MATHematics illuminated

FACILITATOR GUIDE

UNIT 10

UNIT 10

HARMONIOUS MATH

FACILITATOR GUIDE

ACTIVITIES

NOTE: At many points in the activities for *Mathematics Illuminated*, workshop participants will be asked to explain, either verbally or in written form, the process they use to answer the questions posed in the activities. This serves two purposes: for the participant as a student, it helps to solidify any previously unfamiliar concepts that are addressed; for the participant as a teacher, it helps to develop the skill of teaching students "why," not just "how," when it comes to confronting mathematical challenges.

NOTE: Instructions, answers, and explanations that are meant for the facilitator only and not the participant are in grey boxes for easy identification.

ACTIVITY 1

(45 minutes)

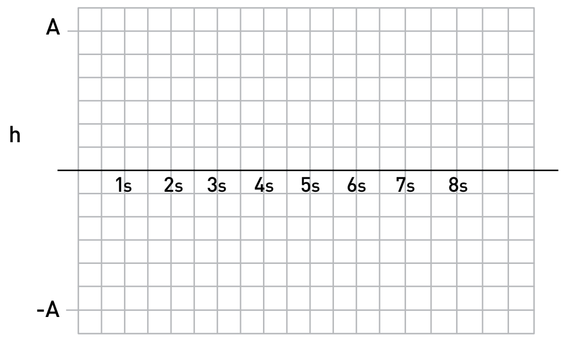
WAVE ARITHMETIC

MATERIALS

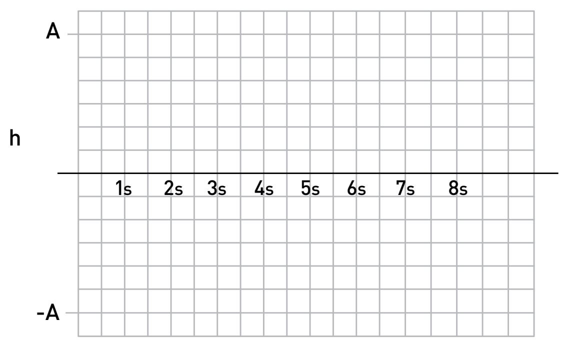
- Wave Arithmetic Worksheet—each person needs four copies
- Graphing calculator (optional)
- Colored pencils

ACTIVITY 1

Wave Arithmetic Worksheet



t	h



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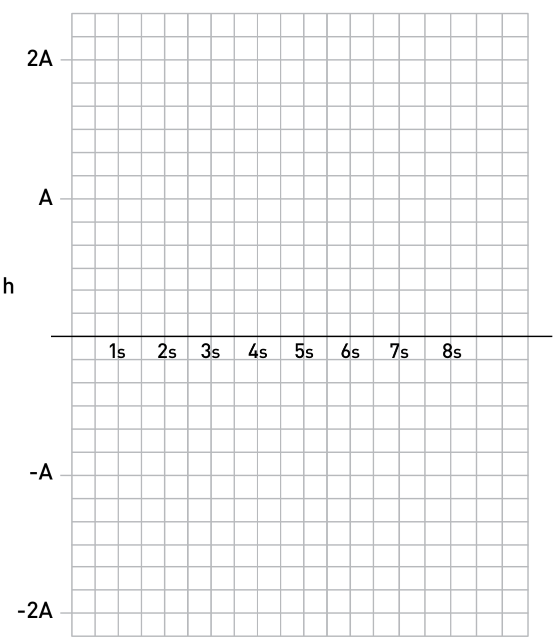


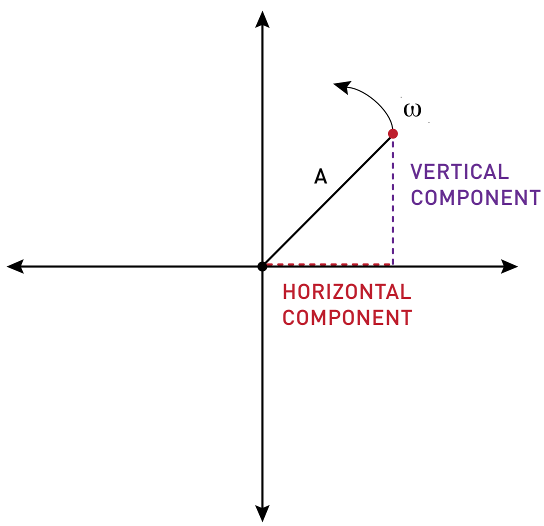
ACTIVITY 1

This activity is to be done in groups of three.

In the text for Unit 10, Harmonious Math, we saw how a periodic function could be modeled by a rotating wheel. In this activity, you will use this model to make “time domain” graphs of periodic functions. You will then use these representations to perform simple arithmetic, addition and multiplication, using waves. Adding waves together is the fundamental idea at the heart of Fourier series and will serve as the basic skill required for all of the activities in this unit.

A

Imagine a spinning bicycle wheel of radius A with one painted spoke (shown below).



1. The spoke completes eight revolutions every 64 seconds. What is its period in seconds?
Answer: T = 8 seconds (one revolution every 8 seconds)

2. The spoke sweeps out an angle equal to 2π radians in every revolution.
What is its angular velocity in radians per second?
Answer: angular velocity = $\omega = \frac{(2\pi \text{ rad})}{(8 \text{ sec})} = \frac{\pi}{4}$ rad/sec

3. What size angle does the spoke sweep out in 3 seconds?
Answer: 3 sec x $\frac{\pi}{4}$ rad/sec = $\frac{3\pi}{4}$ radians

ACTIVITY 1

4. Assume that the spoke starts at time zero with an angle of zero radians. What function gives the vertical component of the tip of the spoke after time t?

Hint 1: Draw a triangle.

Answer: A sin ωt

5. Complete the following chart:

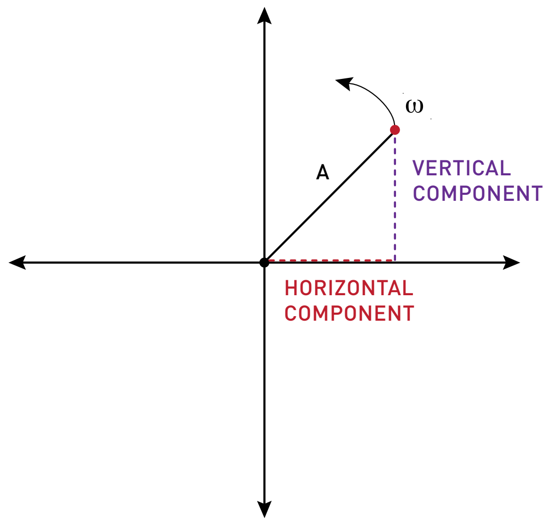
Time (sec)	0	1	2	3	4	5	6	7	8
Angle of spoke (rad)	0	$\frac{\pi}{4}$				$\frac{5\pi}{4}$			
Vertical component of position of the tip of the spoke (as a portion of spoke length, A)	0	A sin $\left(\frac{\pi}{4}\right)$ = $A\left(\frac{\sqrt{2}}{2}\right)$ ~ 0.7A							

Answer:

Time (sec)	0	1	2	3	4	5	6	7	8
Angle of spoke (rad)	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
Vertical component of position of the tip of the spoke (as a portion of spoke length, A)	A sin (0) = 0	A sin $\left(\frac{\pi}{4}\right)$ = $A\left(\frac{\sqrt{2}}{2}\right)$ ~ 0.7A	A sin $\left(\frac{\pi}{2}\right)$ = A x 1 = A	A sin $\left(\frac{3\pi}{4}\right)$ = $A\left(\frac{\sqrt{2}}{2}\right)$ ~ 0.7A	A sin (π) = 0	A sin ($5\pi/4$) = $-A\left(\frac{\sqrt{2}}{2}\right)$ ~ -0.7A	A sin $\left(\frac{3\pi}{2}\right)$ = A x -1 = -A	A sin $\left(\frac{7\pi}{4}\right)$ = $-A\left(\frac{\sqrt{2}}{2}\right)$ ~ -0.7A	A sin (2π)= 0

ACTIVITY 1

6. Use the graph you just completed, along with the Wave Arithmetic Worksheet, (using a color other than black) to make a graph of the vertical component of the position of the tip of the spoke in the time domain (which means that the horizontal axis of your graph should be denominated in seconds). The domain of your graph should be [0,8], and the range should be [-A, A]. Draw your graph on the worksheet where it says "graph 1."



B

1. Write a function that gives the horizontal component of the position of the tip of the spoke at time t.
Answer: $A \cos \omega t$

On your Wave Arithmetic Worksheet, where it says "graph 2," use a different color to graph the horizontal position of the tip of the spoke in the time domain. Note that this graph should have the same domain and range as the previous graph.

If you need help getting started, make a chart similar to the one you made for the previous graph.
Answer: For answer, see the answer to A #6.

ACTIVITY 1

C

On the third set of axes on the Wave Arithmetic Worksheet, graph the sum of the two waves that you graphed in A and B: Asin ωt + Acos ωt. To do this, simply add together the y-values at each time step and graph this composite value. Connect the points you graph using a smooth curve.

For example, at time zero, Asin ωt = 0 and Acos ωt = A, so:

Asin ωt + Acos ωt = 0 + A = A

The first point graphed should therefore be (0,A).

1. Before you graph, why is the range [-2A,2A] instead of [-A,A]?
Answer: Since we are adding values, it is possible to have range values greater than A; if both components of the sum were at their max, the sum would be 2A. Conversely, if both functions were at a minimum, the sum would be –2A.

For the answer to this graph, see A #6.

2. What is the period of this combined function? What is its angular velocity?
Answer: The period is 8 secs; the angular velocity is 2π rad/period = $\frac{(2\pi \text{ rad})}{(8 \text{ sec})} = \frac{\pi}{4}$ rad/sec

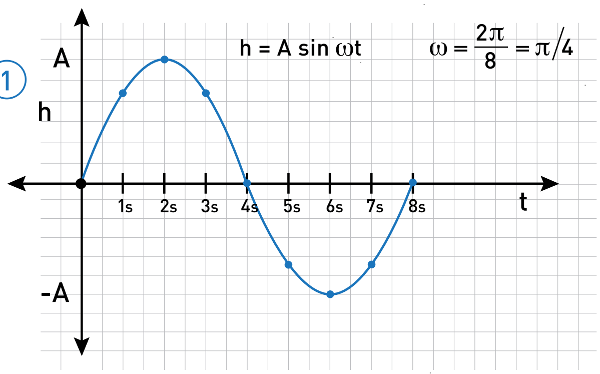
3. How do the period and angular velocity of the combined function compare to the period and angular velocity of the component functions?
Answer: The period and angular velocity of a sum of periodic functions having the same period and angular velocity are equal to the periods and angular velocities of the component functions.

When all groups are finished with C, convene the large group and discuss the results. Note that the period of the composite function is equal to the periods of the component functions.

ACTIVITY 1

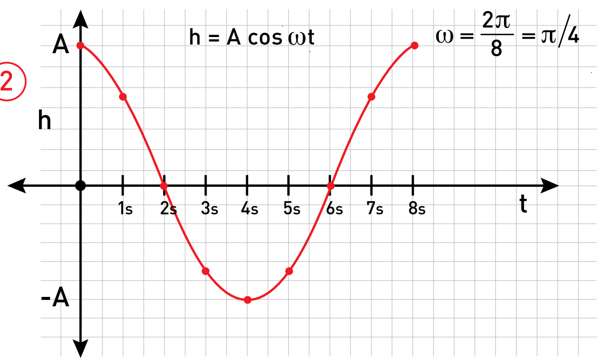
Answer: Wave Arithmetic Worksheet for Sections A through C

Graph 1



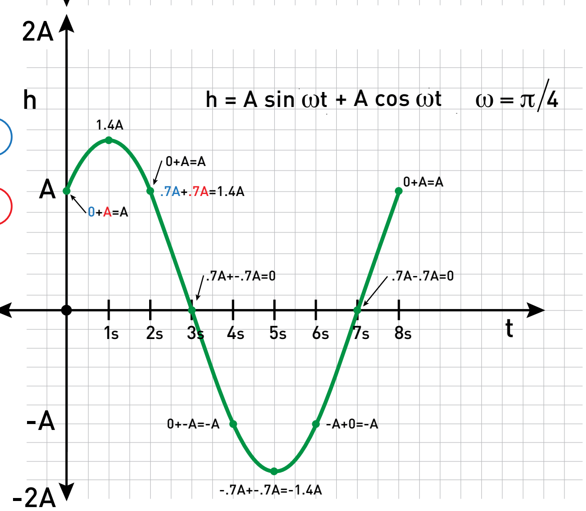
t	h
1s	$A \sin \pi/4 = (\sqrt{2})/2A \sim .7A$
2s	$A \sin \pi/2 = A$
3s	$A \sin 3\pi/4 = (\sqrt{2})/2A \times .7A$
4s	$A \sin \pi = 0$
5s	$A \sin 5\pi/4 = -(\sqrt{2})/2A \sim -.7A$
6s	$A \sin 3\pi/2 = -A$
7s	$A \sin 7\pi/4 = -.7A$
8s	$A \sin 2\pi = 0$

Graph 2



t	h
1s	$A \cos \pi/4 = (\sqrt{2})/2A \sim .7A$
2s	$A \cos \pi/2 = 0$
3s	$A \cos 3\pi/4 = (\sqrt{2})/2A \sim -.7A$
4s	$A \cos \pi = -A$
5s	$A \cos 5\pi/4 = -(\sqrt{2})/2A \sim -.7A$
6s	$A \cos 3\pi/2 = 0$
7s	$A \cos 7\pi/4 = (\sqrt{2})/2 \sim .7A$
8s	$A \cos 2\pi = A$

Graph 1
+
Graph 2



ACTIVITY 1

D

Have plenty of copies of the Wave Arithmetic Worksheet available.

In your group, use the Wave Arithmetic Worksheets to find the following sums and products. Divide the work as you see fit (no one should do all six—each person should do two, one addition and one multiplication). Be ready to analyze and discuss your results as a group.

Graph 1: $h = A \sin \omega t$
Graph 2: $h = A \cos \omega t$
Graph 3: $h = c$ (an arbitrary constant)
Graph 4: $h = A \sin 2\omega t$

Sums:

Problem 1: Graph 3 + Graph 1
Problem 2: Graph 1 + Graph 1
Problem 3: Graph 1 + Graph 4

Note: multiplication of graphs works just like addition, except that you multiply the y-values at each time-step rather than add.

Products:

Problem 4: Graph 3 * Graph 1
What is the area under this curve?

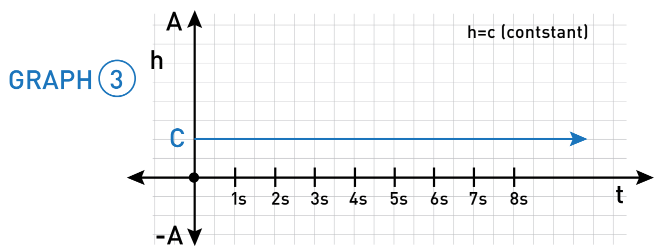
Problem 5: Graph 1 * Graph 4
Problem 6: Graph 1 * Graph 2

Compare the areas under the curves (one period) for products of waves of the same frequency and products of waves of different frequencies.

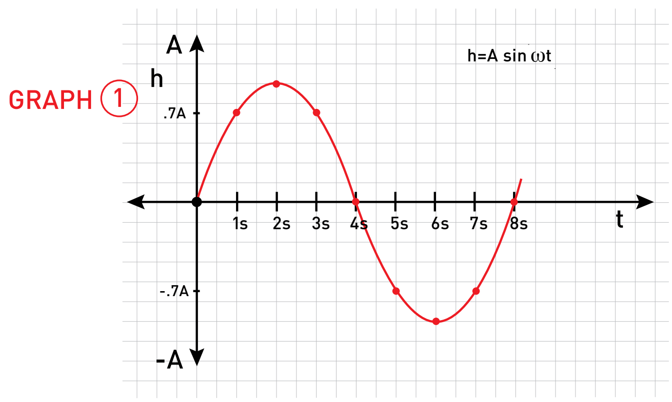
Hint 2: You can do this visually by counting squares. You can think of squares above the x-axis as units of positive area and squares below the x-axis as units of negative area.

