:01 01:12:40:16 So, 14.

291 01:12:40:18 01:12:41:14 Yeah.

292 01:12:42:19 01:12:43:21 Okay.

293 01:12:43:23 01:12:45:11 Are there any tricks
to finding out

294 01:12:45:13 01:12:47:02 what the perimeters
are going to be?

295 01:12:47:04 01:12:50:08 If this is 12 inches
and this is 12 inches,

296 01:12:50:10 01:12:52:12 and you put them together...

297 01:12:52:14 01:12:54:25 if $12 + 12 = 24$,

298 01:12:54:27 01:12:56:16 is the perimeter
going to be 24 inches?

299 01:12:56:18 01:13:00:10 Not necessarily,
because in between here,

300 01:13:00:12 01:13:02:23 they put together
to make one shape.

301 01:13:02:25 01:13:05:18 Okay, and when
you put them together,

302 01:13:05:20 01:13:08:29 then how do you...
what are you taking away?

303 01:13:09:01 01:13:11:02 Probably one or two.

304 01:13:11:04 01:13:14:04 "Probably one or two,"
because here's a side

305 01:13:14:06 01:13:17:22 and there's a side,
so you could make a prediction.

306 01:13:17:24 01:13:20:28 If the two pentominoes
are 24 inches together,

307 01:13:21:00 01:13:22:08 you take away two,

308 01:13:22:10 01:13:23:26 What's your prediction
going to be?

309 01:13:23:28 01:13:24:26 Twenty-two.

310 01:13:24:28 01:13:25:25 Let's see.

311 01:13:25:27 01:13:26:25 Go ahead and count.

312 01:13:26:27 01:13:31:06 One, two, three, four, five...

313 01:13:31:08 01:13:35:12 six, seven, eight, nine, ten,

314 01:13:35:14 01:13:40:13 11, 12, 13, 14, 15, 16,

315 01:13:40:15 01:13:46:25 17, 18, 19, 20, 21, 22.

316 01:13:46:27 01:13:49:28 Okay, so there's a little
math theory right there.

317 01:13:50:00 01:13:51:13 Yup.

318 01:13:51:15 01:13:54:00 You put two
12-inch pentominoes together,

319 01:13:54:02 01:13:57:14 you take away two because
you join two sides: $24 - 2 = 22$.

320 01:13:57:16 01:13:59:24 Maybe that could help you
when join other pentominoes.

321 01:13:59:26 01:14:01:03 Mm-hmm.

322 01:14:01:05 01:14:03:00 Kate, can we share that
discovery with you?

323 01:14:03:02 01:14:04:05 KATE:
Sure.

324 01:14:04:07 01:14:05:28 When we put two
pentominoes together...

325 01:14:06:00 01:14:06:28 Bree, show her.

326 01:14:07:00 01:14:09:13 It takes up
two sides, right?

327 01:14:09:15 01:14:11:21 Yeah.

328 01:14:11:23 01:14:14:01 So this is...
 $12 + 12 = 24$.

329 01:14:14:03 01:14:15:02 KATE:
Yeah.

330 01:14:15:04 01:14:16:02 But you take away

331 01:14:16:04 01:14:17:11 these two sides, right, Bree?

332 01:14:17:13 01:14:18:12 BREE:
And it equals 22.

333 01:14:18:14 01:14:19:12 Equals 22.

334 01:14:19:14 01:14:20:24 These sides
you have touching.

335 01:14:20:26 01:14:22:16 Like, if you have
more sides touching,

336 01:14:22:18 01:14:24:08 then you can make
the shape smaller...

337 01:14:24:10 01:14:25:16 Good.

338 01:14:25:18 01:14:26:29 and you have
less sides touching.

339 01:14:27:01 01:14:28:11 I did that on my first...

340 01:14:28:13 01:14:30:04 So what are some ones
that you got, Kate,

341 01:14:30:06 01:14:31:13 that are different
perimeters?

342 01:14:31:15 01:14:33:19 I got, um...

343 01:14:33:21 01:14:35:09 Wait, for this one I got...

344 01:14:35:11 01:14:36:09 20 inches.

345 01:14:36:11 01:14:37:11 20 inches.

346 01:14:37:13 01:14:38:25 Okay, on that
20-inch one, Kate...

