PROGRAM: 12
In Sync

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TIME CODE	Audio
00:15	OPENING CREDITS
00:40	HOST Many things in the universe behave in a synchronized way — whether manmade, or natural –
00:51	HOST — working together in harmony, moving simultaneously, pulsing with a regular rhythm, coordinated in time and space. Moving in sync we would say.
01:02	HOST We see synchronization, as an emergence of spontaneous order in systems that most naturally should be disorganized. And when it emerges, there is a beauty and a mystery to it, qualities that often can be understood through the power of mathematics.
01:20	**HOST (V.O.)** **How does a symphony orchestra play in sync and harmony? The answer is obvious: the group consists of humans who have the capacity to read music, listen to each other, and follow their leader…**
01:34	**HOST (V.O.)** **But what about a group with no leader or sheet music? Like a jazz jam session? Its members still perform in sync – each performer working off the cues of the others, synchronized via spoken or unspoken communication.**
01:51	**HOST (V.O.)** **But what about schools of fish, flocks of birds? How do they know when to turn left, turn right, move higher or lower — perfectly in sync — each adjusting its movements instinctively, somehow highly sensitive to what its neighbors are doing…**
02;08	HOST While the orchestra and marching band work in a premeditated, planned, even calculated order … the flock of swifts and school of fish, they move in "unison" by a phenomenon called spontaneous biological order. In their collective movement, we see a system of individuals who somehow have managed to synchronize their changes in motion. And through mathematics we can make sense of this group dynamic using the language of calculus.
02;34	HOST To put it simply, calculus allows us to make mathematical sense of change in moving systems, whether gradual or constant. For instance, a car's speedometer may say 55 miles per hour, but that's just a rough approximation of its exact speed at that moment. How can we describe exactly how fast the car is moving at *each instant* it drives along?

02;56	HOST For this answer, we must go back to the 17th century, when the English scientist Robert Hooke challenged his rival Isaac Newton with that kind of question. He asked: How exactly do the planets pull on each other, and does the law of pulling explain the orbits we see? To answer it, Newton realized he had to describe and measure the movement of the planets instant by instant. And so, in 1666, he independently creates a new branch of mathematics, calculus.
03;28	**HOST (V.O.)** **Newton needed a way to notate the incremental changes of where a planet was on its orbit, and then to be able to add up all of those incremental changes to actually construct the orbit.** He also needed a way to figure out how fast it was moving, and where it would move next. He saw unity - gravity didn't end at the atmosphere.
03;45	HOST Newton was the first to show that the motion of objects on Earth and planetary motion are governed by the same set of natural laws.
03;53	HOST His famous equation "Force equals mass times acceleration – "F=ma" neatly sums that up.
04;01	HOST This same math he used to study the movement of inanimate objects in the 17th century, today assists us in calculating the behavior of animate objects.
04:11	HOST Thanks to Newton, calculus provides a mathematical language that allows us to measure a variable, for instance, for a bird, the angle of flight, or for a car, its speed. But birds and drivers are their own entities with reactions and influence factors -- and therefore, represent moving objects with many variables. It's perhaps a bit easier to learn about calculus by examining the solar system, because with planets there's a consistent measurable force at play.
04;41	HOST In order to study the rate of change of an object in motion, like a planet -- we look at the tiny changes in position of the planet in the sky. These tiny changes are then called infinitesimals. They are a theoretical construct and are meant to be the smallest possible increment by which a quantity can change. This idea leads us to derivatives, another important concept in differential calculus. Derivatives allow us to describe how fast something is changing at any particular instant.
05;08	HOST (V.O. CONT) In the case of a planet's orbit around the sun, let's say that its position at any moment in time is described by a variable, we'll call it **"p"** for position. Since this position changes over time, we write this as: **"p(t)"**, or **"p"** of **"t"** that's how mathematicians say it**,** which simply means