ACTIVITY 1

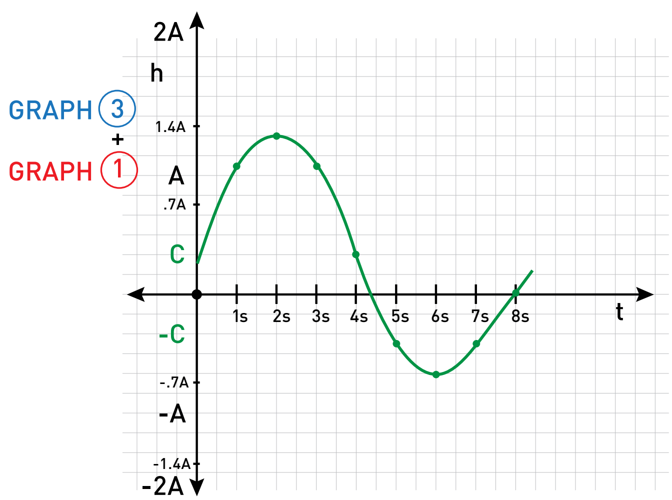
Anwser: Wave Arithmetic Worksheet for Problem 1: Graphic 3 + Graph 1



t	h=c
0	C
1	C
2	C
3	C
4	C
5	C
6	C
7	C
8	C



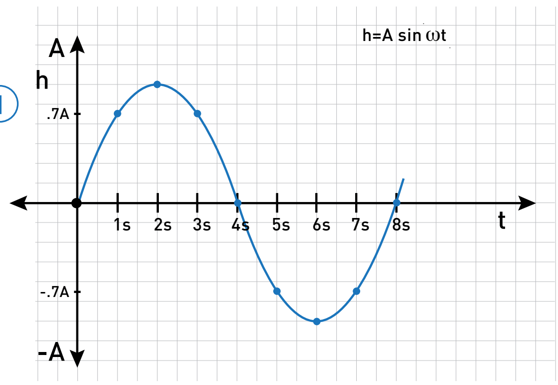
t	h
0	A sin (0) = A×0 = 0
1	A sin (π/4) = A(√2)/2 ~ 0.7A
2	A sin (π/2)=A×1=A
3	A sin (3π/4)=A(√2)/2 ~ 0.7A
4	A sin (π)=A × 0 = 0
5	A sin (5π/4)= A(-√2)/2 ~ -0.7A
6	A sin (3π/2)=Ax-1= -A
7	A sin (7π/4)=A(-√2)/2 ~ -0.7A
8	A sin (2π)= A × 0 = 0



ACTIVITY 1

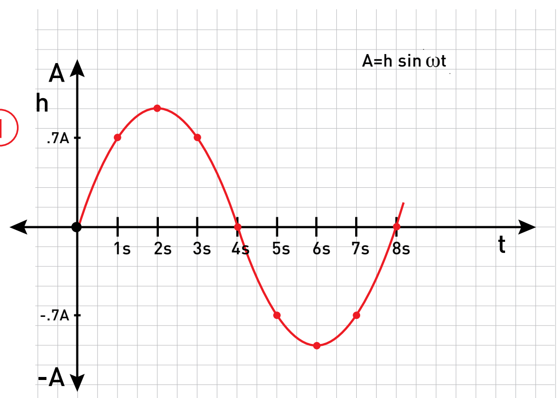
Anwser: Wave Arithmetic Worksheet for Problem 2: Graphic 1 + Graph 1

GRAPH 1



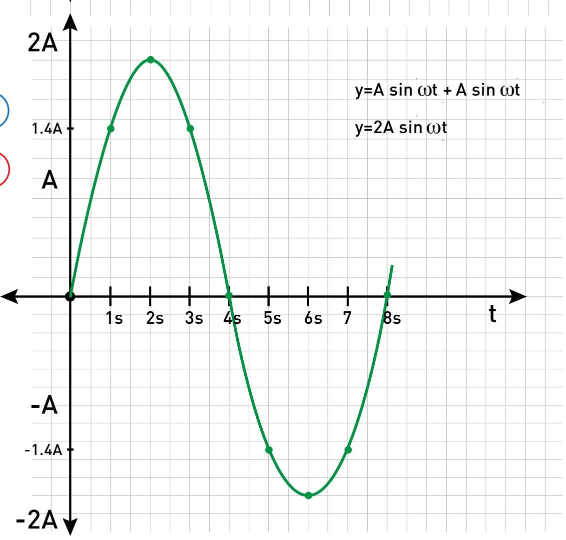
t	h
0	$A \sin(0)=A\times0=0$
1	$A \sin(\pi/4)=A(\sqrt{2})/2 \sim \underline{0.7A}$
2	$A \sin(\pi/2)=\underline{A}\times\underline{1=A}$
3	$A \sin(3\pi/4)=A(\sqrt{2})/2 \sim \underline{0.7A}$
4	$A \sin(\pi)=A \times 0 = \underline{0}$
5	$A \sin(5\pi/4)= A(-\sqrt{2})/2 \sim -\underline{0.7A}$
6	$A \sin(3\pi/2)=\underline{A}\times\underline{-1=-A}$
7	$A \sin(7\pi/4)=A(-\sqrt{2})/2 \sim -\underline{0.7A}$
8	$A \sin(2\pi)= A \times 0 = 0$

GRAPH 1



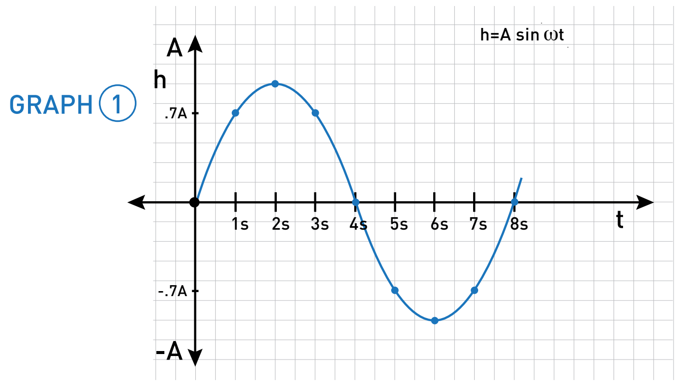
t	h
0	$A \sin(0)=A\times0=0$
1	$A \sin(\pi/4)=A(\sqrt{2})/2 \sim \underline{0.7A}$
2	$A \sin(\pi/2)=\underline{A}\times\underline{1=A}$
3	$A \sin(3\pi/4)=A(\sqrt{2})/2 \sim \underline{0.7A}$
4	$A \sin(\pi)=A \times 0 = \underline{0}$
5	$A \sin(5\pi/4)= A(-\sqrt{2})/2 \sim -\underline{0.7A}$
6	$A \sin(3\pi/2)=\underline{A}\times\underline{-1=-A}$
7	$A \sin(7\pi/4)=A(-\sqrt{2})/2 \sim -\underline{0.7A}$
8	$A \sin(2\pi)= A \times 0 = 0$

GRAPH 1
+
GRAPH 1

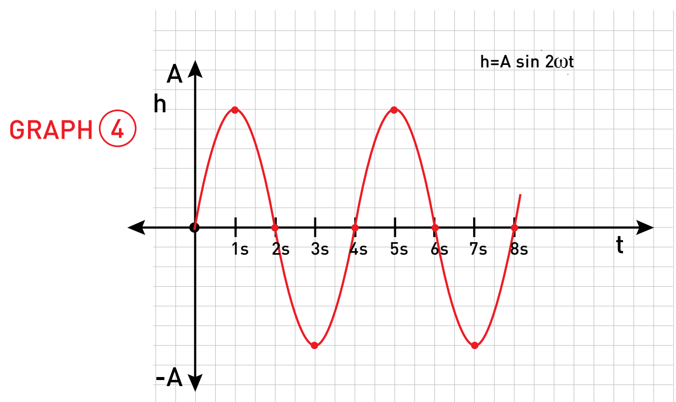


ACTIVITY 1

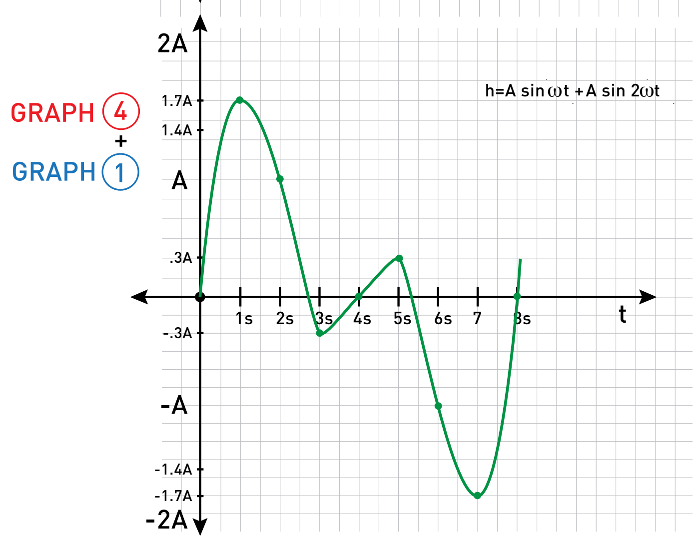
Wave Arithmetic Worksheet for Problem 3: Graph 1 + Graph 4



t	h
0	$A \sin(0) = A \times 0 = 0$
1	$A \sin(\pi/4) = A(\sqrt{2})/2 \sim \underline{0.7A}$
2	$A \sin(\pi/2) = \underline{A} \times \underline{1 = A}$
3	$A \sin(3\pi/4) = A(\sqrt{2})/2 \sim \underline{0.7A}$
4	$A \sin(\pi) = A \times 0 = \underline{0}$
5	$A \sin(5\pi/4) = A(-\sqrt{2})/2 \sim -\underline{0.7A}$
6	$A \sin(3\pi/2) = \underline{A} \times \underline{-1 = -A}$
7	$A \sin(7\pi/4) = A(-\sqrt{2})/2 \sim -\underline{0.7A}$
8	$A \sin(2\pi) = A \times 0 = 0$

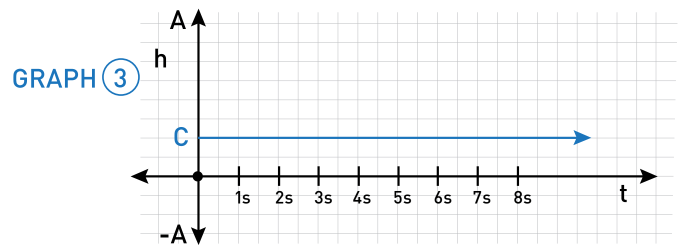


t	h
0	$A \sin(0) = \underline{0}$
1	$A \sin 2(\pi/4) = A \sin(\pi/2) = A \times 1 = \underline{A}$
2	$A \sin 2(\pi/2) = A \sin(\pi) = \underline{0}$
3	$A \sin 2(3\pi/4) = A \sin(3\pi/2) = A \times -1 = -\underline{A}$
4	$A \sin 2(\pi) = A \sin(2\pi) = \underline{0}$
5	$A \sin 2(5\pi/4) = A \sin(5\pi/2) = A \times 1 = \underline{A}$
6	$A \sin 2(3\pi/2) = \sin(3\pi) = \underline{0}$
7	$A \sin 2(7\pi/4) = A \sin(7\pi/2) = A \times -1 = -\underline{A}$
8	$A \sin 2(2\pi) = A \sin(4\pi) = 0$

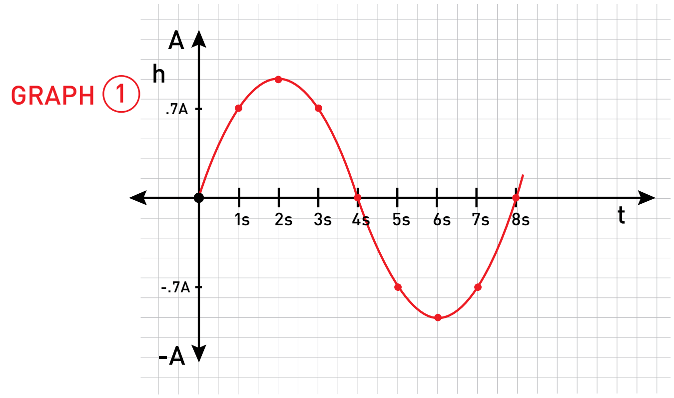


ACTIVITY 1

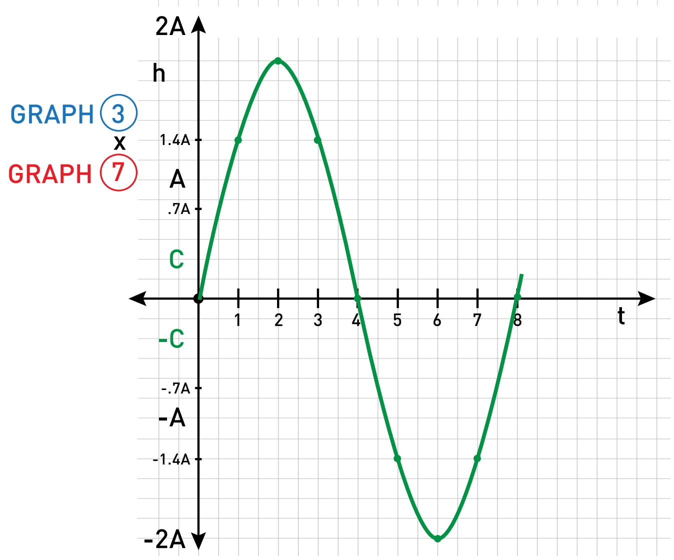
Wave Arithmetic Worksheet for Problem 4: Graph 3 *Graph 1



t	h=c
0	C
1	C
2	C
3	C
4	C
5	C
6	C
7	C
8	C



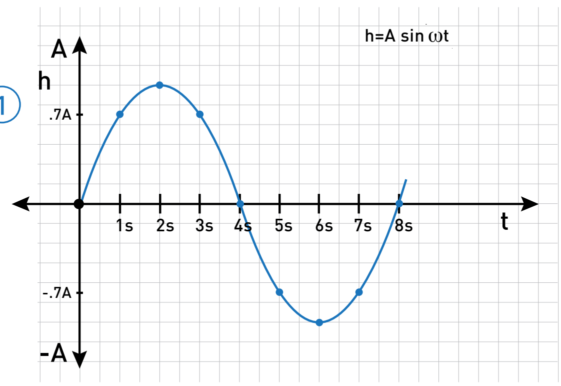
t	h
0	$A \sin(0) = A \times 0 = 0$
1	$A \sin(\pi/4) = A(\sqrt{2})/2 \sim \underline{0.7A}$
2	$A \sin(\pi/2) = \underline{A} \times \underline{1 = A}$
3	$A \sin(3\pi/4) = A(\sqrt{2})/2 \sim \underline{0.7A}$
4	$A \sin(\pi) = A \times 0 = \underline{0}$
5	$A \sin(5\pi/4) = A(-\sqrt{2})/2 \sim -\underline{0.7A}$
6	$A \sin(3\pi/2) = \underline{A} \times \underline{-1 = -A}$
7	$A \sin(7\pi/4) = A(-\sqrt{2})/2 \sim -\underline{0.7A}$
8	$A \sin(2\pi) = A \times 0 = 0$



ACTIVITY 1

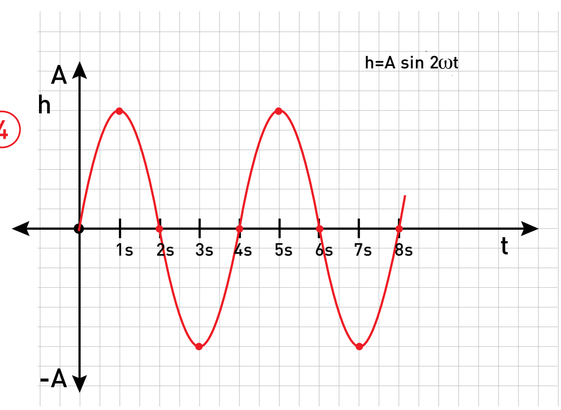
Wave Arithmetic Worksheetfor Problem 5: Graph 1 * Graph 4

GRAPH 1



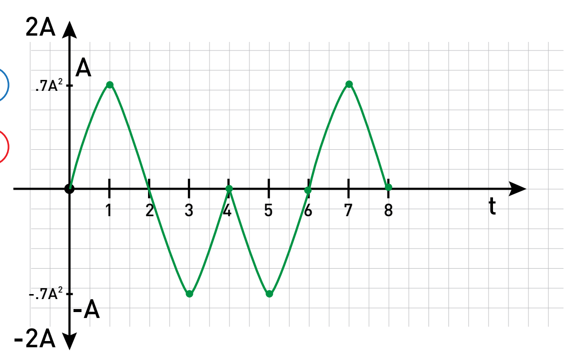
t	h
0	$A \sin(0) = A \times 0 = 0$
1	$A \sin(\pi/4) = A(\sqrt{2})/2 \sim \underline{0.7A}$
2	$A \sin(\pi/2) = \underline{A} \times \underline{1 = A}$
3	$A \sin(3\pi/4) = A(\sqrt{2})/2 \sim \underline{0.7A}$
4	$A \sin(\pi) = A \times 0 = \underline{0}$
5	$A \sin(5\pi/4) = A(-\sqrt{2})/2 \sim -\underline{0.7A}$
6	$A \sin(3\pi/2) = \underline{A} \times \underline{-1 = -A}$
7	$A \sin(7\pi/4) = A(-\sqrt{2})/2 \sim -\underline{0.7A}$
8	$A \sin(2\pi) = A \times 0 = 0$

GRAPH 4



t	h
0	$A \sin(0) = \underline{0}$
1	$A \sin 2(\pi/4) = A \sin(\pi/2) = A \times 1 = \underline{A}$
2	$A \sin 2(\pi/2) = A \sin(\pi) = \underline{0}$
3	$A \sin 2(3\pi/4) = A \sin(3\pi/2) = A \times -1 = -\underline{A}$
4	$A \sin 2(\pi) = A \sin(2\pi) = \underline{0}$
5	$A \sin 2(5\pi/4) = A \sin(5\pi/2) = A \times 1 = \underline{A}$
6	$A \sin 2(3\pi/2) = \sin(3\pi) = \underline{0}$
7	$A \sin 2(7\pi/4) = A \sin(7\pi/2) = A \times -1 = -A$
8	$A \sin 2(2\pi) = A \sin(4\pi) = 0$

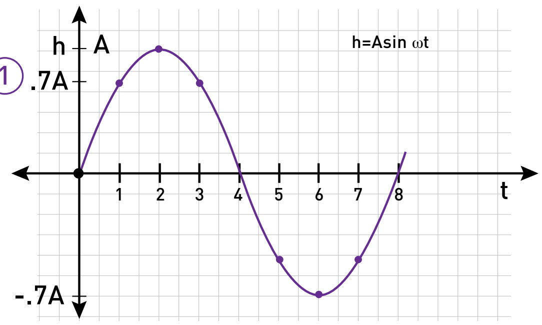
GRAPH 1
x
GRAPH 4



ACTIVITY 1

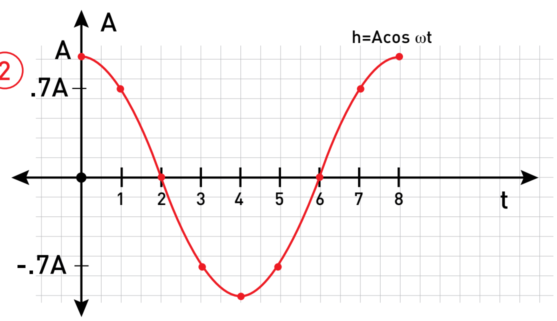
Wave Arithmetic Worksheet for Problem 6: Graph 1 * Graph 2

GRAPH 1



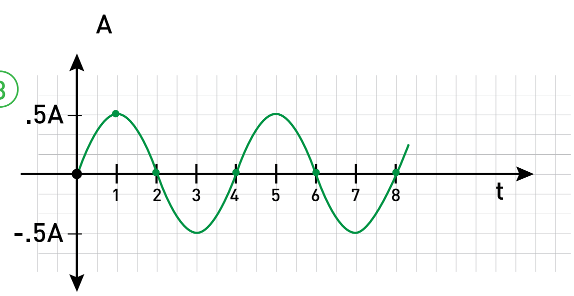
t	h
0	A sin $(0)=A\times0=0$
1	A sin $(\pi/4)=A(\sqrt{2})/2\sim\underline{0.7A}$
2	A sin $(\pi/2)=\underline{A}\times\underline{1=A}$
3	A sin $(3\pi/4)=A(\sqrt{2})/2\sim\underline{0.7A}$
4	A sin $(\pi)=A\times0=\underline{0}$
5	A sin $(5\pi/4)=A(-\sqrt{2})/2\sim-\underline{0.7A}$
6	A sin $(3\pi/2)=\underline{A}\times\underline{-1=-A}$
7	A sin $(7\pi/4)=A(-\sqrt{2})/2\sim-\underline{0.7A}$
8	A sin $(2\pi)=A\times0=0$

GRAPH 2



t	h
0	A cos $(0)=A\times1=A$
1	A cos $(\pi/4)=A(\sqrt{2})/2\sim\underline{0.7A}$
2	A cos $(\pi/2)=\underline{0}$
3	A cos $(3\pi/4)=A(-\sqrt{2})/2\sim-\underline{0.7A}$
4	A cos $(\pi)=A\times-1=\underline{-A}$
5	A cos $(5\pi/4)=A(-\sqrt{2})/2\sim-\underline{0.7A}$
6	A cos $(3\pi/2)=\underline{0}$
7	A cos $(7\pi/4)=A(\sqrt{2})/2\sim\underline{0.7A}$
8	A cos $(2\pi)=A\times1=A$

GRAPH 3



ACTIVITY 2

(30 minutes)

MATERIALS

- Graph paper
- Colored pencils
- Worksheet: Cards for Fourier components for groups 1A through 1E
- Worksheet: Cards for Fourier components for groups 2A through 2E

Be sure to cut out the cards for each group before the activity.

