OF ALL THE EXPERIENCES a student has in kindergarten through twelfth grade, successful completion of the first course in algebra has the greatest impact on future opportunities. Algebra is the fork in the road where one direction leads to opportunity and the other to limited options for further education and promising careers. None of the other disciplines has a similar decision point. For example, in most high schools all students are required to take English every year. In science, the ability to succeed in chemistry or physics depends more on prior mathematics courses than on prior science courses. Social studies courses tend not to depend on knowledge from previous social studies courses.

Whereas some other countries have elaborate testing systems that determine which students will go on to higher education, in the United States and Canada success in precollege mathematics plays a major role in the sorting process. Mathematics has been described as the "critical filter" for determining a student's options (Sells 1978). The elementary algebra course is the most critical point of the critical filter.

In the typical precollege high school sequence (algebra 1, geometry, algebra 2, precalculus), each course successively screens out more students. Only a small number of students complete the full four-year mathematics sequence that prepares them for college majors in mathematics, science, engineering, and other technical fields. We can anticipate that in the information society of the future, a good background in mathematics will be required for most fields. Elementary algebra, geometry, advanced algebra, and additional work in computers, statistics, and probability will be necessary to open doors in practically every field.

Mathematics courses have tended to screen out certain groups, such as minorities, females, and students from low-income families, in excessive numbers. This process has contributed to the inequities in employment and income that exist in society today. To achieve an equitable society, we must change the algebra course from a filter that screens out segments of our population to a pump that propels all students toward opportunity (National Research Council 1990). In the process we can develop the brainpower needed for the society of the future.
Expectations and the Self-fulfilling Prophecy

To help students achieve success in algebra, we need to begin by believing that they can be successful. We can learn something from a look at the difference in expectations between Asian societies and our own. Whereas Asians tend to attribute success in mathematics to effort, we tend to attribute success to ability. (Stevenson et al. 1986.) Unfortunately, many in our society accept the notion that a large segment of our students do not have the ability to learn algebra. This idea leads to a self-fulfilling prophecy—with the result that many students do not successfully complete algebra—and, thereby, limits their opportunities for the future.

A major reason that students continue in mathematics or avoid it is their perception of how good they are at mathematics (Armstrong and Kahl 1979). If they believe that they have ability in mathematics, then they are likely to continue to study mathematics. Students who believe they lack mathematical ability tend to drop out of mathematics. Teachers should take every opportunity to give positive reinforcement to students. Whenever students are successful, they should receive the message that they were successful because they applied themselves and that if they continue to apply themselves, they can expect more success. Students who do poorly should not be allowed to come to the conclusion that they are dumb, that they can never learn mathematics, and, therefore, that they should drop out of mathematics. The difficult job of developing self-confidence for these students can be accomplished by teaching them how to learn mathematics and offering them motivating experiences in which they can have success.

The idea that it is all right not to be good in mathematics should not be acceptable in our society. Today people who specialize in mathematics often receive this reaction to their specialty from others at social occasions: “Oh, well, I was never very good at mathematics.” No one ever says to a reading specialist that they were never very good at reading. To prepare for the future, we need to convey the expectation that everyone can learn mathematics and that avoiding mathematics is unacceptable.

The belief that virtually all students can learn algebra places a whole new perspective on teaching algebra. We cannot write off unsuccessful students as simply not having mathematical ability. Some possible reasons for lack of success in algebra are (1) a lack of prerequisite knowledge, (2) the failure to study properly, and (3) inadequate instruction.

Expectations and Prerequisite Knowledge

Before beginning a lesson, an algebra teacher should take steps to ensure that the students have the necessary prerequisite knowledge. Henry Luce, former publisher of Time and Life magazines, is reported to have told his editors, “Never overestimate what the readers know, and never underestimate what they can learn.” We should replace “readers” with “students” and apply this adage to teaching algebra. If the teacher checks to see that the students have the necessary prerequisite knowledge, the students can be provided with the appropriate advance preparation for successful learning during the lesson.

Throughout the algebra course, teachers continually need to use a variety of
techniques to assess and review prior mathematical knowledge. This approach is especially important early in the course to ensure that students’ first experience with algebra is successful. This success will contribute to high expectations, which will contribute to more success. Expectations illustrate the validity of the cliche “Nothing succeeds like success.”

