PROGRAM: 13
The Concepts of Chaos

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Host: Dan Rockmore

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Time Code	Audio
00:00	OPENING CREDITS
00:40	HOST: Most of us learned at an early age how an apple falling from a tree...
00:44	HOST ...inspired Isaac Newton to describe how the universe behaves by certain predictable rules. But what about when the universe doesn't behave so... predictably?
00:56	HOST: Can mathematics explain the often unpredictable behavior of the physical world --
01:01	HOST: Everything from the weather to ...
01:04	HOST: ...the way a baseball travels through air?
01:08	HOST: Welcome to Chaos Theory.
01:15	RED: Jake's fastball's comin' in about ninety-four. POPS Yep.
01:23	RED: An object at rest will remain at rest...
01:27	POPS: ...unless acted upon by an external and unbalanced force. Newton on your mind again? RED: Yep.
01:36	RED: I've always been partial to his First Law of Motion. POPS: Uh huh. Lots of forces act on a baseball. Gravity, friction...turbulence.
01:50	RED: Turbulence?
01:52	POPS: Knuckleball.
01:58	POPS: Some things, you just can't predict.
02:02	RED:

	Still, every action has a reaction –
02:12	HOST: Pops and Red are discussing an age old dilemma - why in nature, some things like a baseball, can behave both predictably and unpredictably. And it's an exploration that began in earnest and one that used mathematics, with Isaac Newton and his apple in the late 17th Century.
02:29	**HOST (V.O.):** **Newton's revelation about gravity led him to define a set of rules about how the physical world operated, which he published in 1687. His laws were, in fact, so precise that they seemed to describe the world as perfectly balanced -- behaving like "clockwork." Newton's Laws would influence all the sciences, as well as the arts, religion and philosophy for centuries to come -- essentially framing how civilized man understands the universe.**
02:56	HOST: The philosophical movement that arose from Newton's mathematical concepts came to be called Determinism; the belief that future events are necessitated by past and present events, in combination with the laws of nature. Now, some scholars even took this one step further suggesting that once the initial conditions of the universe were set, the rest of the history was inevitable. This is one of those examples where a little bit of mathematics in the wrong hands can be dangerous. But on the other hand the ideas that there's a clockwork universe or that there's a regularity to the world as described by Newton's laws, is in fact something that we know to be true for much of physical phenomena.
03:37	HOST: So, with his Three Laws of Motion -- the laws of inertia, acceleration and reciprocal actions -- Newton was the first to mathematically describe how an object's motion through space and time can be calculated by adding up the infinitesimal changes in its path.
~~*03:55*~~	HOST: In other words: Newton showed us how to use mathematics to predict an object's motion from instant to instant -- given such quantities as acceleration, mass, and gravitational pull.
04:06	HOST: Newton's ideas were revolutionary – an entirely new way of thinking about the universe. After all, he showed us how we can use the mathematics of differential equations to predict the future! Differential equations as a mathematical crystal ball.
04:21	HOST: Let's look at a simple example. Suppose we have an object "X"

	traveling around object "Y", solely under the influence of gravity. Then we know exactly where "X" will be in space ten seconds or ten thousand years from now. Newton's theory efficiently described the interactions of what was called the two-body system, answering the question: what occurs when the forces of 2 masses affect each other.
04:50	HOST: Now we see the power of these ideas some two hundred years later with the advent of more powerful telescopes.
04:56	**HOST (V.O.):** **At *that* time, astronomers noticed that some planets were not following the perfect, Newtonian elliptical orbits -- especially the planet Uranus. So, they theorized that its orbit must have been "upset" by some other body, and then using differential calculus, they were able to actually calculate the orbit of the unknown orb.**
05:16	**HOST (V.O.):** **The mathematicians said: "Point your telescope here and you will find it." And so, in 1846, there it was. Neptune became the first planet to be discovered based on mathematical prediction rather than mere observation.**
05:31	**HOST (V.O.):** **The late 19th century was, in fact, alive with such scientific triumphs -- and royal prizes were offered for solutions to the most challenging mathematical problems. One such challenge was introduced by the King of Sweden in 1888. He offered a prize to anyone who could solve the so-called three-body problem. In layman's terms, the King asked: "Does Newton's two-body solution, the simple elliptical motions of a single planet around a massive sun—can we do the similar prediction for more than two bodies?"**
06:03	**HOST (V.O.):** **One of the greatest mathematicians and scientists of all time took up the King's challenge: Jules Henri Poincare:**
06:11	HOST: Now, Poincare tried to find a closed form solution to the famous problem, using those differential equations. Again essentially looking for a formula like one for an ellipse to describe the motion of several planets all under the influence of gravity. Although Poincare didn't succeed, he came close enough that the King awarded him the prize anyway -- because his explorations had made a significant contribution to classical mechanics. Besides adding a lot of mathematics.
06:38	HOST: However, when a referee asked for a clarification, Poincare discovered