Divide the participants into five groups (A, B, C, D, and E) of approximately four people each, with as little overlap with previous groups as possible.

A (20 minutes)

MAKING WAVES

In the previous activity, you added waves together graphically to get new waves. In this activity, you will use the techniques you used in the last activity to create two well-known periodic functions.

Hand out cards 1A and 2A to group A, 1B and 2B to group B, 1C and 2C to group C, etc.

Each person in your group should have two cards, one with a number 1 and one with a number 2.

Card Number 1:

Graph the "wave-function" shown on your card (number 1), in the suggested domain and range, on a piece of graph paper. As a group, decide on nine specific, evenly spaced, domain values for which each person will find range values. For example, if all four people have a suggested domain of [-1,1] then the agreed upon values could be {-1, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, and 1}. Each person would use these values to plot their individual graphs. Feel free to use a calculator.

ACTIVITY 2

After each person has graphed his or her wave, add the four waves together using the technique from Activity 1 to get a composite wave. Be sure to increase the range of the composite wave. Note: it is much easier to find the new range if you first simplify the expression on your card so that the coefficient is one number.

When all groups have created their composite waves, make a large graph on either the blackboard, whiteboard, or overhead. Lead the group in constructing the sum of the five composite waves. With any luck, you should have a triangle wave!

ACTIVITY 2

Fourier components for groups 1A through 1E
Cards to cut out; page one of two

Group 1A

n=0, $(-5 < x < 5, -1.2 < y < 1.2)$ 1A $y = \frac{(-1)^0}{(2 \times 0 + 1)^2} \sin((2 \times 0 + 1)x)$	n=2, $(-5 < x < 5, -1.2 < y < 1.2)$ 1A $y = \frac{(-1)^2}{(2 \times 2 + 1)^2} \sin((2 \times 2 + 1)x)$
n=1, $(-5 < x < 5, -1.2 < y < 1.2)$ 1A $y = \frac{(-1)^1}{(2 \times 1 + 1)^2} \sin((2 \times 1 + 1)x)$	n=3, $(-5 < x < 5, -1.2 < y < 1.2)$ 1A $y = \frac{(-1)^3}{(2 \times 3 + 1)^2} \sin((2 \times 3 + 1)x)$

Group 1B

n=4, $(-1 < x < 1, -.1 < y < .1)$ 1B $y = \frac{(-1)^4}{(2 \times 4 + 1)^2} \sin((2 \times 4 + 1)x)$	n=6, $(-1 < x < 1, -.1 < y < .1)$ 1B $y = \frac{(-1)^6}{(2 \times 6 + 1)^2} \sin((2 \times 6 + 1)x)$
n=5, $(-1 < x < 1, -.1 < y < .1)$ 1B $y = \frac{(-1)^5}{(2 \times 5 + 1)^2} \sin((2 \times 5 + 1)x)$	n=7, $(-1 < x < 1, -.1 < y < .1)$ 1B $y = \frac{(-1)^7}{(2 \times 7 + 1)^2} \sin((2 \times 7 + 1)x)$

ACTIVITY 2

Fourier components for groups 1A through 1E
Cards to cut out; page two of two

Group 1C

n=8, (-.5 < x < .5, -.01 < y < .01) 1C $y=\frac{(-1)^{8}}{(2\times8+1)^{2}}\sin((2\times8+1)x)$	n=10, (-.5 < x < .5, -.01 < y < .01) 1C $y=\frac{(-1)^{10}}{(2\times10+1)^{2}}\sin((2\times10+1)x)$
n=9, (-.5 < x < .5, -.01 < y < .01) 1C $y=\frac{(-1)^{9}}{(2\times9+1)^{2}}\sin((2\times9+1)x)$	n=11, (-.5 < x < .5, -.01 < y < .01) 1C $y=\frac{(-1)^{11}}{(2\times11+1)^{2}}\sin((2\times11+1)x)$

Group 1D

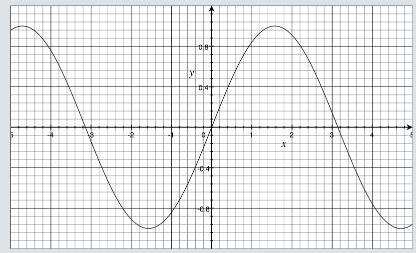
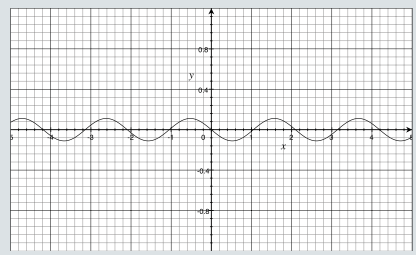
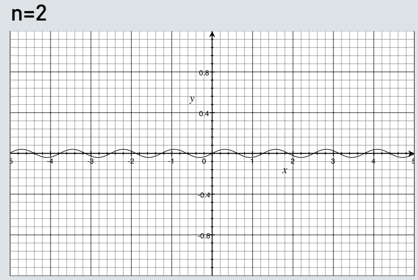
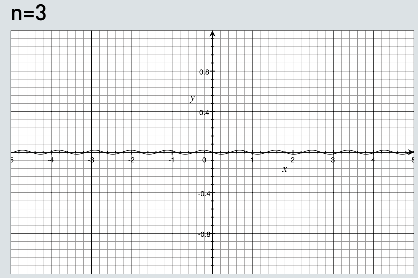
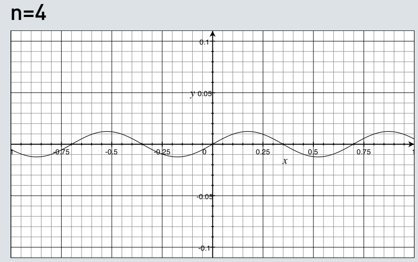
n=12, (-.3 < x < .3, -.005 < y < .005) 1D $y=\frac{(-1)^{12}}{(2\times12+1)^{2}}\sin((2\times12+1)x)$	n=14, (-.3 < x < .3, -.005 < y < .005) 1D $y=\frac{(-1)^{14}}{(2\times14+1)^{2}}\sin((2\times14+1)x)$
n=13, (-.3 < x < .3, -.005 < y < .005) 1D $y=\frac{(-1)^{13}}{(2\times13+1)^{2}}\sin((2\times13+1)x)$	n=15, (-.3 < x < .3, -.005 < y < .005) 1D $y=\frac{(-1)^{15}}{(2\times15+1)^{2}}\sin((2\times15+1)x)$

Group 1E

n=16, (-.2 < x < .2, -.001 < y < .001) 1E $y=\frac{(-1)^{16}}{(2\times16+1)^{2}}\sin((2\times16+1)x)$	n=18, (-.2 < x < .2, -.001 < y < .001) 1E $y=\frac{(-1)^{18}}{(2\times18+1)^{2}}\sin((2\times18+1)x)$
n=17, (-.2 < x < .2, -.001 < y < .001) 1E $y=\frac{(-1)^{17}}{(2\times17+1)^{2}}\sin((2\times17+1)x)$	n=19, (-.2 < x < .2, -.001 < y < .001) 1E $y=\frac{(-1)^{19}}{(2\times19+1)^{2}}\sin((2\times19+1)x)$

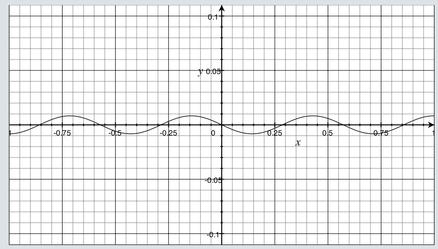
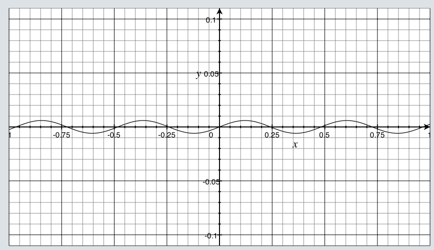
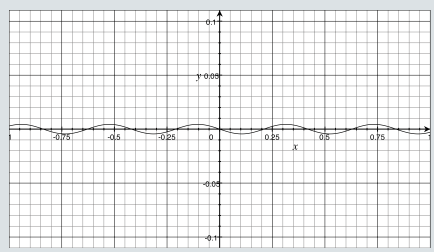
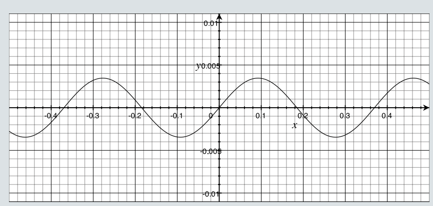
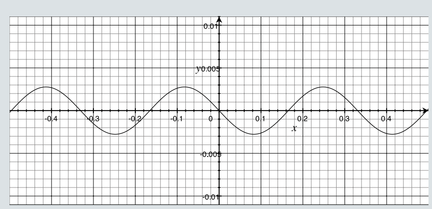
ACTIVITY 2

Answers: Fourier components for groups 1A through 1E – triangle wave

n=0, {-5<x<5, -1.2<y<1.2} $y=\frac{(-1)^0}{(2\cdot0+1)^2}\sin((2\cdot0+1)x)$	n=0 
n=1, {-5<x<5, -1.2<y<1.2} $y=\frac{(-1)^1}{(2\cdot1+1)^2}\sin((2\cdot1+1)x)$	n=1 
n=2, {-5<x<5, -1.2<y<1.2} $y=\frac{(-1)^2}{(2\cdot2+1)^2}\sin((2\cdot2+1)x)$	n=2 
n=3, {-5<x<5, -1.2<y<1.2} $y=\frac{(-1)^3}{(2\cdot3+1)^2}\sin((2\cdot3+1)x)$	n=3 
n=4, {-1<x<1, -.1<y<.1} $y=\frac{(-1)^4}{(2\cdot4+1)^2}\sin((2\cdot4+1)x)$	n=4 

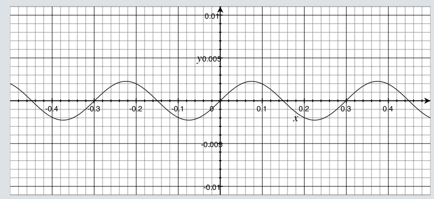
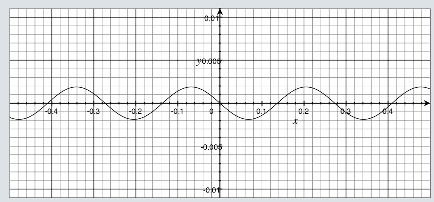
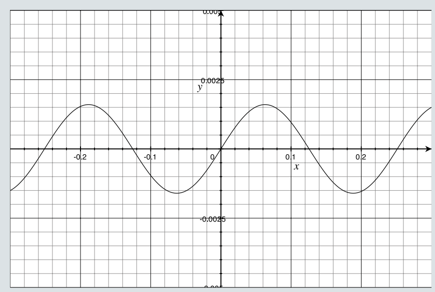
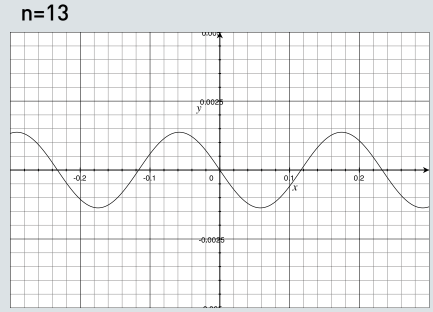
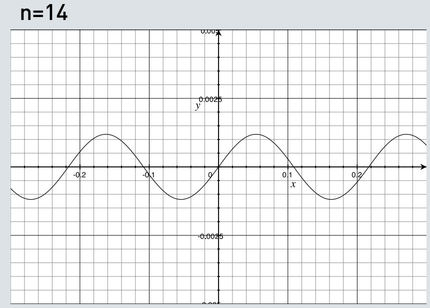
ACTIVITY 2

Answers: Fourier components for groups 1A through 1E – triangle wave

n=5, {-1<x<1, -.1<y<.1} $y=\frac{(-1)^0}{(2\cdot 5+1)^2}\sin((2\cdot 5+1)x)$	n=5 
n=6, {-1<x<1, -.1<y<.1} $y=\frac{(-1)^6}{(2\cdot 6+1)^2}\sin((2\cdot 6+1)x)$	n=6 
n=7, {-1<x<1, -.1<y<.1} $y=\frac{(-1)^7}{(2\cdot 7+1)^2}\sin((2\cdot 7+1)x)$	n=7 
n=8, {-.5<x<.5, -.01<y<.01} $y=\frac{(-1)^8}{(2\cdot 8+1)^2}\sin((2\cdot 8+1)x)$	n=8 
n=9, {-.5<x<.5, -.01<y<.01} $y=\frac{(-1)^9}{(2\cdot 9+1)^2}\sin((2\cdot 9+1)x)$	n=9 

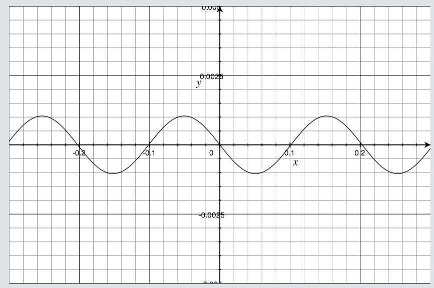
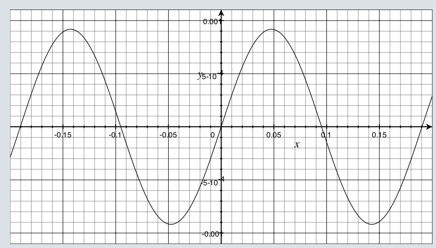
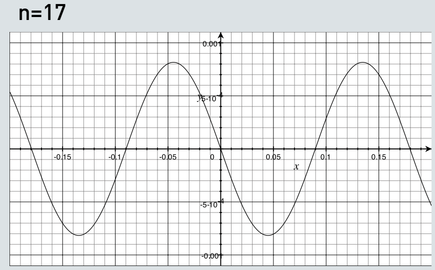
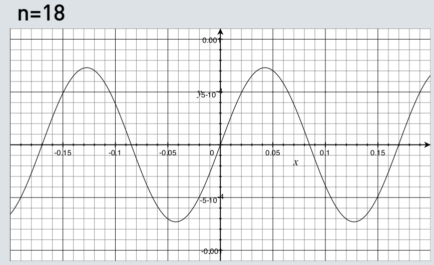
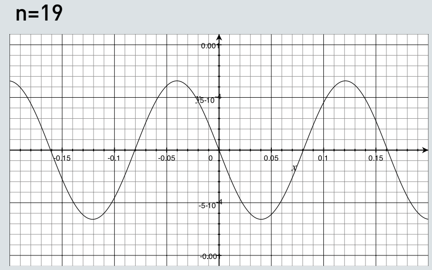
ACTIVITY 2

Answers: Fourier components for groups 1A through 1E – triangle wave

n=10, $\{-.5<x<.5, -.01<y<.01\}$ $y=\frac{(-1)^{10}}{(2\cdot10+1)^2}\sin((2\cdot10+1)x)$	n=10 
n=11, $\{-.5<x<.5, -.01<y<.01\}$ $y=\frac{(-1)^{11}}{(2\cdot11+1)^2}\sin((2\cdot11+1)x)$	n=11 
n=12, $\{-.3<x<.3, -.005<y<.005\}$ $y=\frac{(-1)^{12}}{(2\cdot12+1)^2}\sin((2\cdot12+1)x)$	n=12 
n=13, $\{-.3<x<.3, -.005<y<.005\}$ $y=\frac{(-1)^{13}}{(2\cdot13+1)^2}\sin((2\cdot13+1)x)$	n=13 
n=14, $\{-.3<x<.3, -.005<y<.005\}$ $y=\frac{(-1)^{14}}{(2\cdot14+1)^2}\sin((2\cdot14+1)x)$	n=14 

ACTIVITY 2

Answers: Fourier components for groups 1A through 1E – triangle wave

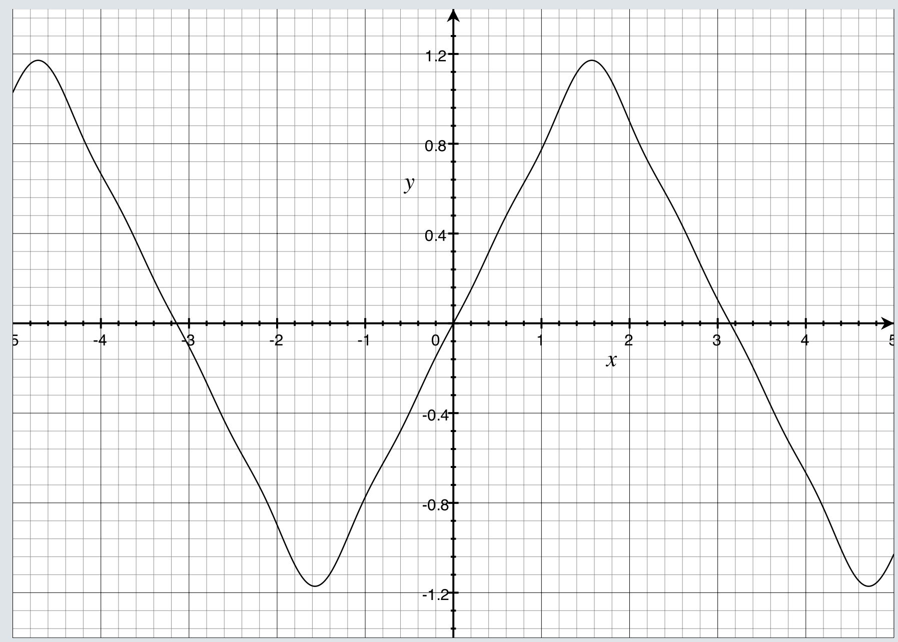
n=15, {-.3<x<.3, -.005<y<.005} $y=\frac{(-1)^{15}}{(2\cdot 15+1)^2}\sin((2\cdot 15+1)x)$	n=15 
n=16, {-.2<x<.2, -.001<y<.001} $y=\frac{(-1)^{16}}{(2\cdot 16+1)^2}\sin((2\cdot 16+1)x)$	n=16 
n=17, {-.2<x<.2, -.001<y<.001} $y=\frac{(-1)^{17}}{(2\cdot 17+1)^2}\sin((2\cdot 17+1)x)$	n=17 
n=18, {-.2<x<.2, -.001<y<.001} $y=\frac{(-1)^{18}}{(2\cdot 18+1)^2}\sin((2\cdot 18+1)x)$	n=18 
n=19, {-.2<x<.2, -.001<y<.001} $y=\frac{(-1)^{19}}{(2\cdot 19+1)^2}\sin((2\cdot 19+1)x)$	n=19 