347 01:14:38:27 01:14:40:00 Yeah?

348 01:14:40:02 01:14:41:13 Bree, take a look
at Kate's there.

349 01:14:41:15 01:14:42:13 How'd you get 20?

350 01:14:42:15 01:14:44:02 How many sides are touching?

351 01:14:44:04 01:14:45:05 Um, well...

352 01:14:45:07 01:14:46:27 Oh, yeah, I used these two.

353 01:14:46:29 01:14:50:21 And then this one is,
um... this one is...

354 01:14:50:23 01:14:51:25 BELBER:
That's 12, right?

355 01:14:51:27 01:14:53:00 Yeah, so that's 12.

356 01:14:53:02 01:14:56:15 And then this one's...

357 01:14:56:17 01:14:57:15 12.

358 01:14:57:17 01:14:58:14 12 inches, great.

359 01:14:58:16 01:14:59:26 So 12 and 12 is 24.

360 01:14:59:28 01:15:02:24 KATE:
Yeah, and then I
put them together.

361 01:15:02:26 01:15:06:22 And, um, there's one side,
two side... wait.

362 01:15:06:24 01:15:10:10 There's one side, two side,
three side, four side.

363 01:15:10:12 01:15:11:15 And so $24 - 4 = 20$.

364 01:15:11:17 01:15:13:12 BELBER:
That's the math,
excellent.

365 01:15:13:14 01:15:15:04 So you can
get different ones.

366 01:15:15:06 01:15:17:15 CHAPIN:
One of the important things

367 01:15:17:17 01:15:20:28 about teaching area
and perimeter to young students

368 01:15:21:00 01:15:23:11 is that they learn
to differentiate

369 01:15:23:13 01:15:25:09 between these two measures.

370 01:15:25:11 01:15:28:15 Once they have had those kinds
of experiences,

371 01:15:28:17 01:15:31:01 that allows them
to look at formulas

372 01:15:31:03 01:15:34:00 and actually think
much more mathematically

373 01:15:34:02 01:15:35:21 in terms of computation:

374 01:15:35:23 01:15:39:14 How do we derive those measures
with different shapes?

375 01:15:39:16 01:15:40:14 I have a question.

376 01:15:40:16 01:15:41:20 Yes, that's good.

377 01:15:41:22 01:15:43:06 How many could there be?

378 01:15:43:08 01:15:45:27 Like, have you figured out
the lowest and the highest?

379 01:15:45:29 01:15:48:02 I have, yeah, because I
spent a lot of time

380 01:15:48:04 01:15:49:15 doing exactly
what you're doing,

381 01:15:49:17 01:15:50:29 which is playing
and finding out.

382 01:15:51:01 01:15:52:24 But I think you're on
to some patterns here.

383 01:15:52:26 01:15:53:28 Okay.

384 01:15:54:00 01:15:55:10 For instance,

385 01:15:55:12 01:15:57:12 when you join two together
and you take away two,

386 01:15:57:14 01:15:59:01 join two together
and take away four.

387 01:15:59:03 01:16:00:04 Yeah.

388 01:16:00:06 01:16:01:18 See if you can
develop a pattern.

389 01:16:01:20 01:16:02:18 Okay.

390 01:16:02:20 01:16:03:27 And find out
the in-betweens.

391 01:16:03:29 01:16:05:05 But I can see
that you found

392 01:16:05:07 01:16:07:01 the lowest and the highest
in 14 and 22.

393 01:16:07:03 01:16:10:09 BELBER:
Okay, we're going to wrap up
our creative discovery

394 01:16:10:11 01:16:13:01 and our putting together
of two pentominoes

395 01:16:13:03 01:16:14:19 to make the new designs,

396 01:16:14:21 01:16:17:05 and I want to hear
a little bit from you

397 01:16:17:07 01:16:18:24 about discoveries you made

398 01:16:18:26 01:16:22:07 about the largest perimeters
and the smallest perimeters.

399 01:16:22:09 01:16:25:18 And I see some wonderful
new designs that you've created

400 01:16:25:20 01:16:27:27 that are colorful
and really show

401 01:16:27:29 01:16:29:12 a lot of your hard work.

402 01:16:29:14 01:16:32:20 Um, and then we'll have a chance
to display our work.