	an error, and the great mathematician went back to the drawing board...
06:50	HOST: Unable to solve the problem as it was originally posed, Poincare made up his own: will the solar system always stay together or will it fall apart?
07:01	HOST: What Poincare discovered was that for more than two bodies certain initial conditions could lead to chaos. You see, with two bodies and Newton's equations, basically only two things could happen. Either the two objects could move apart to infinity, essentially like a meteor passing by the planet…or we could get the familiar old periodic solutions of one planet orbiting around another one. And most importantly, changing the starting position slightly wouldn't change the behavior. But add some more objects to the situation and suddenly all bets were off. Other wild, non-periodic behaviors could happen and moreover slight changes in the starting position could cause great changes in the long term effects. Although roughly the bodies would still move in approximately the same region of space.
07:50	HOST: Poincare's discovery was astounding, but even more the way in which he made this discovery was at least as important as the discovery itself. His epiphany was that the system of equations could be approached visually.
08:03	HOST: To understand what Poincare did, we have to understand a bit more about problem: In classical mechanics, an object's position is recorded using its location in three dimensions: the so-called X, Y, and Z coordinates. When that object is moving, its velocity along each of those axis is also noted: velocity X, velocity Y, and velocity Z.
08:27	HOST: When a second object is added to the system, six more variables must now be calculated. As additional objects are added, tracking all of the variables can be completely unwieldy especially if the calculations are difficult. Still like clockwork. Just a lot of moving parts. What mathematicians call a deterministic dynamical system.
08:49	**HOST (V.O.):** **What Poincare "saw" was, that if we look beyond the numbers that govern their orbits, we would find that the entire system could be viewed as a single point moving through a multidimensional space of very high dimension. We now call this "phase space."**
09:04	**HOST (V.O.):**

	And what the picture told him was that the system as a whole would behave unpredictably, like various leaves floating down a stream.
09:15	HOST: Poincare's revelations put a crack in the foundation of Newton's clockwork universe, and paved the way to a theory of chaos.
09:22	HOST: But no one would really explain the *unpredictability* that Poincare's calculations hinted at until midway through the 20th Century – when one man stumbled on an explanation while using computers to examine a problem much more complex than three bodies moving through phase space: weather patterns on the planet earth.
09:42	**HOST (V.O.):** **At M.I.T the early 1960's, Dr. Edward Lorenz built a simplified mathematical model of the way air moves in the Earth's atmosphere. Working with a twelve-variable computerized weather model, Lorenz repeated a calculation involving a numerical representation of a slightly shifting wind condition. To save computation time he began the simulation in the middle of its course - but entered the data by rounding off the original 6- digit variables to just 3 digits: an adjustment of just one-one thousandth from the original setting.**
10:16	**HOST (V.O.):** **But to his surprise, the weather that the computer predicted on this new run, using these slightly different intermediate values, was completely different than his earlier simulations. Lorenz expected that the miniscule difference would have practically no effect.**
10:32	**HOST (V.O.):** **By iterating this slightly altered calculation, Lorenz realized that minute variations in the initial values in his weather model could result in widely divergent weather patterns.**
10:42	**HOST (V.O.):** **Remember Newton showed how an object's motion through space could be predicted by calculating the infinitesimal changes in its path?**
10:50	**HOST (V.O.):** **Now Lorenz was showing that for certain equations an infinitesimal change in the data could end in a highly unpredictable result.**
11:00	HOST: That's what we mean by sensitive dependence on initial conditions: the error -- the distance between the two "trajectories" -- grows exponentially fast. The fact that small changes in initial conditions