	position at some time, T. Now in some tiny interval of time, described by "**dt**", the position will change some tiny amount, which we describe as "**dp**".
05;37	HOST If we compare the small change in position to the small change in time, we get a ratio, **"dp/dt"** and this is like an instantaneous and infinitesimal calculation of an average velocity...How far did I travel, in how much time? So this ratio is the derivative and it tells us how fast something is changing at any particular moment. In this case, it tells us how fast the planet's position is changing with time, which we commonly refer to as the planet's velocity. If we use this ratio of "**dp/dt"** which represents the planet's velocity, to write a calculation of its velocity in relation to the forces of the sun, we create a differential equation.
06;17	HOST We call it a differential equation because it's an equation relating very small differences or differentials. This kind of equation describes how the planet's motion changes dependent upon its relative distance to the sun, and the other planets.
06;32	HOST Differential equations can be used to understand more than just the movements of the planets. They can be used to study other systems where physics describes the pushing and pulling of objects on each other. Often the scale and complexity of these systems are such that it is only through mathematics that we can simplify and understand them. This simplification will take a big system and turn it into a system of coupled differential equations. These can then be investigated on the computer.
07;00	HOST Although calculus was developed out of Newton's desire to explain the movements of celestial bodies, the same mathematics can be applied to whole realms of science – from solar systems... to the beating of our hearts...
07;14	CHARLES PESKIN: My name's Charles Peskin, I'm a professor of mathematics at the Courant Institute of Mathematical Sciences, New York University. What I do is math and computing applied to biology and medicine. And my kind of main project is the heart. Computer simulation of the heart.
07;37	PESKIN: The question is how can we understand the basic laws that govern the heart well enough that we can make a model heart in the computer that works the way the real heart does.
07;48	PESKIN: The equations which I use to describe the heart are differential equations, so calculus is absolutely fundamental to what I do.
07;55	PESKIN: The heart has its own natural pacemaker, It's called the sinoatrial node; it's a

	clump of cells which send out waves that synchronize the heart. And what's amazing about these cells is if you grown them in tissue culture separately, they beat on their own, but they're not synchronized. And then when they grow and come in contact they synchronize with each other.
08;14	PESKIN: So the question is basically, 'How does synchronization work, and what's involved in synchronization?' It's an amazing fact that I don't know exactly how to explain, but it's mysterious and interesting, that when you get spontaneous oscillators, each of which has its own rhythm – when they are coupled together in some way, when they influence each other, even when the influence is very weak, they have a tendency to synchronize.
08;40	I'm very fortunate to work with Glenn Fishmann studies electrical conduction in the heart.
08;47	GLENN FISHMANN My name is Glenn Fishmann, I'm director of the Division of Cardiology, and head of the Cardiovascular Biology Program here at NYU School of Medicine. Our lab group is interested in understanding the basis for cardiac arrhythmias.
09;05	FISHMANN: Right, so we've taken this out of the mouse and hooked it up through the aorta to profuse it, to keep it happy. We can look at it both in normal conditions, as well as in some of the genetically engineered mice that we've made without gap junctions or with other channel abnormalities, and we try to understand why they get arrhythmias, which is really the main question that's driving us here.
09;20	FISHMANN: If we understand the biology of the pacemaker cells, it's our hope that we can regenerate portions of the conduction system by re-implanting cells that take in this function from those that have degenerated in the patient. More broadly, in terms of the whole heart's electrical system, if we can understand the basis for many forms of lethal cardiac arrhythmias, we can go in and treat those sorts of diseases.
09;46	FISHMANN: Cardiovascular disease is the leading cause of mortality in the United States, and sudden cardiac death from arrhythmia is the leading cause of death within the cardiovascular category. So it's clearly relevant in terms of a public health burden that we see from cardiovascular diseases.
10;02	PESKIN: People want to be able to just say some words which explain what happens but actually what you need is equations and mathematical models and computer simulations, because if you didn't know the basic rules that govern it you could not have made the model.
10;20	HOST It takes 10,000 cells in a tiny corner of the heart, all working in sync, to keep us alive -- to keep a steady rhythm inside us. Each cell is a bioelectrical oscillator, similar in concept to the mechanical oscillator of this clock - meaning that they each obey a basic rhythm, and that rhythm can be understood using the language of calculus. For the clock, the state is the