403 01:16:32:22 01:16:36:17 Owen, can you share with us
a largest or a smallest

404 01:16:36:19 01:16:38:24 or maybe even an in-between?

405 01:16:43:03 01:16:46:14 That one is 22
and the area's ten.

406 01:16:46:16 01:16:47:14 Right...

407 01:16:47:16 01:16:48:14 It has to be.

408 01:16:48:16 01:16:49:22 It has to be.

409 01:16:49:24 01:16:51:03 Because it's two...

410 01:16:51:05 01:16:53:12 the area on one
pentomino is five

411 01:16:53:14 01:16:56:17 so you just double it
and that has to be ten

412 01:16:56:19 01:16:59:14 because 2×5 is ten.

413 01:16:59:16 01:17:03:02 Is there a way that you know
it's the largest, Owen?

414 01:17:03:04 01:17:05:00 Well, there are
a lot of them

415 01:17:05:02 01:17:06:14 that add up to 22...

416 01:17:06:16 01:17:07:20 Yup.

417 01:17:07:22 01:17:11:03 But I just counted
one, two, three, four,

418 01:17:11:05 01:17:14:22 five, six, seven, eight,
nine, ten,

419 01:17:14:24 01:17:19:06 11, 12, 13, 14, 15, 16,
17, 18, 19, 20, 21, 22.

420 01:17:19:08 01:17:20:12 Great.

421 01:17:20:14 01:17:21:13 And I used my numbers,

422 01:17:21:15 01:17:22:23 because the other one
seemed, like,

423 01:17:22:25 01:17:23:20 too complicated and...

424 01:17:23:22 01:17:24:20 Okay.

425 01:17:24:22 01:17:25:20 And...

426 01:17:25:22 01:17:27:03 If you see...

427 01:17:27:05 01:17:28:27 If you take a step back
here with me, Owen,

428 01:17:28:29 01:17:30:09 you'll see
I've got a range here

429 01:17:30:11 01:17:31:15 starting on the left.

430 01:17:31:17 01:17:33:15 Smallest all the way up
to largest.

431 01:17:33:17 01:17:35:09 So if you feel
it's the largest,

432 01:17:35:11 01:17:37:16 can you put that
in the right bracket?

433 01:17:37:18 01:17:39:05 Right column?

434 01:17:39:07 01:17:40:12 Terrific.

435 01:17:40:14 01:17:42:11 Okay, great.

436 01:17:42:13 01:17:43:27 From your desk--

437 01:17:43:29 01:17:46:17 because we'll all have a chance
to go up in a little bit--

438 01:17:46:19 01:17:51:22 from your desk, can you tell me,
um, about another perimeter

439 01:17:51:24 01:17:54:16 or a reason why
22 is the largest

440 01:17:54:18 01:17:56:26 or why 14 is the smallest?

441 01:17:56:28 01:17:59:12 Well, I think that
the largest is 22

442 01:17:59:14 01:18:03:06 because each
of the things here
have 12 inches around

443 01:18:03:08 01:18:06:16 and if you put them
together with another one,

444 01:18:06:18 01:18:09:09 at least two sides
have to be touching,

445 01:18:09:11 01:18:12:21 so, um, so you can't
make it any bigger
than 22.

446 01:18:12:23 01:18:13:26 BELBER:
Exactly.

447 01:18:13:28 01:18:15:21 Okay, so, having said that,

448 01:18:15:23 01:18:19:05 um, who thinks they have
another one other than 22?

449 01:18:19:07 01:18:20:21 Um, and there are ways

450 01:18:20:23 01:18:24:03 to make 22-inch perimeters
with these new designs,

451 01:18:24:05 01:18:26:15 but I'm wondering
what's the range.

452 01:18:26:17 01:18:30:07 Okay, you can see there's
a range up here on the board,

453 01:18:30:09 01:18:32:14 but I'd love to see it visually.

454 01:18:32:16 01:18:33:14 16.

455 01:18:33:16 01:18:36:26 16, okay, and how'd
you get 16?

456 01:18:36:28 01:18:38:18 Well... well, 12 plus...

457 01:18:38:20 01:18:40:16 You know how 12 + 12 is 24?

458 01:18:40:18 01:18:42:00 Yep.