ACTIVITY 2

Fourier triangle wave composite

When all groups combine their graphs, this is one way it could look:
Fourier components – Group results:

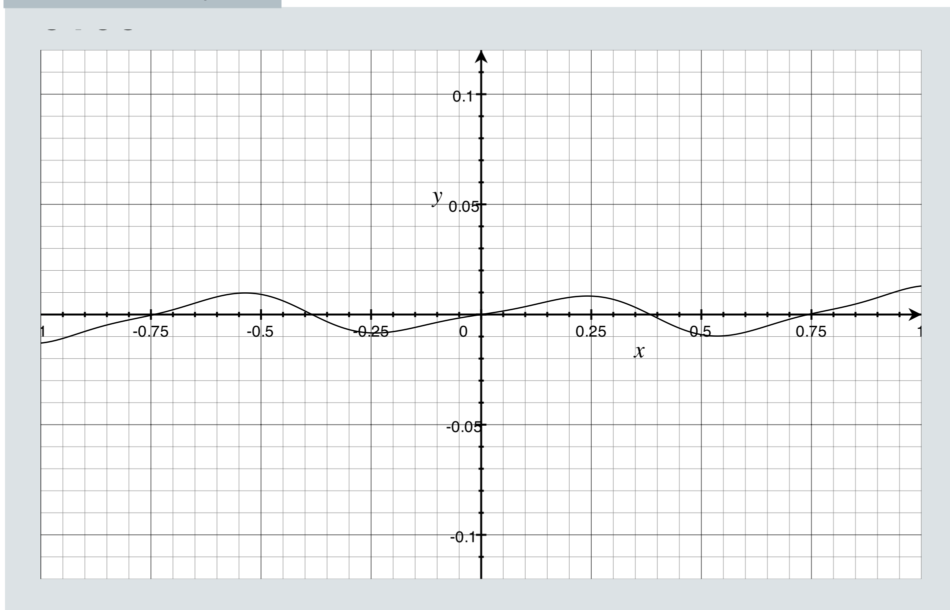
Group 1A: n = 0 through n = 3
{ -5←x←5, -1.2←y←1.2 }; to accommodate: { -5←x←5, -1.6←y←1.6 }



ACTIVITY 2

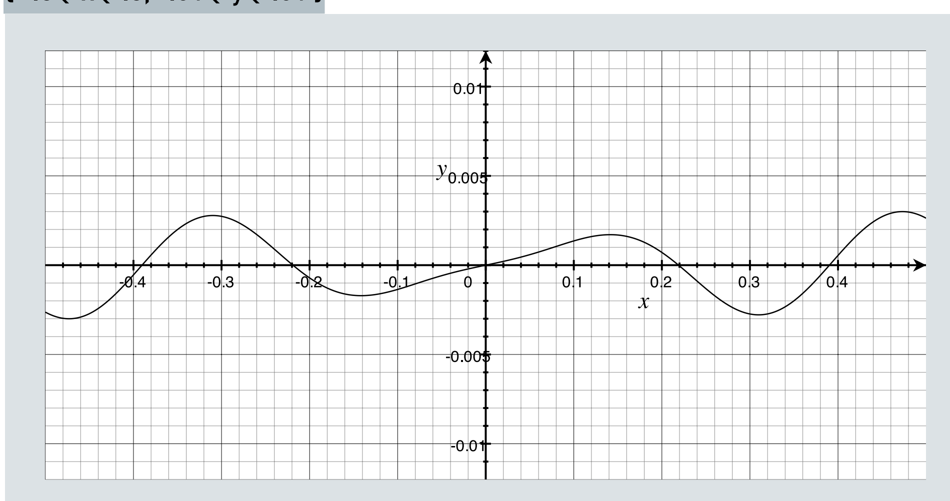
Group 1B: n = 4 through n = 7

{ -1←x←1, -.1←y←.1 }



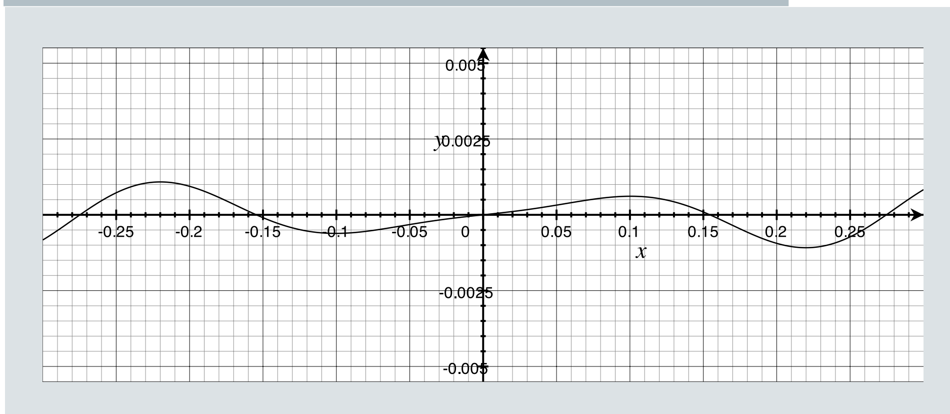
Group 1C: n = 8 through n = 11

{ -.5←x←.5, -.01←y←.01 }

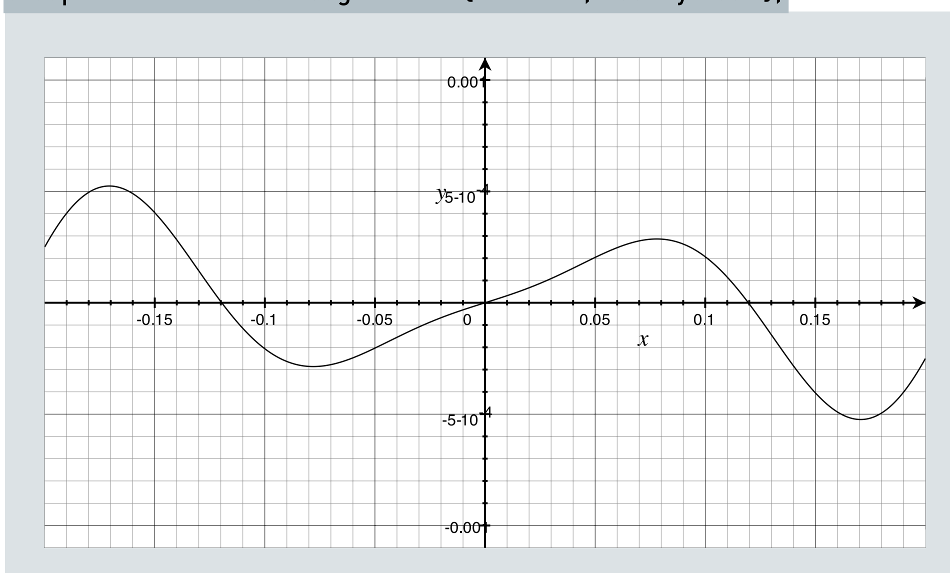


ACTIVITY 2

Group 1D: n = 12 through n = 15 { -.3<x<.3, -.005<y<.005 };



Group 1E: n = 16 through n = 19 { -.2<x<.2, -.001<y<.001 };

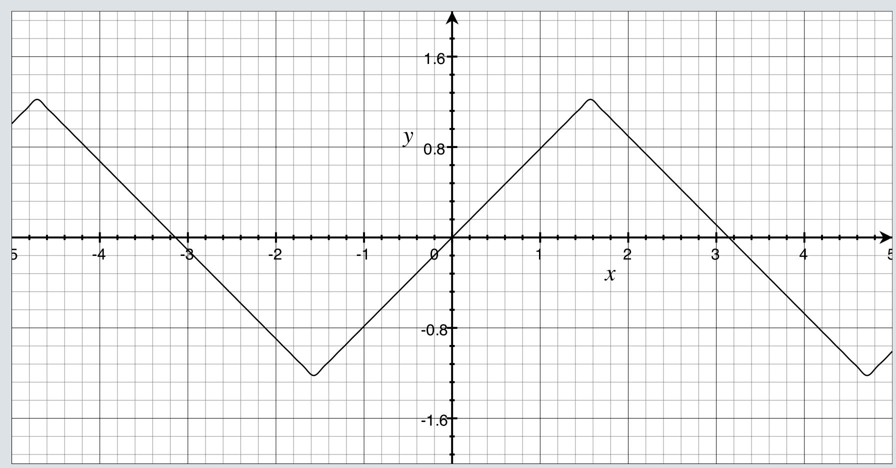


ACTIVITY 2

When all groups combine their data, the formula is this:

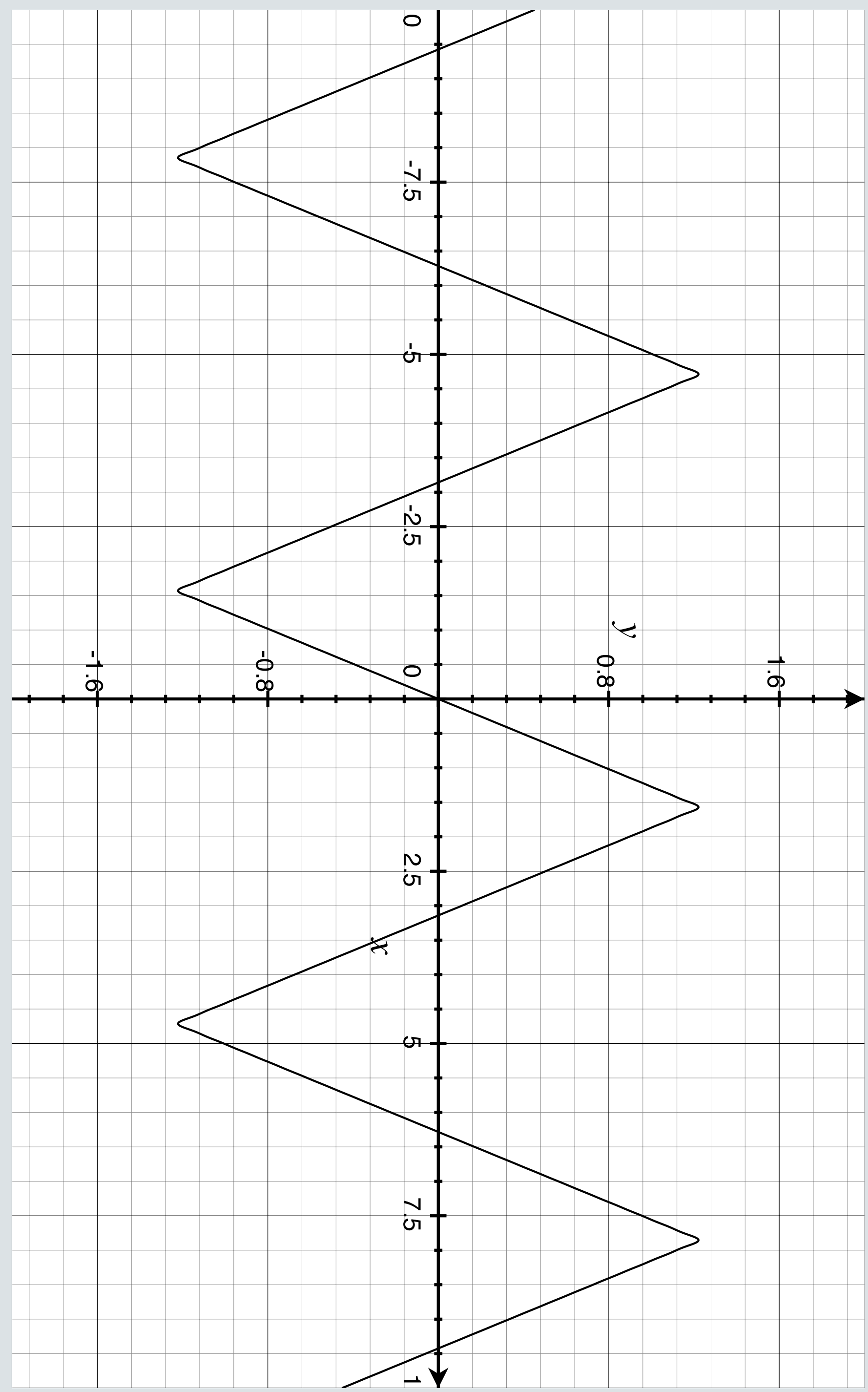
$$y = \sum_{n=0}^{19} \frac{(-1)^n}{(2n+1)^2} \sin\left(\left(2n+1\right)x\right)$$

A graph could look like this: { -5<x<5, -2<y<2 }



ACTIVITY 2

If we zoom out, to the range and domain { -10<x<10, -2<y<2 }, a graph could look like this:



ACTIVITY 2

Cards for Fourier components for groups 2A through 2E
Cards to cut out; page one of two

Group 2A

n=16, ($-.15 < x < .15$, $-.05 < y < .05$) 2A $y = \frac{1}{2\times 16+1} \sin((2\times 16+1)x)$	n=18, ($-.15 < x < .15$, $-.05 < y < .05$) 2A $y = \frac{1}{2\times 18+1} \sin((2\times 18+1)x)$
n=17, ($-.15 < x < .15$, $-.05 < y < .05$) 2A $y = \frac{1}{2\times 17+1} \sin((2\times 17+1)x)$	n=19, ($-.15 < x < .15$, $-.05 < y < .05$) 2A $y = \frac{1}{2\times 19+1} \sin((2\times 19+1)x)$

Group 2B

n=12, ($-.25 < x < .25$, $-.08 < y < .08$) 2B $y = \frac{1}{2\times 12+1} \sin((2\times 12+1)x)$	n=14, ($-.25 < x < .25$, $-.08 < y < .08$) 2B $y = \frac{1}{2\times 14+1} \sin((2\times 14+1)x)$
n=13, ($-.25 < x < .25$, $-.08 < y < .08$) 2B $y = \frac{1}{2\times 13+1} \sin((2\times 13+1)x)$	n=15, ($-.25 < x < .25$, $-.08 < y < .08$) 2B $y = \frac{1}{2\times 15+1} \sin((2\times 15+1)x)$

ACTIVITY 2

Cards to cut out; page two of two

Group 2C

2C n=8, ($-.25 < x < .25$, $-.08 < y < .08$) $y=\frac{1}{2\times8+1}\sin((2\times8+1)x)$	2C n=10, ($-.25 < x < .25$, $-.08 < y < .08$) $y=\frac{1}{2\times10+1}\sin((2\times10+1)x)$
2C n=9, ($-.25 < x < .25$, $-.08 < y < .08$) $y=\frac{1}{2\times9+1}\sin((2\times9+1)x)$	2C n=11, ($-.25 < x < .25$, $-.08 < y < .08$) $y=\frac{1}{2\times11+1}\sin((2\times11+1)x)$

Group 2D

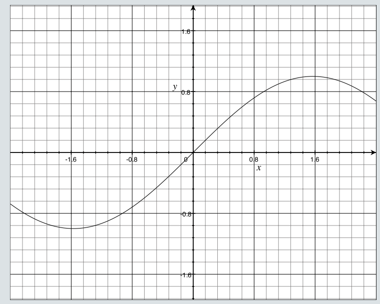
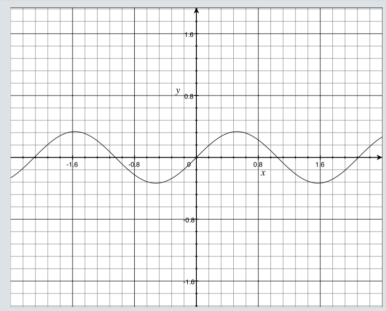
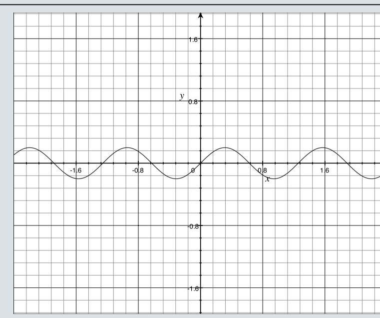
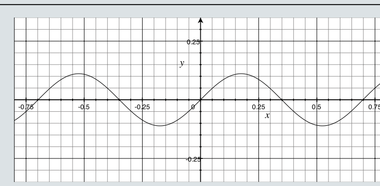
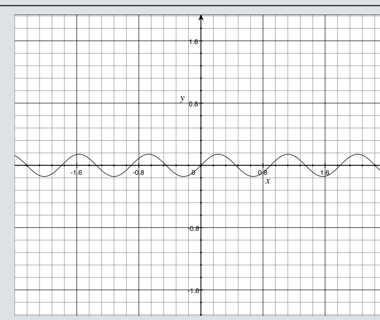
2D n=4, ($-.75 < x < .75$, $-.35 < y < .35$) $y=\frac{1}{2\times4+1}\sin((2\times4+1)x)$	2D n=6, ($-.75 < x < .75$, $-.35 < y < .35$) $y=\frac{1}{2\times6+1}\sin((2\times6+1)x)$
2D n=5, ($-.75 < x < .75$, $-.35 < y < .35$) $y=\frac{1}{2\times5+1}\sin((2\times5+1)x)$	2D n=7, ($-.75 < x < .75$, $-.35 < y < .35$) $y=\frac{1}{2\times7+1}\sin((2\times7+1)x)$

Group 2E

2E n=0, ($-2.4 < x < 2.4$, $-1.6 < y < 1.6$) $y=\frac{1}{2\times0+1}\sin((2\times0+1)x)$	2E n=2, ($-2.4 < x < 2.4$, $-1.6 < y < 1.6$) $y=\frac{1}{2\times2+1}\sin((2\times2+1)x)$
2E n=1, ($-2.4 < x < 2.4$, $-1.6 < y < 1.6$) $y=\frac{1}{2\times1+1}\sin((2\times1+1)x)$	2E n=3, ($-2.4 < x < 2.4$, $-1.6 < y < 1.6$) $y=\frac{1}{2\times3+1}\sin((2\times3+1)x)$