	position of the mechanism and for the heart cell, it is how close it is to firing an electrical signal. However, mathematically, these different mechanisms are the same.
10;58	HOST So, any single oscillator has a simple description, but now we are interested in a more complicated problem: what happens if there's a group of oscillators with a means of communicating with one another but with no overall authority dictating an overall plan – Be they oscillators or birds, heart cells or fish. Can differential equations help us understand that kind of phenomena?
11;19	Dan Rockmore: So to get the answer, we're going to talk to Steve Strogatz, a professor of Applied Mathematics at Cornell University. Steve, how do we use math to get from heart cells to pendulums?
11;31	Steve: Well, they're both oscillators. That is they both move in cycles. You know, there's an electrical cycle in the -- the heart cell where it's voltage goes up and back down, and up and back down. So -- so that's a rhythm, that's a cycle. And, of course, a pendulum is a cycle…
11;44	Daniel: It swings back and forth.
11;46	Steve: …in that it swings back and forth. So, they're really not that different. I mean, it happens one is an inanimate thing, one is living; that doesn't really matter to a mathematician at the abstract level.
11;55	Steve: And so -- so what we love in math is that there's a unity, that you can see the connection between pendulums and heart cells, if you maybe have the same equation that can describe both, but just with different interpretations of what that…
12;06	Daniel: Of just one -- one abstract framework and then you fill in the phenomenon for the variable spaces. Right. Yeah.
12;10	Steve: Right. Right. So that's -- that's what we like to do. And in the case of pendulums, which we understand much better than heart cells, it gives us a way in to -- to understanding these mysterious phenomena of life by thinking about mechanical things that we've understood for several hundred years.
12;23	Steve: So one connection that we -- we can make here is when we think about a -- an enormous collection of cells in the heart, this is -- you could think of it s a kind of population of oscillators. That is, there's maybe ten thousand of these cells, in the case of the pacemaker of the heart. And we're going to describe each of them by a differential equation that tells how it -- it changes its voltage from time to time.
12;46	But what's difficult about this is we need to understand their behavior as a group. But it's not enough to look at each cell in isolation, or…
12;52	Daniel: Because each cell in isolation is actually pretty well understood, right?
12;57	Steve: Okay. Understanding one oscillator, one rhythmic thing, is no problem. And, as I say, the challenge is to understand the -- the cooperative or collective behavior of hundreds or thousands of them. This -- this really was an outstanding challenge, even as late as 1960.
13;15	Steve: In this case Art Winfrey was then a college senior at Cornell, interested in biology. He knew that he was interested in it, but he was in the