459 01:18:42:02 01:18:45:08 Well, more than two sides
are touching,

460 01:18:45:10 01:18:49:11 so there are six sides touching,
so it's like...

461 01:18:49:13 01:18:51:01 Six sides touching?

462 01:18:51:03 01:18:54:02 Okay, so what's your math there
to get to 16?

463 01:18:54:04 01:18:56:08 Because...
Okay, well...

464 01:18:56:10 01:18:58:23 Um, let me try to find
what shapes I used.

465 01:18:58:25 01:19:01:09 Yup, put your shapes
again on there, yup.

466 01:19:01:11 01:19:03:09 GIRL:
I think those two
are these.

467 01:19:03:11 01:19:04:24 Okay.

468 01:19:04:26 01:19:07:19 Okay, I used these two shapes

469 01:19:07:21 01:19:11:13 and one, two, three,
four, five... wait.

470 01:19:11:15 01:19:14:17 Yeah, six... six sides
are touching

471 01:19:14:19 01:19:19:06 because there's three
and then three, so...

472 01:19:19:08 01:19:20:27 Put that down on your paper
and recount,

473 01:19:20:29 01:19:22:23 because I think
you're on to something there

474 01:19:22:25 01:19:24:08 about the amount of sides
touching.

475 01:19:24:10 01:19:25:13 Oh, no-- it's eight.

476 01:19:25:15 01:19:26:13 It's eight.

477 01:19:26:15 01:19:27:13 Eight sides touching.

478 01:19:27:15 01:19:28:17 Right, so if each
pentomino

479 01:19:28:19 01:19:29:20 was 12 inches
in perimeter,

480 01:19:29:22 01:19:32:07 right, and you
put them together,

481 01:19:32:09 01:19:33:08 that's 24 inches.

482 01:19:33:10 01:19:34:08 Right.

483 01:19:34:10 01:19:35:08 How do you get to 16?

484 01:19:35:10 01:19:37:15 State the math there.

485 01:19:37:17 01:19:38:24 Because $24 - 8$ is 16

486 01:19:38:26 01:19:40:06 Excellent.
Can you post that up

487 01:19:40:08 01:19:42:16 on the board?

488 01:19:42:18 01:19:46:24 And I'm going to ask someone
to be thinking about...

489 01:19:46:26 01:19:50:03 going lower than 16.

490 01:19:50:05 01:19:52:05 Um, I got a 14.

491 01:19:52:07 01:19:53:21 BELBER:
Okay.

492 01:19:53:23 01:19:55:13 Which was this and this.

493 01:19:55:15 01:19:56:12 All right.

494 01:19:56:14 01:19:57:26 And how'd you get that?

495 01:19:57:28 01:19:59:18 Um...

496 01:19:59:20 01:20:01:21 (*chuckling*);
You just put them
together, I know.

497 01:20:01:23 01:20:04:01 And I put sides...

498 01:20:04:03 01:20:06:18 Think about what Maya said about sides touching.

499 01:20:06:20 01:20:08:06 How many sides are touching?

500 01:20:11:22 01:20:12:25 Eight.

501 01:20:12:27 01:20:13:29 Eight.

502 01:20:14:01 01:20:15:18 Put them down on the paper

503 01:20:15:20 01:20:18:04 and I'm going to ask you to do the same thing that Maya did.

504 01:20:18:06 01:20:20:15 And maybe you'll need to do a marker around one of them

505 01:20:20:17 01:20:21:29 so you can see the touching sides.

506 01:20:22:01 01:20:23:08 Can you take a shape away

507 01:20:23:10 01:20:26:12 and put a marker around the existing one there?

508 01:20:28:10 01:20:32:03 And double-count your... your sides touching.

509 01:20:32:05 01:20:33:05 So I count these?

510 01:20:33:07 01:20:34:13 Yep.

511 01:20:34:15 01:20:36:05 One, two, three, four.

512 01:20:36:07 01:20:37:08 Okay.

513 01:20:37:10 01:20:38:15 Now, is it just four?

514 01:20:38:17 01:20:40:12 I'm going to put this shape back here.

515 01:20:40:14 01:20:42:01 Is it just one, two, three, four?

516 01:20:43:22 01:20:45:22 Put the other shape back on.

517 01:20:50:12 01:20:52:16 How many sides are touching total, would you think?

518 01:20:55:18 01:20:56:16 Eight?