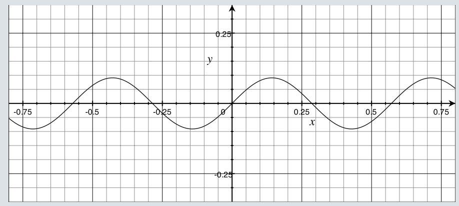
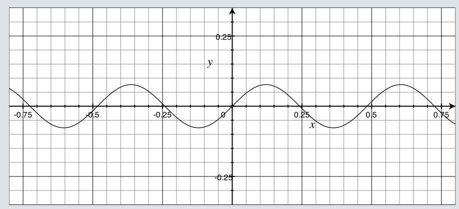
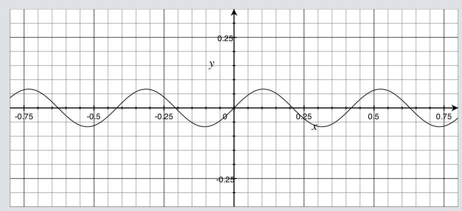
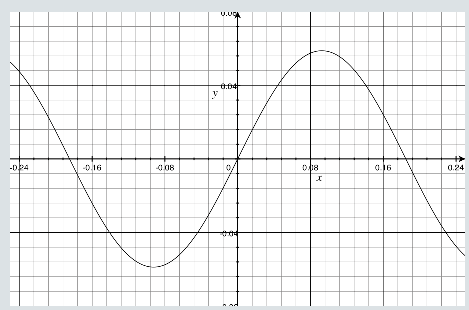
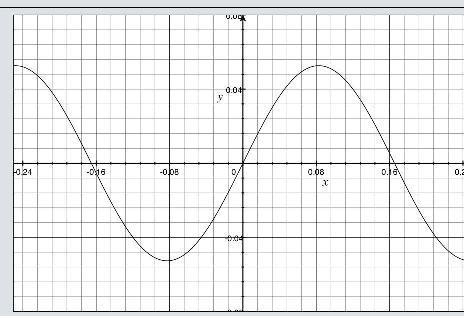
ACTIVITY 2

Answers: Fourier components for groups 2A through 2E – square wave

n=0, {-2.4<x<2.4, -1.6<y<1.6} $y=\frac{1}{2\cdot0+1}\sin((2\cdot0+1)x)$	n=0 
n=1, {-2.4<x<2.4, -1.6<y<1.6} $y=\frac{1}{2\cdot1+1}\sin((2\cdot1+1)x)$	n=1 
n=2, {-2.4<x<**2.4**, -1.6<y<1.6} $y=\frac{1}{2\cdot2+1}\sin((2\cdot2+1)x)$	n=2 
n=3, {-2.4<x<2.4, -1.6<y<1.6} $y=\frac{1}{2\cdot3+1}\sin((2\cdot3+1)x)$	n=3 
n=4, {-.75<x<.75, -.35<y<.35} $y=\frac{1}{2\cdot4+1}\sin((2\cdot4+1)x)$	n=4 

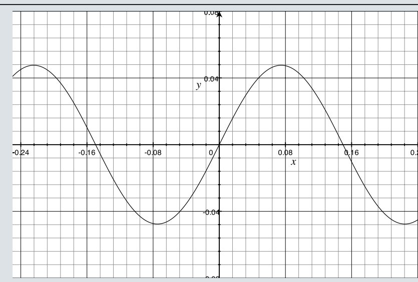
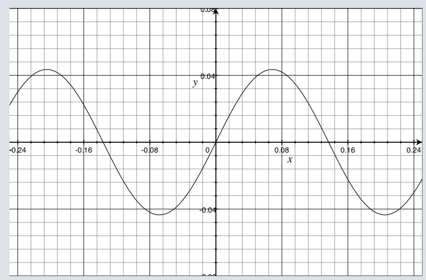
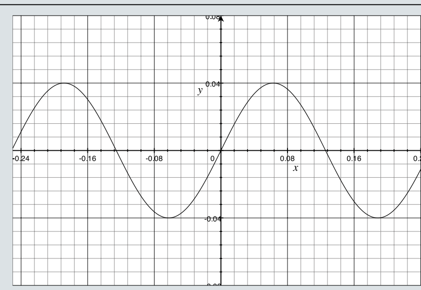
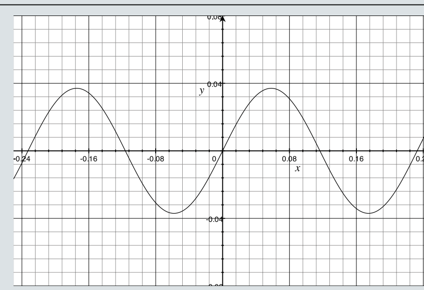
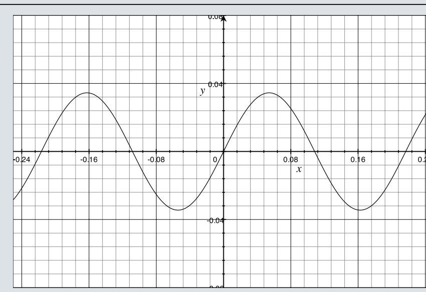
ACTIVITY 2

Answers: Fourier components for groups 2A through 2E – square wave

n=5, $\{-.75<x<.75, -.35<y<.35\}$ $y=\frac{1}{2\cdot 5+1}\sin((2\cdot 5+1)x)$	n=5 
n=6, $\{-.75<x<.75, -.35<y<.35\}$ $y=\frac{1}{2\cdot 6+1}\sin((2\cdot 6+1)x)$	n=6 
n=7, $\{-.75<x<.75, -.35<y<.35\}$ $y=\frac{1}{2\cdot 7+1}\sin((2\cdot 7+1)x)$	n=7 
n=8, $\{-.25<x<.25, -.08<y<.08\}$ $y=\frac{1}{2\cdot 8+1}\sin((2\cdot 8+1)x)$	n=8 
n=9, $\{-.25<x<.25, -.08<y<.08\}$ $y=\frac{1}{2\cdot 9+1}\sin((2\cdot 9+1)x)$	n=9 

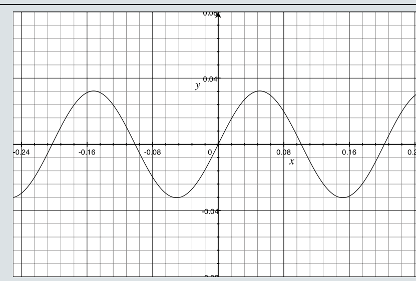
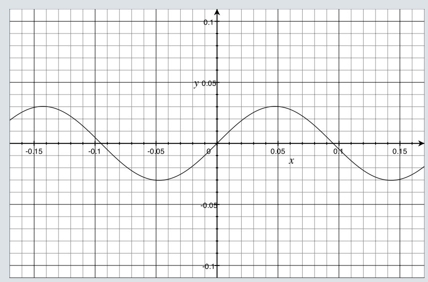
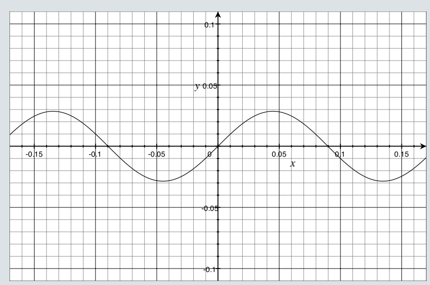
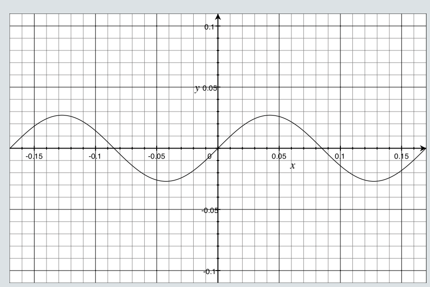
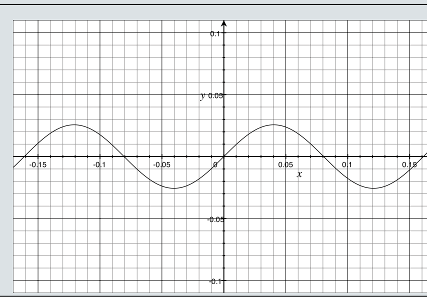
ACTIVITY 2

Answers: Fourier components for groups 2A through 2E – square wave

n=10, {-.25<x<.25, -.08<y<.08} $y=\frac{1}{2\cdot 10+1}\sin((2\cdot 10+1)x)$	n=10 
n=11, {-.25<x<.25, -.08<y<.08} $y=\frac{1}{2\cdot 11+1}\sin((2\cdot 11+1)x)$	n=11 
n=12, {-.25<x<.25, -.08<y<.08} $y=\frac{1}{2\cdot 12+1}\sin((2\cdot 12+1)x)$	n=12 
n=13, {-.25<x<.25, -.08<y<.08} $y=\frac{1}{2\cdot 13+1}\sin((2\cdot 13+1)x)$	n=13 
n=14, {-.25<x<.25, -.08<y<.08} $y=\frac{1}{2\cdot 14+1}\sin((2\cdot 14+1)x)$	n=14 

ACTIVITY 2

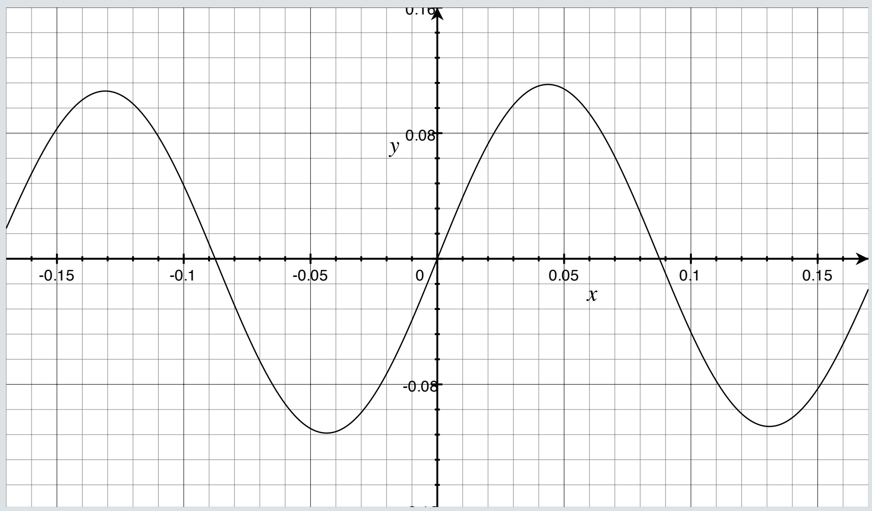
Answers: Fourier components for groups 2A through 2E – square wave

n=15, $\{-.25<x<.25, -.08<y<.08\}$ $y=\frac{1}{2\cdot 15+1}\sin((2\cdot 15+1)x)$	n=15 
n=16, $\{-.15<x<.15, -.05<y<.05\}$ $y=\frac{1}{2\cdot 16+1}\sin((2\cdot 16+1)x)$	n=16 
n=17, $\{-.15<x<.15, -.05<y<.05\}$ $y=\frac{1}{2\cdot 17+1}\sin((2\cdot 17+1)x)$	n=17 
n=18, $\{-.15<x<.15, -.05<y<.05\}$ $y=\frac{1}{2\cdot 18+1}\sin((2\cdot 18+1)x)$	n=18 
n=19, $\{-.15<x<.15, -.05<y<.05\}$ $y=\frac{1}{2\cdot 19+1}\sin((2\cdot 19+1)x)$	n=19 

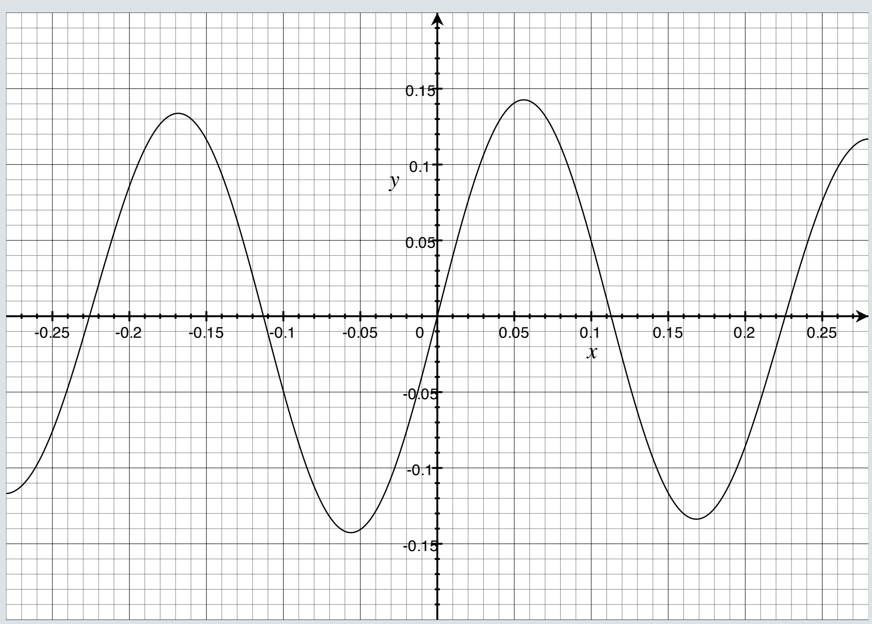
ACTIVITY 2

Fourier components – Group results:

Group 2A
n = 16 through n = 19, {-.15<x<.15, -.15<y<.15}



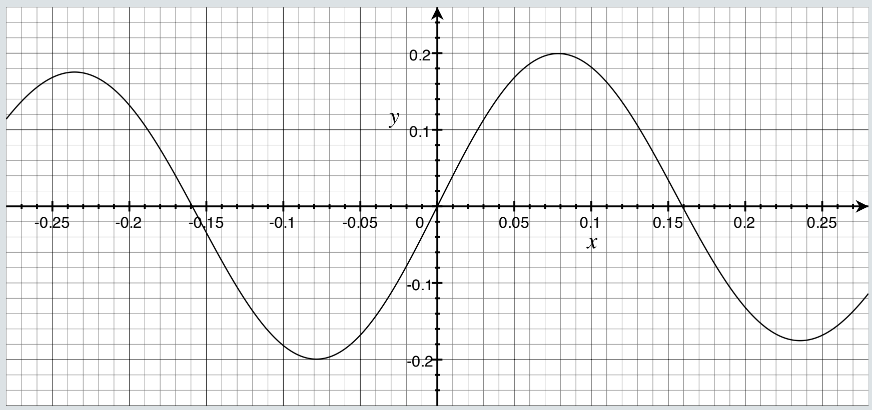
Group 2B
n = 12 through n = 15, {-.25<x<.25, -.2<y<.2}



ACTIVITY 2

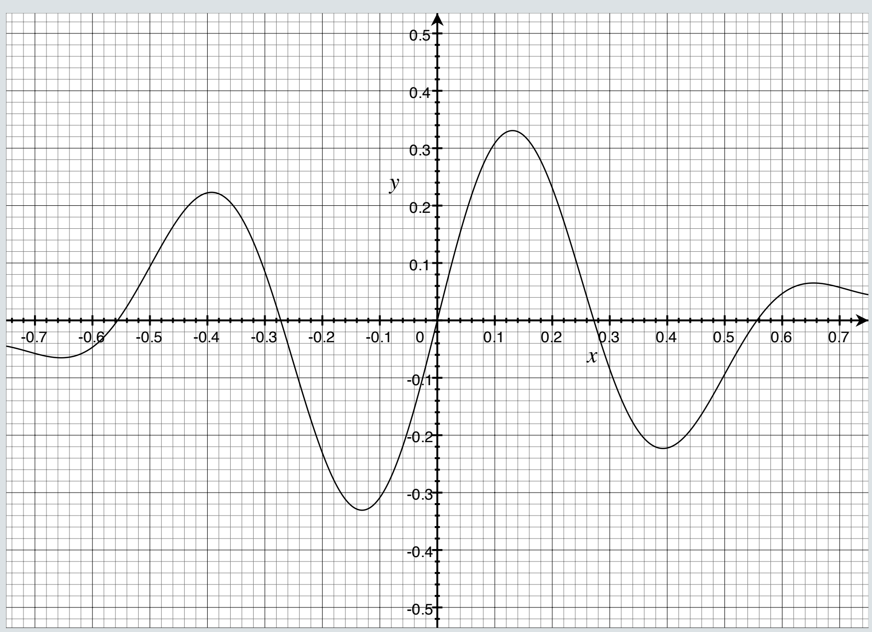
Group 2C

n = 8 through n = 11, {-.25<x<.25, -.25<y<.25}



Group 2D

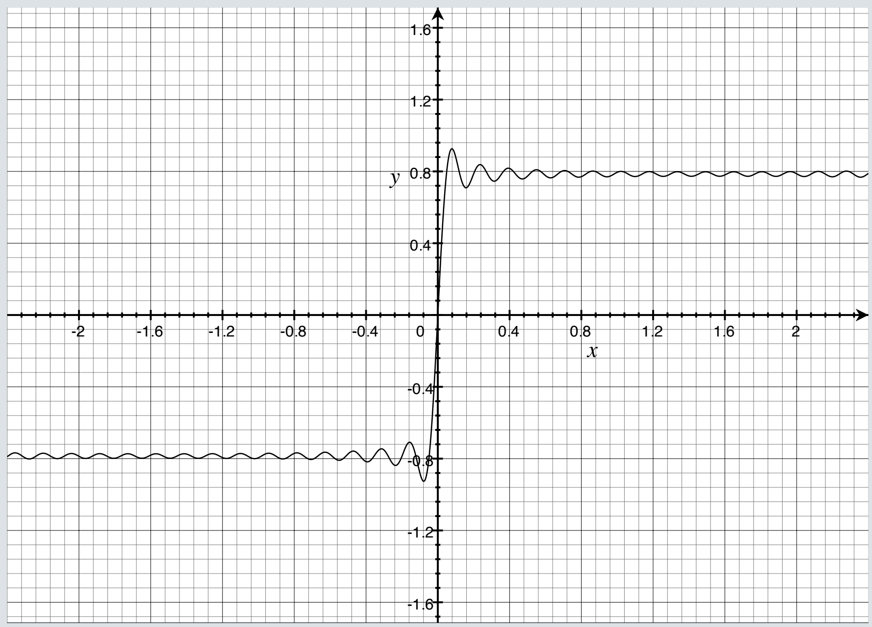
n = 4 through n = 7, {-.75<x<.75, -.5<y<.5}