	produce large changes in the long-term outcome is the trademark of a chaotic system, and is called "sensitivity to initial conditions," or "sensitive dependence."
11:23	HOST: And this process of iteration can be thought of as an amplifier, or mechanism that reveals sensitive dependence.
11:30	**HOST (V.O.):** **Using the computer, which made it possible to rapidly repeat and accumulate the infinitesimally small changes over and over again, Lorenz was able to actually chart this sensitivity.**
11:42	**HOST (V.O.):** **The resulting graph, called the Lorenz Attractor, is in fact a phase space representation of this simplified model of the weather. The graph's similarity to the shape of a butterfly caught on as a way to illustrate the concept of "sensitive dependence."**
11:58	HOST: So it is magical coincidence that we explain this by sometimes saying that a butterfly flapping its wings in China might cause an infinitesimal change in wind current that could lead to a hurricane in Florida several months later: the "Butterfly Effect."
12:13	POPS: First butterfly of the season. Kinda warms the heart, don't you think? Did I ever tell you about how chaos theory's connected to heart dynamics...?
12:23	RED: Think it's going to rain? POPS Feels humid, all right. Good air for a knuckleball...
12:30	RED: That wouldn't have something to do with "Turbulence", would it? POPS: Well, a ball that spins, say, a fastball if thrown correct, can take a more predictable uh, Newtonian path as it moves through the air.
12:43	RED: But a ball that doesn't spin? POPS: Well, that's where celestial mechanics meets chaos theory
12:57	Dan Rockmore: Baseball and mathematics, two great American pastimes. So we're here today with Steve Strogatz…
13:03	Rockmore: …author of the book Nonlinear Dynamics in Chaos and also a professor

	of Theoretical and Applied Mechanics at Cornell University.
13:10	Rockmore: And Steve's going to help us make sense out of chaos on the baseball field.
13:14	Rockmore: Steve, ready?
13:15	Steve Strogatz: Hey, you know, your friends, Pops and Red are having a pretty interesting conversation out there.
13:19	Rockmore: They're quite the grizzled old philosophers.
13:21	Strogatz: So the thing about a knuckle ball, you know, is that
13:23	Strogatz: --that the pitcher grips it with two fingers in this flat part here.
13:29	Strogatz: And the -- and the trick is to throw it so that it has very little spin.
13:32	Strogatz: And what happens then is that the -- the airflow around the -- the pitch starts to create vortices, little whirlpools of air behind the ball…
13:41	Rockmore: It's actually almost pushing the ball forward, is that right?
13:44	Strogatz: Yeah, sort of pushing. Well, I mean, mainly the ball has its own inertia carrying it forward, but these whirlpools do
13:49	Strogatz: -- If they're -- in -- in the wake of knuckleball sort of push it around in a funny and unpredictable way.
13:56	Strogatz: Whereas a fastball, which would have much more spin, has a much more predicable wake and it leads to a -- a pitch that's more predictable, except of course it's fast.
14:04	Rockmore: I mean, so -- wake, so just like a boat moving through the water, you see something behind it and that's the wake.
14:09	Rockmore: And that's exactly what this is doing in the air. And depending on whether or not it's spinning, you get different wake patterns in the back, is that right?
14:16	Strogatz: That's right. And so the -- the chaotic wake behind the -- the knuckleball is more turbulent and it makes the pitch less predictable, in the sense that the next time the pitcher throws it, even if he just

	changes the angle a little bit, of release or the speed, it'll end up looking like a totally different pitch.
14:31	Rockmore: So that's an example of the sensitive dependence on initial conditions that we're going to be hearing about.
14:35	Strogatz: Right. It's -- it's chaos at work on the baseball field.
14:37	Strogatz: Just a tiny change in the pitch makes a big change by the time it crosses the plate.
14:42	Rockmore: These slight changes with the knuckleball are going to affect, well, very different paths that still end up at home plate, or somewhere around home plate.
14:50	Strogatz: Right. Yeah, that's the idea. And so what we want to try to understand a little better is how is it that something that starts like a tiny difference…
14:58	Strogatz: …how can that tiny -- those tiny differences get amplified and grow and grow exponentially fast, leading to very different outcomes. And so it seems like one way that we might want to look at it is what we're showing here on the screen now. We could take a look at just doing this with numbers. Okay? Not with…
15:14	Rockmore: Very -- very simple numerical examples…
15:17	Strogatz: Yeah, just the number line that you've been using.
15:18	Rockmore: …complicated physical phenomena. Right? Strogatz: Right. So let's just focus on numbers, and we'll do a certain operation on the numbers, which is to… Rockmore: So this number is between zero and one. We only have that piece of the -- of the lines -- on the line. Strogatz: Yeah. So not the whole number line. Just start from zero to one and pick some number in there. Let's say .632.
15:34	Rockmore: Okay. Strogatz: Okay. And you could imagine that there's more digits after that, if you want.
15:37	Strogatz:

	But let's just .632 and then we multiply by ten. This is the operation we're going to do. And so then we would get the number 6.32. And what I want to do next is an operation that a mathematician calls Mod One, which means just drop off the number before the decimal point. So a 6.32 would then become just .32. Rockmore: "Mod 1"
16:02	Strogatz: OK, now what I want to do is compare that to what would have happened if we had originally started with .633; making only a difference in the one thousandth place, right? I mean, in science, normally if you have something that's good to three digits, you'd say that's pretty good.
16:17	Rockmore: I'm done.
16:18	Strogatz: Okay. Yeah, so but watch what happens if we do our operation on .633. Then, we multiply by ten and we get .33 after… Rockmore: Right. We multiply by ten and we lob off the integer.
16:29	Strogatz: But that differs from our original number. Not in the thousandths place, but now in…
16:34	Strogatz: …the hundredths place. We've amplified the error, the difference, by ten. Rockmore: Right.
16:38	Strogatz: Okay. And if we did this one more time…
16:39	Rockmore: Right. So we're doing both of them. We multiply them both by ten. We lop off the integer and now we compare.
16:44	Strogatz: Yeah. Rockmore: And now they differ just in the tenths place. There's another multiple of ten in the error.
16:48	Strogatz: Yeah. So that's the thing that the -- this -- the different between these two outcomes is growing by a factor of ten. It's growing exponentially fast as we go forward in time. And that's what happens in chaotic systems. But you might think -- and so this is the other interesting point, why we need the mod. That the error, if it kept growing

	exponentially, that would be like one of the baseball pitches going out of the stadium. That doesn't happen here, because the mod keeps things bounded to always lie between zero and one.
17:14	Rockmore: Right. Always on our little segment here. Strogatz: Yeah. Always stays in our segment, eventually.
17:17	Strogatz: So things can get far apart, but not too far apart, and that's what we see in chaotic systems too.
17:22	Rockmore: So this an interesting point that I'm -- again, people think of chaos as, you know, totally unpredictable, uncontrollable…
17:28	Strogatz: Yeah, that's right. And -- and the way the mathematicians use chaos -- it may be not the best word for the subject in a way, because what we should think of is that there's a whole spectrum of disorder
17:37	Strogatz: Yeah. If we keep sort of like raising the -- the heat, you know, to wilder and wilder behavior, then you would start to see something like turbulence where it's complicated not just in time, like chaos, but also in space
17:49	Strogatz: There's a counterpart to the turbulence that -- that arises in living things. It's -- it would really be a matter of life and death, which is complicated behavior in your own heart in space and time, which we call fibrillation. It's the…
18:02	Rockmore: Fibrillation is actually turbulence in my heart, is that…
18:04	Strogatz: It's a kind of electrical turbulence. Instead of the organized rhythmic flow of electricity that triggers the ventricles to beat properly in sync, you -- you find that -- you start to get a -- something like electrical vortices, electrical whirlpools on the heart causing different parts to beat at different times. You get uncoordinated beating. then no blood gets pumped. And you're -- when people die suddenly in, you know, a matter of minutes, from cardiac arrest, that's what's happening at the electrical level.
18:31	Strogatz: So, mathematicians nowadays are just starting to -- to work with cardiologist to try to figure out this most deadly arrhythmia, using modern day versions of chaos theory. The -- the cutting edge of chaos.

18:42	Rockmore: I see. So a nice -- I mean, a beautiful and important intersection of mathematics and medicine.
18:47	Rockmore: Well, Steve thanks a lot. This has been a really exciting tour of chaos and all sorts of different venues, and I appreciate the tour.
18:53	Strogatz: Sure. Thanks. My pleasure.
18:55	Rockmore: So now we're going to look a little bit more closely at the use of chaos in cardiac dynamics.
18:59	**HOST (V.O.):** **When mathematicians talk about Chaos Theory, what they're talking about is mild/wild: poised between metronomic regularity and the craziness of turbulence. Scientists have begun to explore chaos for such practical applications as the treatment of heart disease. Chaos is giving us a lot of new insight into heart dynamics --helping us understand the arrhythmias that can, in the worst cases, lead to sudden cardiac death.**
19:26	**HOST (V.O.):** **More than 300,000 people in the United States die of cardiac arrest every year. Most attacks are brought about by an abrupt change from rhythmic pumping of the heart muscle to spasmodic convulsions.**
19:38	**HOST (V.O.):** **Scientists discovered that the unstable palpitations, known as cardiac fibrillation are a form of chaos. Like all chaotic occurrences it isn't completely random.**
~~19:49~~	**HOST (V.O.):** **The heart can become arrhythmic because of stress, an injury or some abnormality in the muscle tissue. The electronic impulses of the heart begin rotating in a kind of spiral wave. This rotating disturbance can travel, its chaotic impulses circulating through the heart tissue. It may also break up into a small number of added spiral waves, all rotating and diverging causing the system to destabilize.**
20:13	**HOST (V.O.):** **By using the math of chaos, researchers were able to calculate how to give small electrical pulses to test animals and then, eventually, humans. These pulses induced premature beats which cardiologists were able to use to coerce the heart tissue back into a healthy rhythm. Scientists have yet to utilize chaos**