519 01:20:56:18 01:20:57:16 Eight.

520 01:20:57:18 01:20:59:03 Let's count.

521 01:20:59:05 01:21:01:03 Because there's four in this

522 01:21:01:05 01:21:03:05 and four on that.

523 01:21:03:07 01:21:08:11 Okay, so that's eight sides touching total, so you're...

524 01:21:08:13 01:21:09:26 Are these both

525 01:21:09:28 01:21:11:19 12 inches in perimeter?

526 01:21:13:03 01:21:16:14 Um... no, they're 14.

527 01:21:16:16 01:21:18:29 Okay, together they're 14,

528 01:21:19:01 01:21:22:18 but individually, is this 12 inches?

529 01:21:22:20 01:21:24:28 No, they're... it's, uh...

530 01:21:29:11 01:21:30:16 Yeah, this is 12.

531 01:21:30:18 01:21:32:01 Okay, and how about the other one

532 01:21:32:03 01:21:33:24 which we've called "the squished one"?

533 01:21:38:09 01:21:39:14 This is ten.

534 01:21:39:16 01:21:40:28 Okay, so if you add them together

535 01:21:41:00 01:21:42:16 and you just do
the straight math,

536 01:21:42:18 01:21:44:23 what's $12 + 10$?

537 01:21:44:25 01:21:45:23 Uh, 22.

538 01:21:45:25 01:21:47:06 22, great.

539 01:21:47:08 01:21:50:04 And then you said there
are eight sides touching,

540 01:21:50:06 01:21:51:28 so do the subtraction
from 22.

541 01:21:54:10 01:21:55:21 Excellent.

542 01:21:55:23 01:21:56:26 Good side-work.

543 01:21:58:12 01:22:00:06 And you can be thinking
about this also,

544 01:22:00:08 01:22:02:03 because she's put the...
I know you can't...

545 01:22:02:05 01:22:03:03 14.

546 01:22:03:05 01:22:04:04 Right, 14.

547 01:22:04:06 01:22:05:22 She put
the squished-up one...

548 01:22:05:24 01:22:07:08 Can I hold them up?

549 01:22:07:10 01:22:09:05 She put the squished-up one

550 01:22:09:07 01:22:12:09 and the L-shaped pentomino,
put them together.

551 01:22:12:11 01:22:14:22 So this is ten inches
in perimeter,

552 01:22:14:24 01:22:16:11 12 inches in perimeter.

553 01:22:16:13 01:22:18:01 The logical perimeter

554 01:22:18:03 01:22:21:15 would be 22 inches
around the outside, right?

555 01:22:21:17 01:22:25:29 But since this is...
eight sides touching,

556 01:22:26:01 01:22:28:04 $22 - 8$ is indeed 14.

557 01:22:28:06 01:22:30:10 John, I know I talked to you

558 01:22:30:12 01:22:32:27 about, um, the range
for 14 to 22.

559 01:22:32:29 01:22:35:10 Is there a measurement that's
not represented that you have?

560 01:22:35:12 01:22:36:10 JOHN:
Uh, yeah, um...

561 01:22:36:12 01:22:37:23 What do you have?

562 01:22:37:25 01:22:38:23 20, 20 inches.

563 01:22:38:25 01:22:39:23 Twenty.

564 01:22:39:25 01:22:41:12 Want to come
put that up?

565 01:22:41:14 01:22:42:25 And where does it go?

566 01:22:42:27 01:22:45:04 JOHN:
It's just like...

567 01:22:45:06 01:22:47:13 It's... it's 20
because there's just...

568 01:22:47:15 01:22:50:01 they're just touching
each other in one way

569 01:22:50:03 01:22:52:13 and you minus two
because it's each side

570 01:22:52:15 01:22:53:22 and that's $20 - 2$...

571 01:22:53:24 01:22:56:05 $22 - 20$... minus two
would just be 20.