ACTIVITY 2

Group 2E

n = 0 through n = 3, { -2.4<x<2.4, -1.6<y<1.6 }

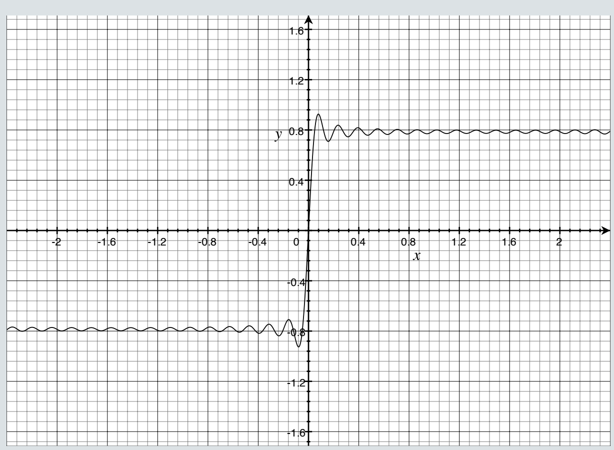


When all groups combine their data, the formula is this:

$$y = \sum_{n=0}^{19} \frac{1}{2n+1} \sin\left(\left(2n+1\right)x\right)$$

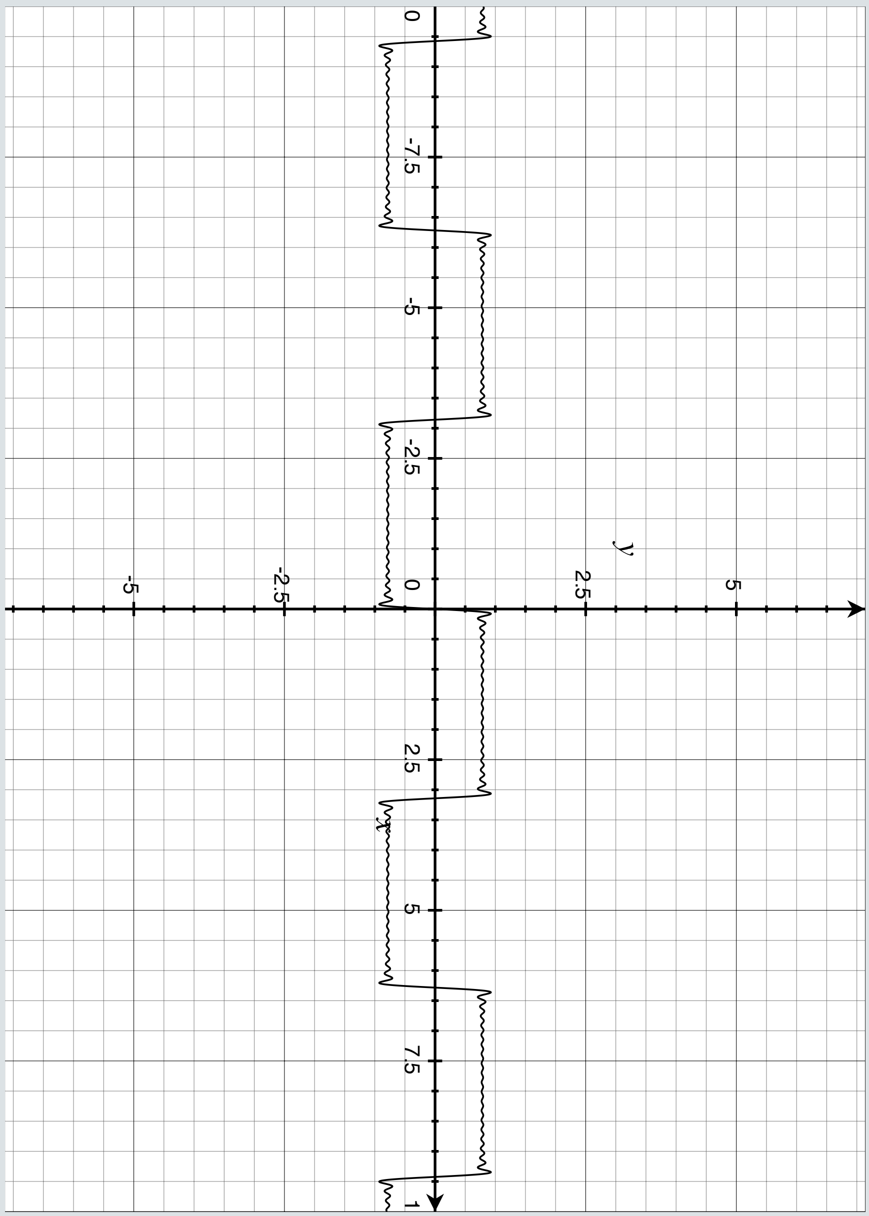
A graph could look like this:

{ -2.4<x<2.4, -1.6<y<1.6 }



ACTIVITY 2

If we zoom out, to the range and domain $\{-10<x<10, -7.5<y<7.5\}$, a graph could look like this:



ACTIVITY 2

Facilitator's note: Not all groups will have selected the same domain values to graph. Before the five composite waves can be added, you should come to a consensus as a large group as to which domain values to use. The best way to do this is to use the domain values from group C. The other groups will have to do a few more calculations so that every domain value of the sum of composite waves has a contribution from every group.

This demonstration will have the most impact if you emphasize the idea that you have taken 20 "curvy" waves and made a more-or-less "rectilinear" triangle wave. This demonstrates the power of Fourier analysis to represent ANY periodic function as a sum of sine waves.

Card Number 2:

Repeat the process you used for card number 1.

Group C's domain should again provide the domain for the final group sum. When you are finished, you should have a square wave!

ACTIVITY 2

B (10 minutes)

THINKING FURTHER

1. Look at the functions on the "number 1" cards. They should all be very similar except that one number is different in each. This number is the index value of the sum of waves that add together to make a triangle wave. Use summation notation to write the general formula for the sum of waves that form the triangle wave.

Answer:

$$\left[y = \sum_{n=0}^{n_{max}} \frac{(-1)^n}{(2n+1)^2} \sin\left(\left(2n+1\right)x\right)\right]$$

2. Do the same for the functions on the "number 2" cards.

Answer:

$$\left[y = \sum_{n=0}^{n_{max}} \frac{1}{2n+1} \sin\left(\left(2n+1\right)x\right)\right]$$

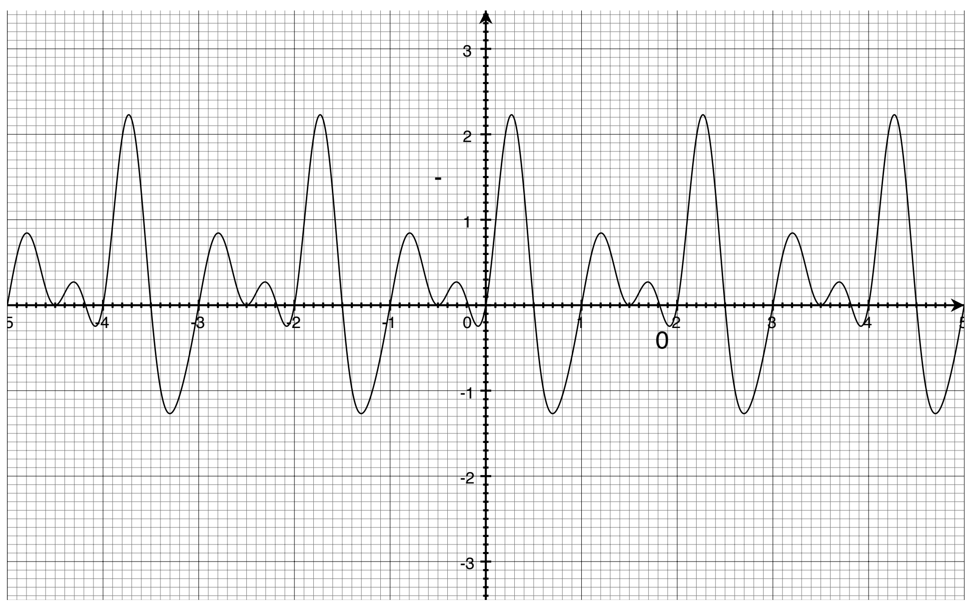
ACTIVITY 3

(40 minutes)

Facilitator's note: Be sure that both Activity 1 and Activity 2 are complete before attempting this.

The power of Fourier analysis lies in its ability to take a complicated periodic signal and break it down into the component frequencies that make it up. It's a bit like a professional taste-tester who can sample a complex sauce and tell the component ingredients that went into it—often he or she can even get a rough sense of the relative amounts of each component used. In this activity, you will be given a relatively complicated waveform and be asked to deduce its ingredient frequencies and their relative amounts, also known as Fourier coefficients. In essence, you will be deducing a complicated "recipe" by "tasting," and adding a dash of reason.

Take a look at this wave:



Call it Wave 1.

In the graph, t is the time, in seconds, and h is the height of the wave, in generic units.

Notice that this wave is neither a pure sine wave nor a pure cosine wave. It is actually a composite of three "ingredient" waves. To find the equations for the ingredient waves, we need to know three things: 1) whether they are sine or cosine waves; 2) the frequencies of the ingredient waves, and 3) the amplitudes of the ingredient waves.

ACTIVITY 3

To keep things simple, we will assume that the ingredient waves are sine waves.

At this point, have the participants work in groups.

PART 1: FINDING FREQUENCIES

1. Recall from the text that a general Fourier expansion looks like this:

$f(t) = a_0 + a_1\cos \omega t + b_1\sin \omega t + a_2\cos 2\omega t + b_2\sin 2\omega t + a_3\cos 3\omega t + b_3\sin 3\omega t + \ldots$

In this example we are concerned only with sine waves, so we can simplify the expansion to:

$f(t) = a_0 + b_1\sin \omega t + b_2\sin 2\omega t + b_3\sin 3\omega t + \ldots$

where ω is the fundamental angular velocity in radians/second. To start off, in your own words, explain the relationship between a wave's period and its fundamental angular velocity. Give an example.

Answer: ω is the angular velocity or how fast a wave oscillates. The period is the time it takes a wave to make one full oscillation, which is the same as saying it's the time it takes a "spoke" to sweep out an angle of 2π radians. One can find ω by dividing 2π by the period. For example, if a wave has a period of 10 seconds, ω will be $\frac{2\pi}{10} = \frac{\pi}{5}$ rad/sec.

2. Look at the graph of Wave 1. What is the period of this wave?

Answer: The period is the time it takes for the wave to complete one cycle. The x-axis is denominated in seconds, so one can see that it takes 2 seconds for Wave 1 to complete one cycle; therefore, the period is 2 seconds.

3. Note that ω is the fundamental frequency—the frequency of the first sine term in the Fourier expansion. The frequencies of all the other sine terms are whole number multiples of ω. Using the value you found for the period of Wave 1, find ω for Wave 1.

Answer: $\omega = 2\pi/\text{period} = \frac{2\pi}{2}$ secs $= \pi$ rad/sec

4. Write the first three terms of the Fourier expansion for Wave 1 using the value of ω you just found. Leave the coefficients as unknowns for the time being.

Answer: $f(t) = a_0 + b1\sin \pi t + b2\sin 2\pi t$

ACTIVITY 3

Now that you have found the frequencies of the ingredient waves, it's time to find the coefficients, which determine the amount that each ingredient wave contributes to Wave 1.

Convene the large group and discuss.

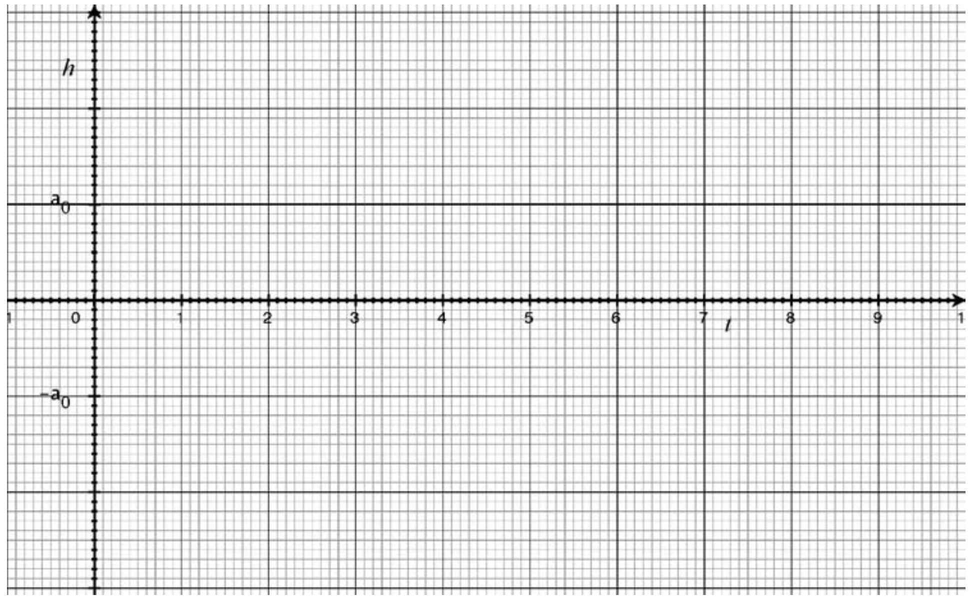
PART 2: FINDING COEFFICIENTS

Because there are three ingredient waves, we need to find three coefficients, a_0, b_1, and b_2. To find these, we need "filters" that can determine the effect that each has on Wave 1. Imagine that Wave 1 is a soup made of broth, large chunks of potatoes, and medium chunks of beef. To separate out the potatoes, we could run the soup through a strainer (a type of filter) that has holes large enough to let the beef chunks and broth pass through. Then to separate out the beef, we could run the now potato-less soup through another strainer, this one with holes small enough to detain the beef chunks and let only the broth pass through. Once the potatoes, beef, and broth have been separated, it is then easy to figure out the relative amounts of each.

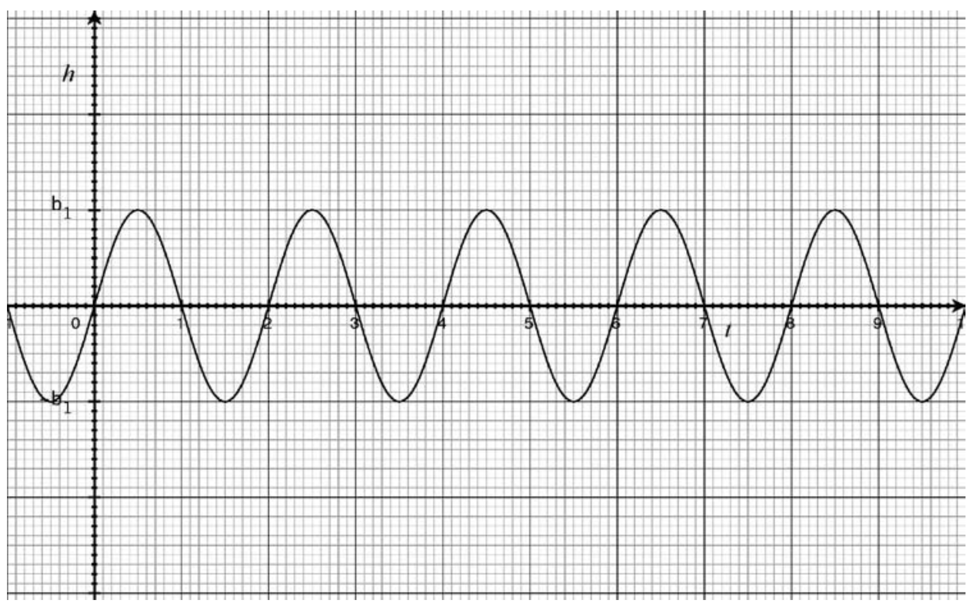
The first filter for Wave 1 should tell us what a_0 is. To make this filter, let's think about the area under a curve.

1. On a piece of graph paper, graph each of the three ingredient waves separately in the domain of [0,2] seconds. Label the vertical axes in terms of a_0, b_1, and b_2, respectively.
Answer: (the three ingredient waves shown on the following page)

ACTIVITY 3

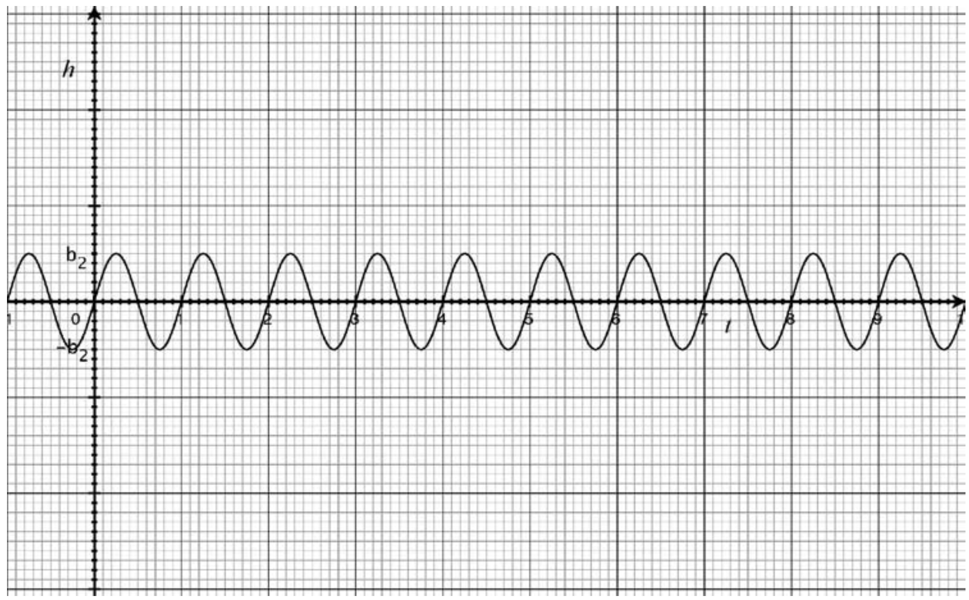


The h value is a_0.



The Amplitude is b_1.

ACTIVITY 3



The amplitude is b_2.

2. Find the areas underneath each curve in the given domain. Count area below the t-axis as negative.

(Note to those who might be tempted to use integration to find these areas: you do not need to use calculus for this; it can be done just by looking at the graphs.)
Answer: The area under $y = a_0$ is $2a_0$ (This can be found by looking at the rectangle, which is 2 seconds in length and a^0 in height.) The areas under both $y = b_1 \sin \pi x$ and $y = b_2 \sin 2\pi x$ are zero, because they have equal areas above and below the x-axis, which cancel out each other.

3. To find the area of a complicated shape, you can break it into simple shapes, find the area of each, and then add their areas together to find the area of the whole shape. Because Wave 1 is made up of three ingredient waves, only one of which, $y= a_0$, has any area, it stands to reason that any area under Wave 1 is equal to the area under $y = a_0$. Explain how this could be used to find the value of a_0.

Facilitator's note: Encourage participants to discuss this in their groups before writing an answer.
Answer: If we could find the area under Wave 1 for one period, this area must be equal to the area under $y = a_0$ for one period, which is in turn equal to a_0 x T, where T is the period. So, we can find the area under Wave 1 in one period and divide this area by the period to find a_0.

ACTIVITY 3

Convene the large group and make sure everyone understands the answer to the preceding question before continuing.