	in specific medical applications, but perhaps the theory will someday lead to better anti-arrhythmia drugs, gentler defibrillators, and other beneficial technologies.
20:42	HOST: Beyond heart dynamics and computer animation, scientists and mathematicians use Chaos to explore evolutionary biology, economics, population growth, artificial intelligence, gaming and probability ... even making more efficient fuel injectors.
21:00	HOST: But perhaps one of the most interesting explorations is one that takes us again into outer space...
21:07	MARTIN LO: The interplanetary superhighway is a network of ultra-low energy orbits generated by the five LaGrange points that connect the entire solar system…
21:26	LO: …and it explains how things can move back and forth using chaotic dynamics.
21:30	LO: I'm Martin Lo. I work as a mission designer designing trajectories for space missions at JPL.
21:38	LO: The very specific branch of mathematics that we use to study the interplanetary Superhighway is called Dynamical Systems Theory.
21:46	LO: It gives us a very different picture of the solar system. Instead of just isolated planets in near circular orbits around the sun…
21:56	LO: …this Interplanetary Superhighway concept says that all the planets, the moons, the asteroid belts the comets, the cyper belt, they're all actually dynamically connected and linked.
22:05	LO: Even though you don't see the orbits connecting them there's a web underneath …
22:19	LO: LaGrange Points are what I call the seeds of the interplanetary superhighway. There locations where all the forces, the gravitational forces, are balanced with the rotational forces…
22:29	LO: …so that if you put a particle there it would just remain there. But if you just have the slightest motion on it, breathing on it, would cause it to drift away.
22:42	LO:

	It's as if you were starting on street # 1 and then you go down it will take you to one place but if you start on street # 2 not only will it take you a different place but it'll be actually to another city. So in our case it could be actually to another planet.
23:06	LO: Starting form these LaGrange Points or the equilibrium points, they generate families of periodic orbits. So these are orbits that close on themselves and they surround the La Grange points and they get bigger, bigger, and bigger. And what happens, these are very special types of periodic orbits that are very sensitive. Or people call them unstable.
23:26	LO: But this sensitive dependence is really the definition of Chaos.
23:34	LO: The very first mission that used this type of very sensitive orbits – it was called the ISE3: International Sun-Earth Explorer 3 was launched in 1978. By using the sensitive dependence on slight changes it was able to reorient itself, move away from the L1 LaGrange point and actually go to follow ah to study a comet. And so it's this type of sensitivity and energy savings that makes these orbits very, very powerful.
24:05	LO: Here we are in the JPL space museum and behind me is a life size model of the Galileo spacecraft. What's really exciting for me is that even though the original design used very classical theories to come up with the trajectory, we now can show that it's actually following the pathways of the interplanetary superhighway.
24:31	LO: In terms of the future research on the concept of the interplanetary superhighway. There's really two sides. From the scientific point of view, by understanding these pathways and mapping them out, it will help us understand how solar systems form how the transport of material that builds life comes to the earth. On the second vein on a more practical for humans is that it will help us find ways to fly cheaply from A to B. It might help us to deflect, detect and even perhaps capture rogue asteroids that might otherwise hit the earth.
25:08	RED: You see that? POPS: You mean that shooting star? RED: Yeah. Beautiful.
25:13	POPS:

	Could'a come all the way out from the hyper belt, caught the interplanetary superhighway -- RED: I know, I know. You and your LaGrange tubes. Bunch of convoluted mathematical mumbo jumbo if you ask me, Chaos Theory.
25:32	RED: ...Gimme ol' Isaac Newton anytime.
25:34	POPS: I don't think the weather has anything to do with clockwork. RED: You wouldn't be suggesting that were gonna get rained out on account of some butterfly flapping its wings in China – would you? POPS: The Butterfly Effect? Nah.
25:58	HOST: A butterfly flapping it's wings in China causing a hurricane over Florida? Might be a stretch. But mathematically, well, as we've seen, anything's possible.
26:08	HOST: And maybe that butterfly is the right metaphor for mathematics and Mathematicians. Small discoveries over time as well as the big ones amplified through history done by some of the greatest thinkers of all time. Pythagoras to Euclid, Newton, Poincare, Lorenz and countless others whose journeys travel amazing intellectual trajectories through history. And with chaos, we have just begun to explore the unpredictable, on our way to perhaps discovering other superhighways of knowledge that might some day lead us to the end of the unknown.
26:50	CREDITS