572 01:22:56:07 01:22:57:23 Excellent, so you...
 573 01:22:57:25 01:22:59:16 BELBER:
 When students come to the board
 574 01:22:59:18 01:23:02:01 it's very important they feel
 ownership of their work
 575 01:23:02:03 01:23:04:05 and their peers can see
 the work they've done
 576 01:23:04:07 01:23:06:00 and the way
 they've looked at something.
 577 01:23:06:02 01:23:07:29 So it's a great way to share
 578 01:23:08:01 01:23:10:27 and for me to see
 how they put it together.
 579 01:23:10:29 01:23:13:00 Did they challenge themselves
 580 01:23:13:02 01:23:15:29 with some of the designs
 they came up with?
 581 01:23:16:01 01:23:17:24 Okay, this is a great
 representation
 582 01:23:17:26 01:23:20:04 and only a small representation
 of the range
 583 01:23:20:06 01:23:22:20 from 14 inches up to 22 inches.
 584 01:23:22:22 01:23:24:26 I want to ask one question
 585 01:23:24:28 01:23:27:26 before I talk about
 homework tonight,
 586 01:23:27:28 01:23:31:29 which is, what's the pattern
 of skipping by two?
 587 01:23:32:01 01:23:36:13 Why isn't there a total
 perimeter of 15 or of 17,
 588 01:23:36:15 01:23:38:27 what seems like odd numbers?
 589 01:23:38:29 01:23:41:02 How come we don't have,
 590 01:23:41:04 01:23:44:18 um, representation
 of those perimeters?
 591 01:23:44:20 01:23:47:17 When you put two of these
 together, like this,
 592 01:23:47:19 01:23:51:02 there has to be an even side
 touching because there's,
 593 01:23:51:04 01:23:53:18 like, two things,
 so it has to be even.
 594 01:23:53:20 01:23:56:11 So it goes from 14 to 16
 to 18 to 20 to 22.
 595 01:23:56:13 01:23:57:16 Great.
 596 01:23:57:18 01:23:58:23 Tonight for homework,
 597 01:23:58:25 01:24:00:21 as many of you
 could easily predict...
 598 01:24:00:23 01:24:03:04 Okay, I just randomly put three
 together right here.
 599 01:24:05:04 01:24:07:15 Ethan, based on
 your work today...
 600 01:24:07:17 01:24:09:21 Um, I know your hand isn't up,
 601 01:24:09:23 01:24:12:04 but I'd love to hear
 your thoughts
 602 01:24:12:06 01:24:13:29 on just a good estimation
 603 01:24:14:01 01:24:16:21 on a perimeter
 with three pentominoes.
 604 01:24:16:23 01:24:19:18 Uh, I still think the perimeter
 will be even,
 605 01:24:19:20 01:24:23:14 because, um, because those two

sides are touching each other
606 01:24:23:16 01:24:25:00 and so it would be like
607 01:24:25:02 01:24:27:28 if there were three sides
touching each other,
608 01:24:28:00 01:24:32:00 those three sides...
and 3 + 3 is 6-- that's even.
609 01:24:32:02 01:24:34:06 Okay, so every time
it will be even.
610 01:24:34:08 01:24:37:10 Um, if I were to add
the three perimeters together,
611 01:24:37:12 01:24:40:15 if each one is 12, Ethan,
what is that going to be?
612 01:24:40:17 01:24:42:16 12, 12 and 12?
613 01:24:42:18 01:24:43:20 Uh, 36.
614 01:24:43:22 01:24:45:21 36 inches.
615 01:24:45:23 01:24:48:11 Is it going to be more than 36
inches or less than 36 inches?
616 01:24:48:13 01:24:49:14 Less.
617 01:24:49:16 01:24:50:21 Why?
618 01:24:50:23 01:24:52:24 Because you, um,
minus them for touching.
619 01:24:52:26 01:24:54:26 You minus sides that
are touching-- great.
620 01:24:54:28 01:24:58:22 CHAPIN:
This was a very successful
lesson for a number of reasons.
621 01:24:58:24 01:25:01:25 It posed a task that was
very accessible to students
622 01:25:01:27 01:25:04:12 and was based on
sound mathematics.
623 01:25:04:14 01:25:06:06 Second, it used materials
624 01:25:06:08 01:25:10:24 that enabled students to engage
in the task at different levels.
625 01:25:10:26 01:25:13:27 Finally, there was very good
recording of results
626 01:25:13:29 01:25:17:21 so that students had a way
to reflect on what they've done
627 01:25:17:23 01:25:21:13 and to share what they have
learned with their classmates.
628 01:25:29:02 01:25:33:22 Captioned by
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