	engineering physics program,
13;25	So, anyway, Winfrey thought about this question of synchronization of – it could be heart cells, it could be fireflies flashing, it could be crickets chirping, cells in the brain. He abstracted all of them as these differential equations, math that describes rhythmic motion and put in the computer thousand of them together.
13;46	So, in his math he allowed for some of these oscillators to be inherently faster, or slower than others.
13;51	So that, you see, is a challenge for synchronization because how are the slow guys going to keep up with the fast guys? And the answer is, because they pay attention to each other. Now, what does that mean for a cell which can't think? It means that it can feel electricity, in the case of the heart. Cells send each other electrical currents that can cause one to fire faster than it would have otherwise or that can retard it. So by this chemical and electrical communication, or the math that corresponds that sort of interaction, Winfrey was able to make this population behave as a cohesive unit. Sometimes.
14;26	Daniel: Correct. Right.
14;27	Steve: Here's a metaphor. Imagine that you have runners on a track. Steve: It's a track like you'd see at a football stadium and…
14;35	Daniel: So maybe a track like this one.
14;35	Steve: Now, in any system where you've got runners, there's going to be fast ones and slow ones. And they're going to be analogous to our inherently faster or slower oscillators. Okay? And what -- we're going to color code them so you can see who's inherently fast or slow.
14;47	Steve: First, if there's not interaction. They're ignoring each other and you can see that the -- that the fast ones are running away…
14;57	Steve: Okay, but now suppose that it's a running club rather than people who don't know each other. And maybe, you know, like you -- I know you're a pretty good runner, and I'm not. So -- but maybe if you and I were running, I would want to keep up with you.
15;11	Daniel: You'd want to keep up and I'd want to be a social guy and I wouldn't want to outpace my buddy…
15;15	Steve: Right. So there -- that -- that's coupling. Okay? That you'll slow yourself down, as you feel me on your shoulder, and I'll be huffing and puffing to keep up.
15;22	Now, in Winfrey's computer simulation he's -- he imagines conceptually turning up the knob so that now maybe they care about each other more, or they're listening. Like maybe they're shouting to each other, hey, speed up! Slow down! Okay. So, as that interaction strength builds up, at first nothing happens; they're still desynchronized. That's a little bit surprising. You might think that with more interaction they'd get a little bit synchronized, but they don't. Nothing happens until a critical phase transition point is reached. A kind of tipping point when…
15;50	Daniel: So even -- even though they're talking, the fast guys are still

	going….
15;52	Steve: Yeah!
15;53	Steve: That's -- that was a little bit of a surprise. I mean, normally in this world you think that if you change one thing it produces a response in something else. So here, if you would increase the strength of how much they care, they should start clumping more, but they don't. And that's what we mean by phase transitions. Like you can cool water and it's still water. And you can cool it more, and it's still water. But then when you hit the magic point, the phase -- in that case freezing point, it changes qualitatively. Okay? It becomes ice. And that's what happens here. Now here's the second scenario. Now we've crossed this tipping point, the phase transition…
16;27	Steve: …and can you see there's a little clump. The middling oscillators, the guys that are not too fast and not too slow – and there are a lot of them – start to lock together, run as a group in lockstep.
16;38	Okay. And then finally, if we make -- make the coupling even stronger – so I'm turning up this conceptual knob even more – now the whole population starts to run in sync.
16;46	Steve: But there's some distribution which we can draw as a curve. And -- and what is this curve showing? It's like in the case of they were talking about now, how fast are you, as a -- as a runner would be graphed on this axis. And then, how many people are that fast would be graphed on this axis.
16;57	And the reason it's bell shaped is that most people are in the middle range of speeds. And there are some very fast, but not so many. And some very slow, but not so many. So you have this characteristic bell shape.
17;14	Daniel: Yeah. So you really -- I mean, so you really need mathematic -- I mean, probability statistical tools to understand about life phenomena, right?
17;18	Steve: Yes. And that's what makes them so hard mathematically, that you have to combine many things. And we see this personified by Art Winfrey himself. That he was able to use probability theory to talk about the distribution of the speeds. Fantastic work.
17;31	Daniel: But of course, I mean, the -- the most beautiful thing about this, or one of the great things, that as much as this is about heart oscillators or runners, it's actually about a multitude of things, because mathematics is this language that can describe everything by putting different phenomena into the variables.
17;46	**HOST (V.O.)** **Humans, animals, and even our own heart pacemaker cells are capable of, and have the natural tendency toward synchronization. But does synchronization exist on a non- biological level? Could the impulse toward spontaneous order be more primal, more fundamental, than what seems apparent in the biological world?**
18;08	**HOST (V.O.)** **Christiaan Huygens, the Dutch mathematician, physicist and**