4. Find a_0.
Answer: The area under Wave 1 for one period is 2 unit-seconds. Divide this by the period of 2 seconds to get a_0 = 1 unit.

5. Write the Fourier expansion for Wave 1 using the information you have found so far.
Answer: $f(t) = 1 + b_1 \sin \pi t + b_2 \sin 2\pi t$

6. Now, to find b_1 and b_2, we're going to need two more filters. Unfortunately, we can't use the same "area" filter that we used to find a_0 because both b1sin πt and $b_2 \sin 2\pi t$ have areas of zero. However, we can use a trick. Look at your conclusion from Activity 1 regarding the areas under curves of products of waves having the same frequency and products of waves having different frequencies. Summarize your findings.
Answer: The product of two waves having the same frequency will yield a curve that has a non-zero area. The product of two waves having different frequencies will yield a curve that has zero area.

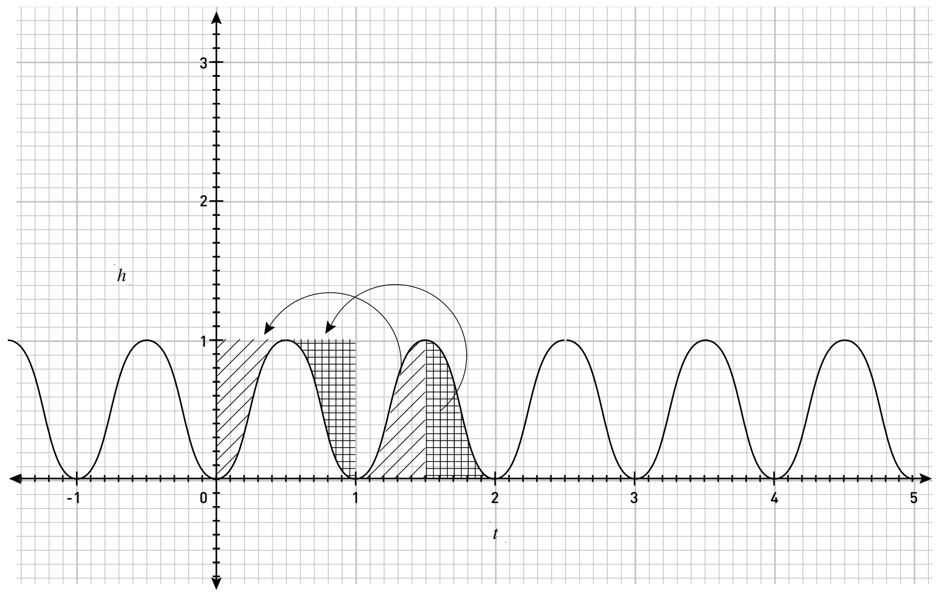
7. Write out f(t) x sin πt.
Answer: $f(t) \times \sin \pi t = (1 + b1 \sin \pi t + b_2 \sin 2\pi t)(\sin \pi t) = \sin \pi t + b_1(\sin \pi t)(\sin \pi t) + b_2(\sin 2\pi t)(\sin \pi t)$

8. For each term of f(t) x sin πt, discuss its contribution to the area under the curve over one period. There is no need to find exact values yet—just say whether or not the term contributes to the area.
Answer: sin πt has an area of zero; $b_1(\sin \pi t)(\sin \pi t)$ has a non-zero area; and $b_2(\sin 2\pi t)(\sin \pi t)$ also has an area of zero. The only term that contributes any area to f(t) x sin πt is $b_1(\sin \pi t)(\sin \pi t)$.

9. Look at the graph of y = (sin πt)(sin πt). How does the period of this wave compare to the period of y = sin πt? How does it compare to the period of f(t)?

ACTIVITY 3

Graph: h = (sin πt)(sin πt)



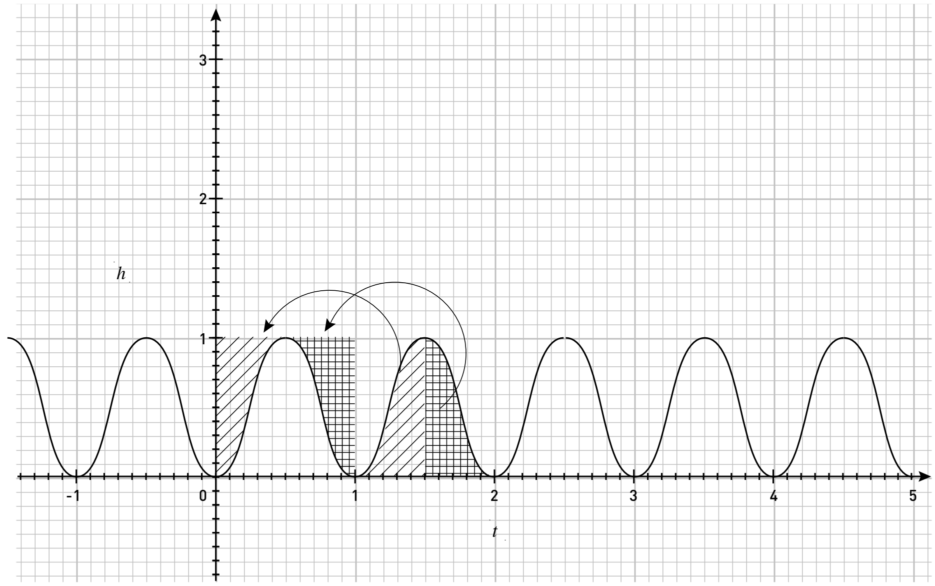
Answer: The period is half that of sin πt. It is also half the period of f(t)

10. What is the area under h = (sin πt)(sin πt) from t = 0 to t = 2 seconds (one period of f(t)?

Hint 1: Think of a jigsaw puzzle—no calculus is required! Try to make a rectangle.

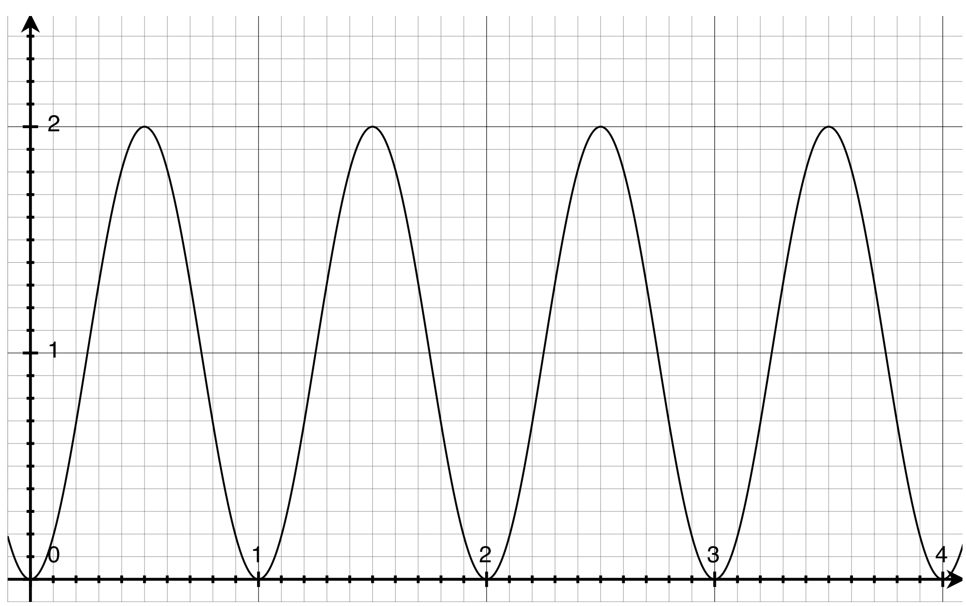
Answer: The area is 1 unit-second. To find this, leave the "hump" between 0 and 1 second. Take the "hump" between 1 and 2 seconds and split it in half along its vertical axis of symmetry. Fit these two pieces (you'll need to turn them upside down first) into the interval from 0 to 1 second to make a 1 x 1 square.

ACTIVITY 3



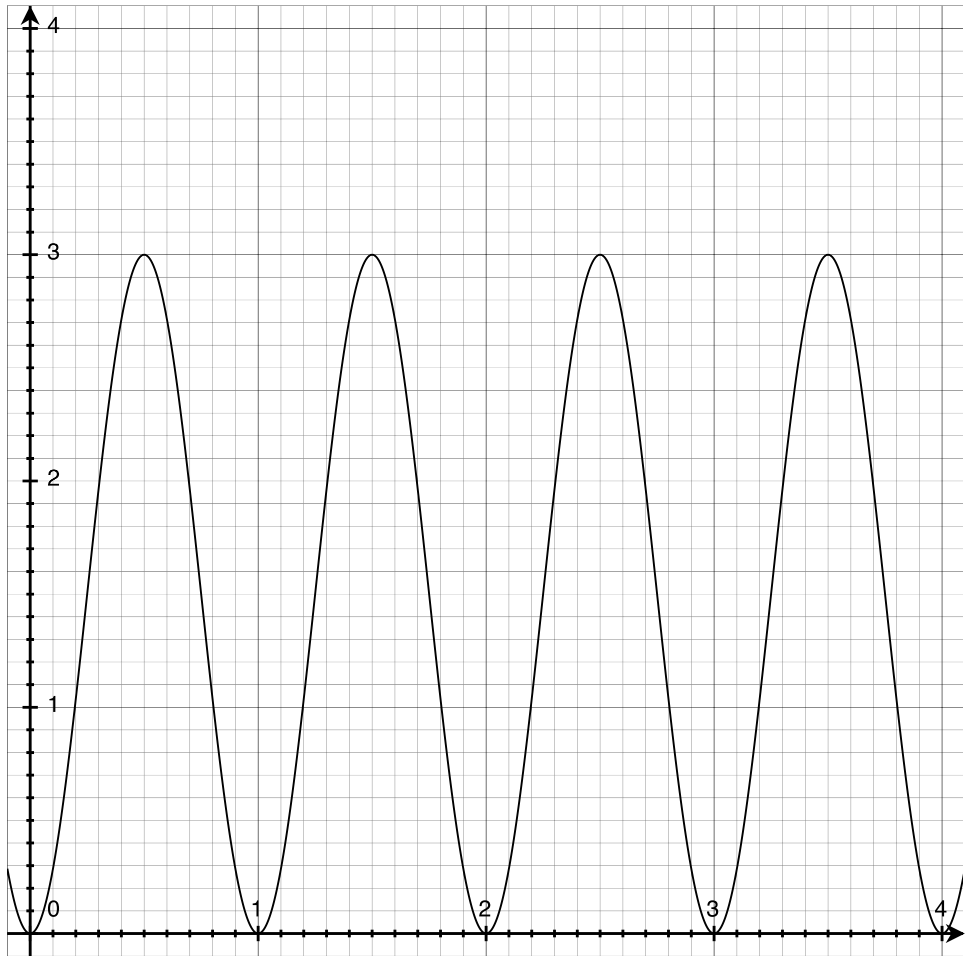
11. What is the area under h = 2(sin πt)(sin πt)? How about the area under h = 3(sin πt)(sin πt)?

Graph: h = 2(sin πt)(sin πt)



ACTIVITY 3

Graph: h = 3(sin πt)(sin πt)



Answer: 2 unit-seconds for the first and 3 unit-seconds for the second, by the method used in the previous problem.

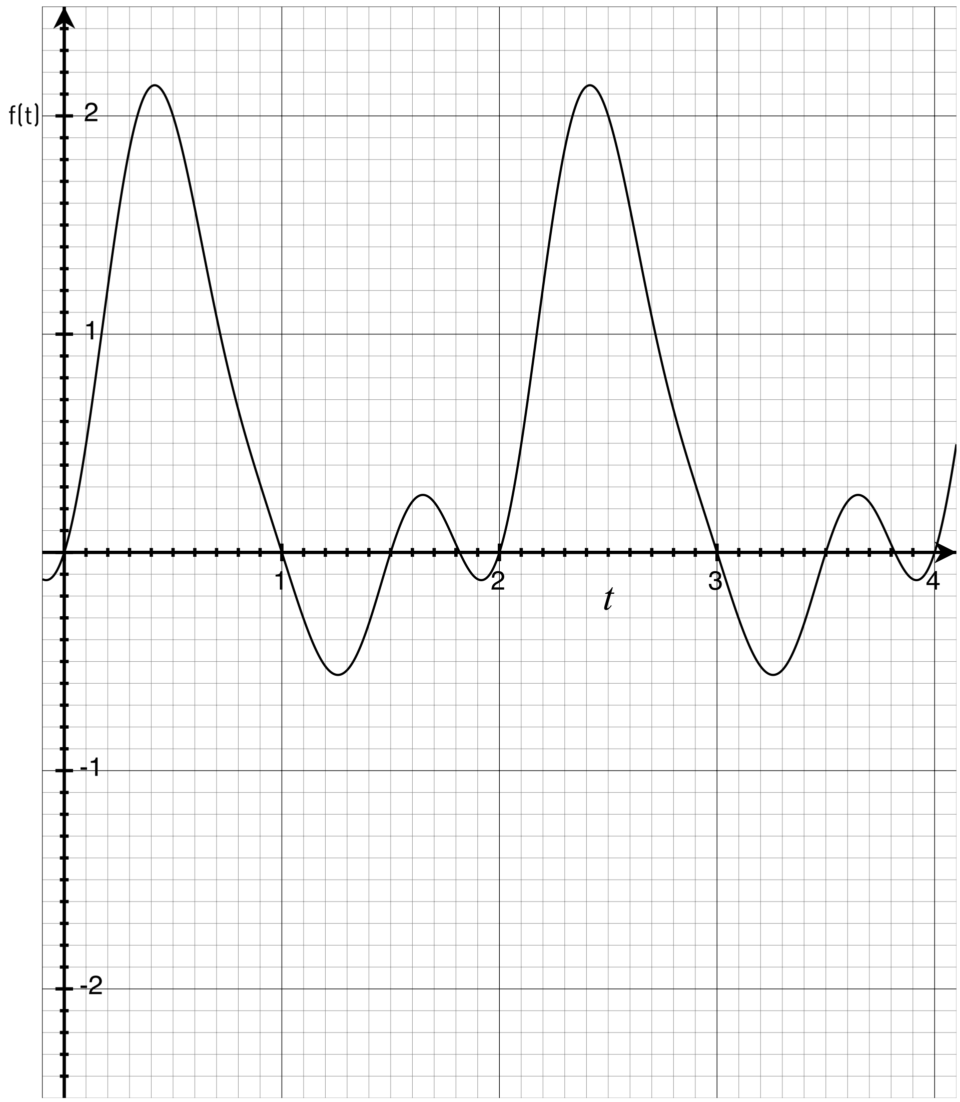
12. What is the relationship between n and the area underneath h = n(sin πt)(sin πt)?
Answer: The two quantities are the same.

13. So, the area underneath f(t) × sin πt is entirely due to the contribution from b_1(sin πt)(sin πt). Also, as you just discovered, the area under b_1(sin πt)(sin πt) is equal to b_1. Use these two pieces of information, in conjunction with the given graph of f(t) × sin πt, to find b_1.

Hint 2: To find the area under f(t) x sin πt, just count squares!

ACTIVITY 3

Graph: $h = (\sin \pi t)(1 + b_1 \sin \pi t + b_2 \sin 2\pi t)$



Answer: The area under $h = (\sin \pi t)(1 + b_1 \sin \pi t + b_2 \sin 2\pi t)$ is equal to 1 unit-second. This means that b_1 equals 1.

14. Now that you have b1, write out the Fourier expansion of Wave 1 so far:
Answer: $f(t) = 1 + (1)\sin \pi t + b_2 \sin 2\pi t$

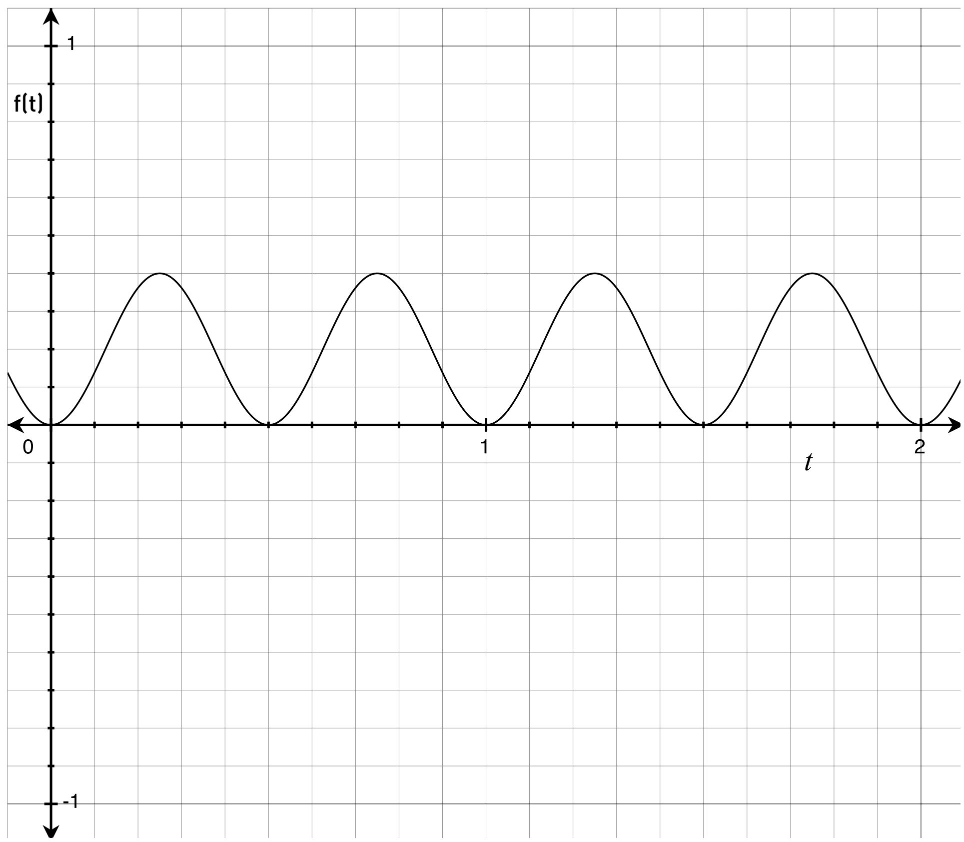
ACTIVITY 3

15. So, if the filter to find b_1 was to multiply f(t) by sin π and find the area, by what should we multiply f(t) to find b_2? Why?

Answer: We should multiply f(t) by sin $2\pi t$, because it will make all the terms except for $b_2(\sin 2\pi t)(\sin 2\pi t)$ give an area of zero.

16. Before you find b_2, take a look at the graph of $h = \left(\frac{2}{5}\right)(\sin 2\pi t)(\sin 2\pi t)$.

Graph of $h = \left(\frac{2}{5}\right)(\sin 2\pi t)(\sin 2\pi t)$.