**Expectations and Studying Properly**

Initially, the teacher should establish clear expectations with respect to the classroom environment, the completion of homework, and the assumption of responsibility by students for their own learning. Students should begin with the assumption that they must apply themselves if they are to learn algebra. They should recognize that algebra is a logical subject with understandable reasons for everything. They should approach algebra with confidence that they have the ability to understand those reasons.

When exploration and reasoning do not lead to the expected results, students must be willing to go back and resolve the differences between their results and the expected results. They may have to start over again, or they may have to seek help. They should recognize that trial and error is a valid mathematical technique. They should not allow fear of errors to deter them from making attempts.

Some students may be studying hard but still not be learning; those students probably need to study smarter rather than harder. They need to be able to talk to someone about algebra. They should begin by doing their homework as early as possible and seeking help when they encounter something they do not understand. The first source of help is the teacher. Students should not be afraid to ask questions. In algebra, the “dumb questions” are the ones that should be asked but are not asked. Students should find other students with whom to study. Then when they get stuck on a problem, they have someone to talk to about it at school or someone to call from home.

Recently at the University of California, Uri Treisman conducted a study of factors that determined success of calculus students at the Berkeley Campus. He observed that race, sex, and socioeconomic status are not critical factors. The one critical factor was whether the students participated in study groups. He observed that whereas Asian students tended to work together in study groups, black students tended to study alone. He was able to get black students to work together through forming the Black Honors Calculus Society, and members’ grades increased dramatically (Shanker 1988). A lesson can be learned here in expectations and communication that could apply to everyone who takes algebra.

**Expectations and Responsibility for Learning Algebra**

Beginning with the expectation that nearly all students can learn algebra puts more pressure on teachers. If students fail, teachers can’t just shrug their shoulders and say that the students did not have the necessary ability because learning algebra is a shared responsibility between the teacher and the student. The teacher needs to assess and maintain prerequisite knowledge. The teacher needs to motivate students to study algebra through informing them of the importance of algebra to their future,
through teaching them study skills, and through establishing an instructional setting in which students who apply themselves can be successful. Teachers need to use a variety of techniques to assess continually what students know, including the techniques of questioning and observation, as well as correction of homework and tests. Teachers must start from where students are to bring them to an understanding of algebraic concepts. Teachers can make algebra come alive by moving away from teaching for rote learning of mindless manipulation toward inspired teaching that conveys the power, the utility, and the beauty of algebra.

A teacher who has high expectations for students shares the responsibility for their success. Rather than ask why a student is unable to learn a concept, the teacher asks what changes in teaching strategy would enable the student to become successful. The teacher gives explanations and presentations that are adapted to the students' needs with a sensitivity to the levels of abstraction at which the students are functioning. The teacher makes liberal use of concrete materials and visuals to help the students understand the concepts being presented.

**Expectations and Structuring Lessons for Success**

A teacher with high expectations teaches lessons that are structured for success. A chapter or a lesson can be started by stating what is to be learned and why it is to be learned and by relating it to what came before and what will come later, with many of the lessons motivated by interesting applications or problems. The teacher ensures that the students have the necessary prerequisite knowledge for the lesson, and the direct-instruction portion of the lesson relates the new knowledge to the students' existing knowledge. Sample problems are demonstrated and discussed. The teacher checks for understanding. The teacher offers guided practice in class to ensure that students can successfully complete their assignments independently. Then the assignment should result in the students' having a successful experience applying the knowledge that was presented in the lesson. All too often, the assignment is made before the students have internalized the new material to be learned; then the teacher needs to explain the lesson while going over the previous day's homework. In the process, students begin with an unsuccessful experience that can contribute to lowered expectations by both the students and the teacher. Ideally, students should have success with every lesson; they should also be given some challenges with which they may not initially be successful. They need to know that the answers do not always come simply and easily; they should be encouraged to develop persistence and problem-solving skills to enable them to attack and ultimately to solve difficult problems.

The teacher should conduct frequent systematic reviews of previously learned material where individual students are held accountable for maintaining previous learning. A systematic review-and-maintenance program will help ensure that students always have the necessary prerequisite knowledge for future lessons.

**Expectations and Assessment**

A teacher with high expectations ensures that the tests measure objectives that have been taught and that the students have learned the material before being tested.