	astronomer who invented the pendulum clock in 1656 made a startling discovery about his clocks while ill and bedridden.
18;19	**HOST (V.O.)** ***~~Noticing two adjacent pendulums swinging in perfect opposition, he wondered what caused this surprising form of synchronization. Was there some mysterious force locking them in rhythm?~~***
18:30	**HOST (V.O.)** **Huygens discovered that the clocks, if taken out of synch, created a disturbance making the surface on which they rested tremble.** **Then something remarkable happened.**
18:41	**HOST (V.O.)** **The pendulums began to sync up in their rhythm, swinging precisely opposite each other like a pair of clapping hands.**
18;50	**HOST (V.O.)** **Huygens assumed that the proximity of the clocks to each other and the air disturbance created was responsible for synchronizing the pendulums. To test his theory, he placed the two clocks on a plank, and the plank lay atop two chairs positioned back to back. He disrupted the pendulum's sync as before, and immediately the chairs and the plank began to shake.**
19;11	**HOST (V.O.)** **The physical disturbance operating on the chairs continued for another 30 minutes, until the clocks restored themselves, and the chairs and plank stabilized.**
19;21	**HOST (V.O.)** **Huygens discovered that the rocking disturbance caused by the pendulums swinging out of synch eventually brought them back into sync.**
19;29	**HOST VO** **The explanation for this involves the equal and opposite forces in play When the clocks were in sync, the force they exerted onto the chairs and plank canceled out any physical disturbance that might have happened. Once that force was disrupted, the trembling started.**
19;45	Rockmore: So the movement of the chairs and the planks stabilize the pendulums. And then that in turn stabilized the chairs. Is that right?
19;53	Steve: So, these pendulums swing as they will. And as they -- if they're swinging in some sort of strange way, not synchronized, what happens is that they end up putting peculiar forces on the plank that makes the plank jiggle and the chairs start clattering on the floor. Now, that then back, in turn, put forces on the pendula that defects their motion and the whole thing eventually settles down to a state; that is there's negative feedback onto the pendula, that's really what's happening – until the pendula get like this, anti-phase, as -- as we saw, okay? And when the

	pendula are like that, they're not putting any net force on the plank, because when I'm pushing this way, you're pushing that way and it cancels. So the plank gets still, the chairs get quiet and we have synchrony.
20;39	Daniel: Voila. Yes.
20;40	Steve: Okay. So negative feedback through the pendula onto the planks and the chairs is -- that's what stabilizes the system.
20;46	Rockmore: This is making me think of this very recent sort of, well near-disaster that we saw in London, right? With the Millennium Bridge, is that right? Strogatz: Alright, watch.
20;57	**HOST (V.O.)** **London's Millennium Bridge, 325 meters long, links the City of London at St. Paul's Cathedral to the Tate Modern Gallery across the Thames.** **It opened to the public on June 10, 2000.**
21;10	**HOST (V.O.)** **Designed by architect Lord Norman Foster with sculptor Sir Anthony Caro and engineering firm Arup, it was described as "a blade of light" that would cross the Thames and create "an absolute statement of our capabilities at the beginning of the 21st century".**
21;25	**HOST (V.O.)** **Approximately 80,000 people crossed the bridge in the opening days --many more than anticipated. Early on vibrations of the bridge were being detected by the visitors. The engineers planned for some natural movement of course, the structure was designed for it. But the bridge began to wobble and sway enough to concern public officials, so they eventually closed it.**
21;48	Steve: Thousands of Londoners showed up to walk across this beautiful new footbridge, the Millennium Bridge. And you can think of those people as analogous to the pendulums.
21;56	Daniel: Okay. Alright.
21;56	Steve: One thing that's going to be different is that these pendulums, that is these people aren't going to stabilize this bridge, this plank. In fact, they're going to destabilize it.
22;07	Steve: Yes. I think -- I think the thing that's important in keeping in mind here that's different is that people don't like to walk on something that's wobbling. Pendulums don't think.
22;15	Daniel: Right. Right, right.
22;16	Steve: But people do and people feel off balance and feel uncomfortable.
22;19	Daniel: So they're actually reacting to each other…
22;20	Steve: Okay. And what was peculiar here is that as the bridge started to wobble, people did what they would naturally do to keep their balance, which is they separated their feet a little wider, they start walking like a novice ice skater, right, with their legs out. And they sort of have this penguin motion, which helps them feel steadier. But this is the part that no one would have