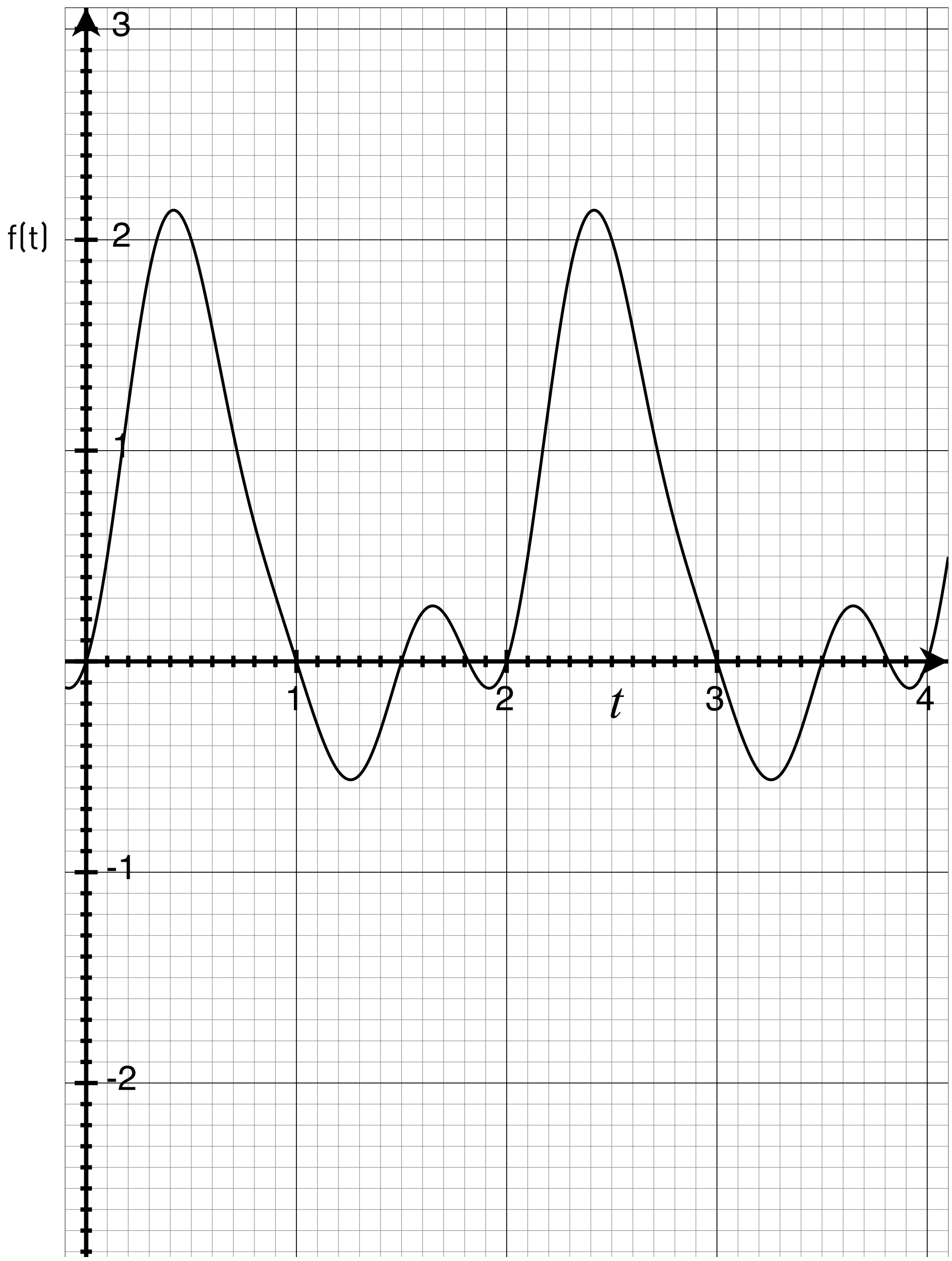
ACTIVITY 3

17. What is the area under this curve for the period of f(t) (0->2 secs)? Use the “jigsaw” method.

Answer: The area is $\frac{2}{5}$ of a unit-second.

18. Use the graph of f(t) × sin $2\pi t$ to find b_2.

Graph of h = (sin $2\pi t$)(1 + $b_1$sin πt + $b_2$sin $2\pi t$)



ACTIVITY 3

Answer: By counting squares, the participants should find that the area under $h = (\sin 2\pi t)(1 + b_1 \sin \pi t + b_2 \sin 2\pi t)$ is ½ of a unit-second. This makes $b_1 = ½$.

19. At last, you finally have all of the information necessary to write the complete Fourier expansion of f(t). Do it!
Answer: $f(t) = 1 + (1)\sin \pi t + ½ \sin 2\pi t$

You now know the frequencies and amounts of all the ingredient waves—in other words, you have the complete recipe to make Wave 1!

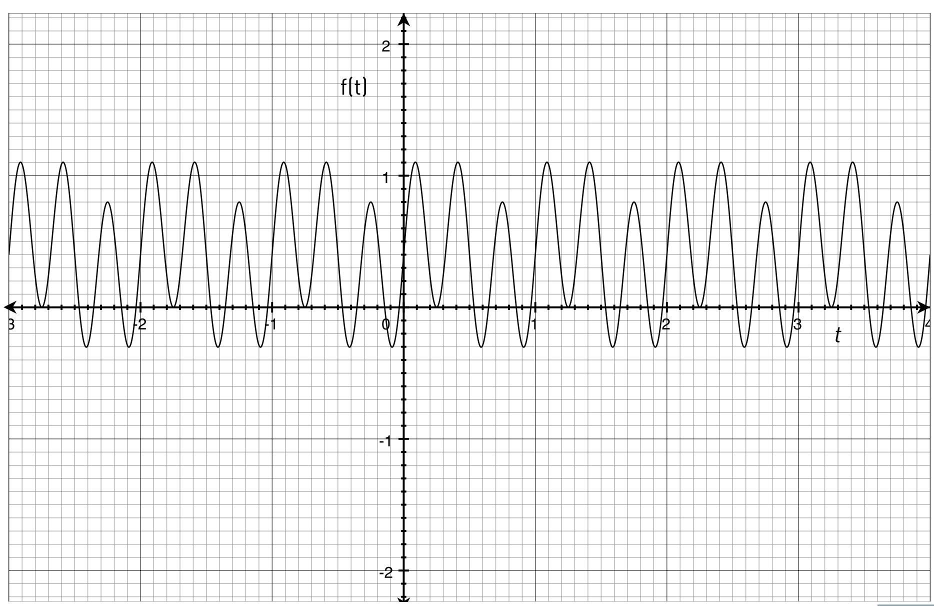
IF TIME ALLOWS:

Use the following graphs to find the "recipe" for Wave 2. Start with four terms of the general Fourier expansion (using only sine waves again).

$f(t) = a_0 + b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t$

Be careful when finding b_1, b_2, and b_3; the fundamental frequency is different in this example, which has implications for the "area" filters.

Facilitator's note: to find the b_n, the participants will have to find the areas of the various product graphs over one period and then divide the result by ½ of a period. The period is 1 second. The fundamental frequency is 2π. The answer is: $f(t) = \left(\frac{2}{5}\right) + \left(\frac{1}{5}\right) \sin 2\pi t + (0)\sin 4\pi t + \left(\frac{3}{5}\right) \sin 6\pi t$.

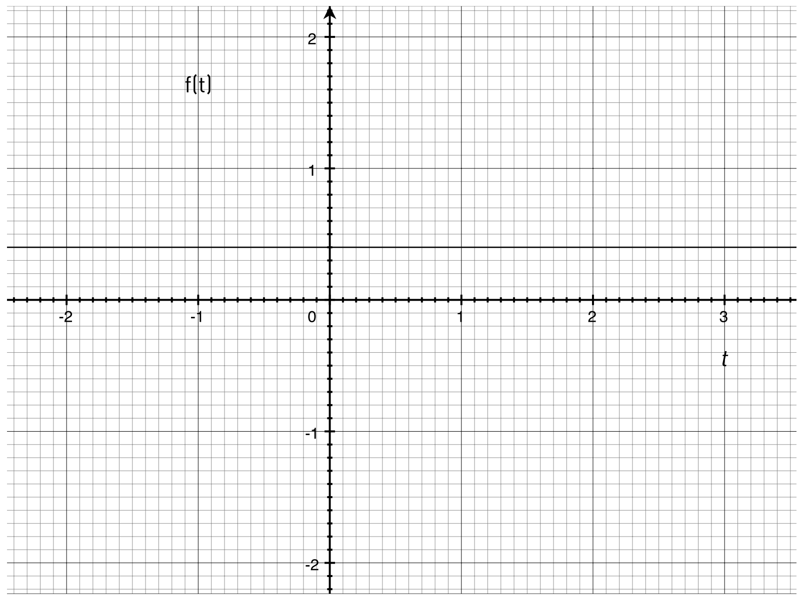


ACTIVITY 3

Let's call it Wave 2.

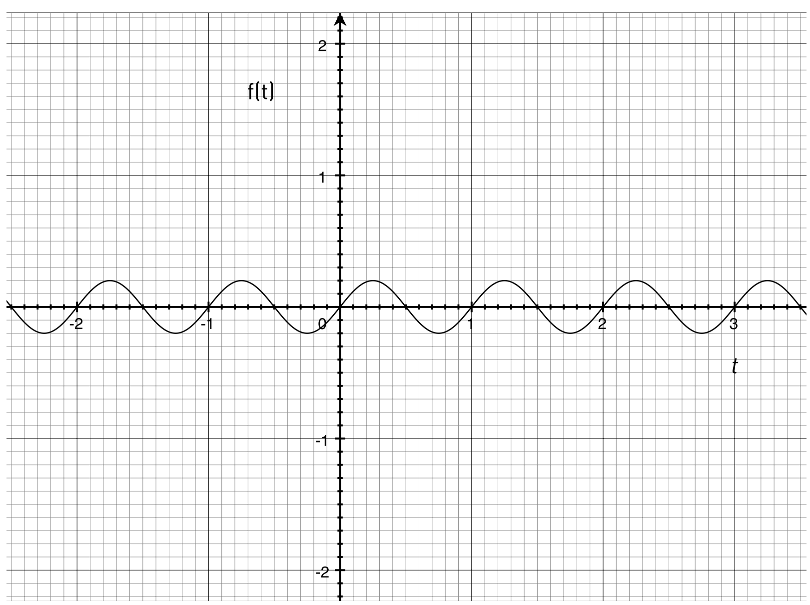
Answer: Ingredient Waves

Graph: $h = \left(\frac{2}{5}\right)$



The h value is a_0

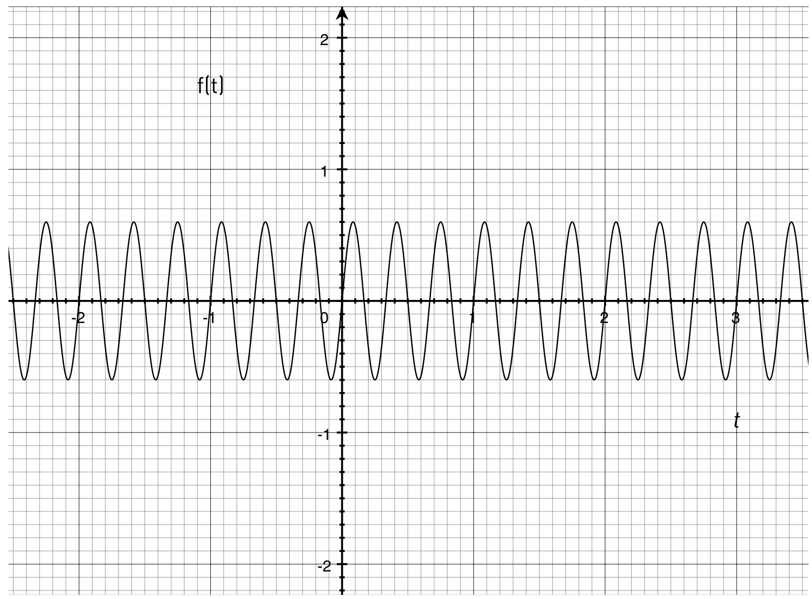
Graph: $h = \left(\frac{1}{5}\right) \sin 2\pi t$



Amplitude is b_1

ACTIVITY 3

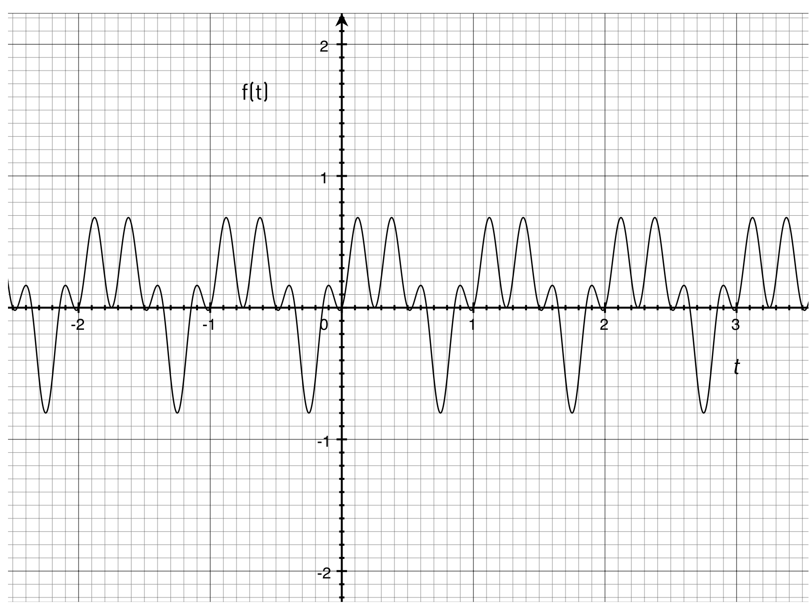
Graph: $h = \left(\frac{3}{5}\right) \sin 6\pi t$



The amplitude is b_2.

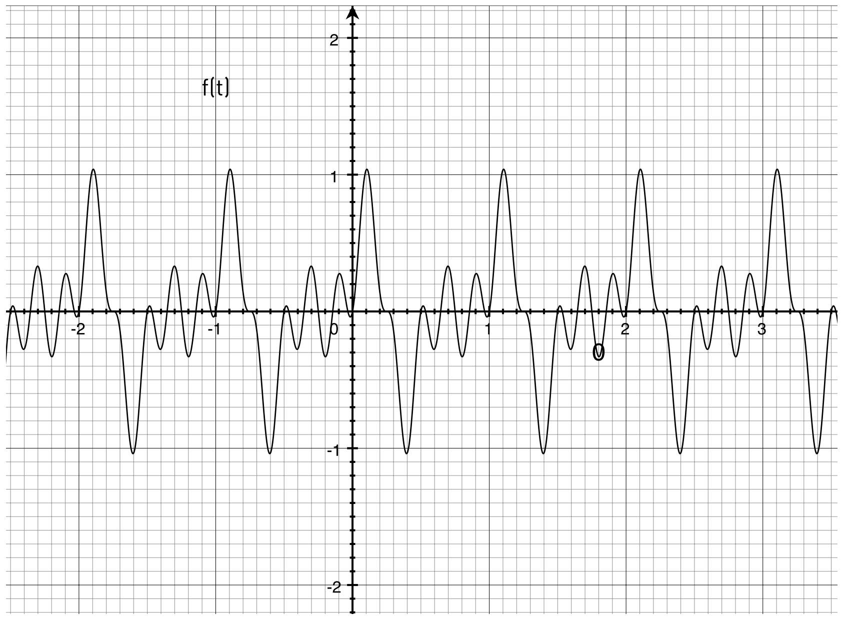
Participant, Product Graphs:

Graph: $f(t) \times \sin 2\pi t$

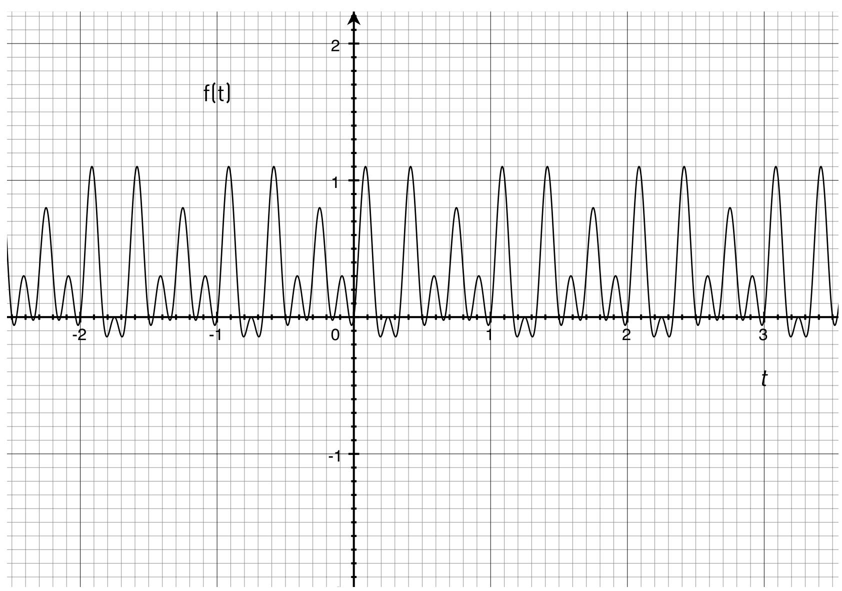


ACTIVITY 3

Graph: f(t) x sin 4πt



Graph: f(t) x sin 6πt



CONCLUSION (30 minutes)

DISCUSSION

HOW TO RELATE TOPICS IN THIS UNIT TO STATE OR NATIONAL STANDARDS

Facilitator's note:
Have copies of national, state, or district mathematics standards available.

Mathematics Illuminated gives an overview of what students can expect when they leave the study of secondary mathematics and continue on into college. While the specific topics may not be applicable to state or national standards as a whole, there are many connections that can be made to the ideas that your students wrestle with in both middle school and high school math. For example, in Unit 12, In Sync, the relationship between slope and calculus is discussed.

Please take some time with your group to brainstorm how ideas from Unit 10, Harmonious Math could be related and brought into your classroom.

Questions to consider:

Which parts of this unit seem accessible to my students with no "frontloading?"

Which parts would be interesting, but might require some amount of preparation?

Which parts seem as if they would be overwhelming or intimidating to students?

How does the material in this unit compare to state or national standards?
Are there any overlaps?

How might certain ideas from this unit be modified to be relevant to your curriculum?

WATCH VIDEO FOR NEXT CLASS (30 minutes)

Please use the last 30 minutes of class to watch the video for the next unit: Connecting with Networks. Workshop participants are expected to read the accompanying text for Unit 11, Connecting with Networks before the next session.

NOTES