	expected. By doing that, they started to pump energy into the bridge. They started to make the bridge move worse, which caused more people to adopt this weird gate, which pumped more energy into the bridge and made it worse. So you had runaway feedback effect, which it turns out occurred through a phase transition very similar to what we talked about in the case of Winfrey and his biological oscillators. Except that here the phase transition had to do with the number of people on the bridge.
23;04	Strogatz: After the problem happened, ARUP, the people that built the bridge, the engineering firm put its own employees onto the bridge to try to diagnose what the problem was. First they put fifty of them and told them to walk around in a circle, and the bridge didn't move. And then they put sixty, walking around, the bridge is still motionless. And then somewhere between a hundred fifty and a hundred sixty people, the bridge started to move, the people started to walk like that and so there was, there was another, almost literally, tipping point, in this case…
23;34	Daniel: So this -- so this clearly looks like synchronization. So I'm assuming that it is and how do we get there mathematically?
23;39	Steve: Uh huh. Well, it definitely is synchronization. I mean, we can see in that footage that you see whole sections of the crowd rocking from side to side in unison. So absolutely it's synchronization among the people. It's also synchronization between the people and the vibrations of the bridge. That is -- that's what makes this phenomenon possible, that -- that the people are wobbling in order to stay comfortable in the way that they walk on this wobbling bridge. So we have two kinds of synchronization happening at the same time, and neither was anticipated.
24;08	Steve: I would give the engineering firm, ARUP, a lot of credit because they figured out what was happening, to a large extent. I mean, it's not totally understood…
24;156	Daniel: Actually, they are mathematically sophisticated in fact.
24;17	Steve: So one of the things they discovered through these studies where they put their employees on the bridge and -- and caused them to excite the bridge in a controlled way, was they found this -- this very simple, shockingly simple formula, F = K x V.
24;35	Steve: Now F just means the sideways force that a typical person, or the whole crowd, puts on the bridge. Okay, because that's what's driving the bridge.
24;43	Steve: Yeah. The K is just a mathematical constant that relates these two different things – force and velocity. But -- but what it's saying is the more the bridge moves, the more force the people end up putting on the bridge to stabilize themselves, which ends up making the bridge move more. So that F = KV is the heart of the positive feedback loop that caused the bridge to start moving.
25;03	Daniel: So now we've been able to turn the interactions of the people and the bridge into mathematics, and this has now allowed the engineers to find a

	way fix it, is that right?
25;11	Steve: Yes. Right. Now when a bridge is wobbling, engineers know there are really two things you can do to stop it. You can make it stiffer in some way by reinforcing it, putting trusses on it. They -- they thought about it and decided that wasn't the right approach, for various reasons, but they were able to use the second standard strategy for curing the vibration, which is to make the bridge more heavily damped. You can put the equivalent of shock absorbers underneath the bridge.
25;38	So they put something like seventy or eighty of these viscous dampers underneath the bridge; very unobtrusive so the bridge is still beautiful, you don't notice. And that solved the problem. And it's just a terrific example of how rational thinking coming from math combined with engineering insight can cure what was this very peculiar strange phenomenon.
25;49	Daniel: Math to the rescue.
26;00	Steve: It's an uplifting subject, in a way, because it gives us a sense that there is -- there's the possibility for cooperation in the natural world, and it's really built into -- to existence. It comes as a relief sometimes when you think of, you know, all the disorder and disharmony that we see around us to think that there is at least this important side of nature where -- where everything becomes harmonious.
26;21	Daniel: Terrific. Well, thanks so much, Steve. It's been fun.
26;23	Steve: Yeah. Thanks.
26;24	HOST And so, our world is a symphony of movement. When the power and beauty of the abstract language of mathematics - of calculus and differential equations – combine with the insights of the physical sciences, we begin to understand the how and why of that symphony... from the movement of the planets to the ticking of a clock to the flight of swifts... to the beating of our own hearts
26;50	CLOSING CREDITS