Unit 23: Control Charts

SUMMARY OF VIDEO

Statistical inference is a powerful tool. Using relatively small amounts of sample data we can figure out something about the larger population as a whole. Many businesses rely on this principle to improve their products and services. Management theorist and statistician W. Edwards Deming was among the first to champion the idea of statistical process management. Initially, Deming found the most receptive audience to his management theories in Japan.

After World War II, Japanese industry was shattered. Rebuilding was a daunting challenge, one that Japanese business leaders took on with great determination. In the decades after the war, they transformed the phrase “Made in Japan” from a sign of inferior, cheaply-made goods to a sign of quality respected the world over. Deming’s emphasis on long-term thinking and continuous process improvement was vital in bringing about the so-called “Japanese Miracle.”

At first, Deming’s ideas were not as well received in America. Deming criticized American managers for their lack of understanding of statistics. But as time went on – and competition from Japan grew – companies in the U.S. began to embrace Deming’s ideas on statistical process control. Now his principles of total quality management are an integral part of American business, helping workers uncover problems and produce higher quality goods and services.

In statistics, a process is a chain of steps that turns inputs into outputs. A process could be anything from the way a factory turns raw iron into a finished bolt to the way you turn raw ingredients into a hot dinner. Statisticians say a process that is running smoothly, with its variables staying within an expected range, is in control. Deming was adamant that statistics could help in understanding a manufacturing process and identifying its problems, or when things were out of control or about to go out of control. He advocated the use of control charts as a way to monitor whether a process is in or out of control. This technique is widely used to this day as we’ll see in the video in a visit to Quest Diagnostics’ lab.

Quest performs medical tests for healthcare providers. So, for example, at Quest a patient’s blood sample is the input of the process and the test result is the output. A courier picks up specimens and transports them to the processing lab, where they are sorted by time of arrival and urgency of test. Technicians verify each specimen and confirm the doctor’s orders. Then the specimens are barcoded and are ready to be passed on for testing. Quest’s Seattle processing lab aimed to get all specimens logged in and ready by 2 a.m. so the sample could move on to the technical department for analysis. However, until a few years ago, they were rarely meeting that 2 a.m. goal. Their lateness was leading to poor customer and employee satisfaction and moreover, it was wasting corporate resources. Enter statistical process control!

Quest needed to know where the process stood at present: How close were they to hitting the 2 a.m. target and how much did finish times vary? Keep in mind that all processes have variation. Common cause variation is due to the day-to-day factors that influence the process. In Quest's case, it could be things like a printer running out of paper and needing to be refilled, or a worker calling in sick. It is the normal variation in a system.

Processes are also susceptible to special cause variation – that's when sudden, unpredictable events throw a wrench into the process. Examples of special cause variation would be blackouts that shut down the lab's power, or a major crash on the highway that would keep the samples from being delivered to the lab. Quest needed to figure out how their process was running on a day-to-day basis when they were only up against common cause variation.

Quest used six months of finish-time data to set up control limits and then created a control chart, which is a graphic way to keep track of variation in finish times. Figure 23.1 shows a control chart for month 1. The center line is the target finish time. The control limits at 12:00 a.m. and 4:00 a.m. are set three standard deviations above and below the center line. The data points are the finish times that Quest tracked over a one-month period.

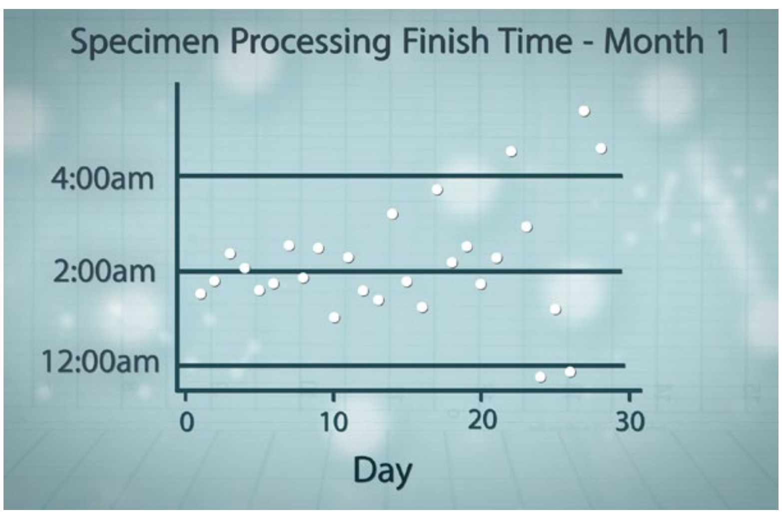

Figure 23.1. Control chart for month 1.

Quest assumed that their nightly finish times are normally distributed. In Figure 23.2, we add a graph of the normal distribution to the control chart. Remember, in a normal distribution 68% of your data is within one standard deviation of the mean, 95% is within two standard deviations, and 99.7% is within three standard deviations.

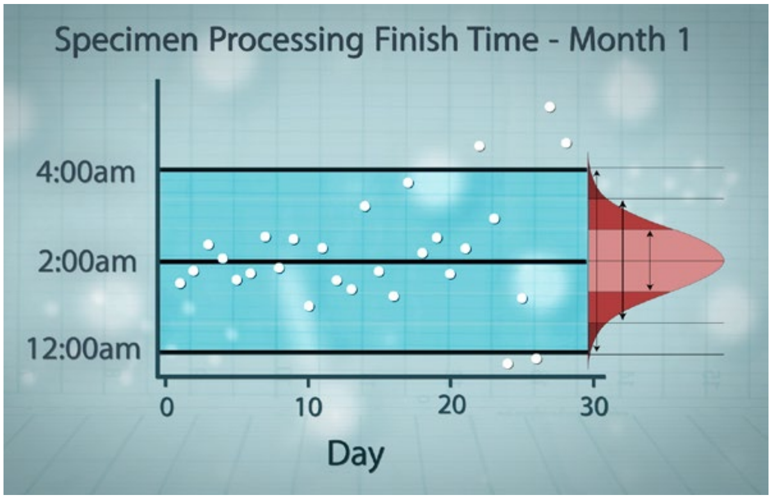

Figure 23.2. Adding an assumption of normality.

Using the control chart Quest was able to figure out when their process had been disturbed and gone out of control, or was heading that way. One dead giveaway that the finish times are out of control is if a point falls outside the control limits. That should only happen 0.3% of the time if everything is running smoothly. Take a look at Figure 23.3, which highlights what happened toward the end of the one-month cycle.

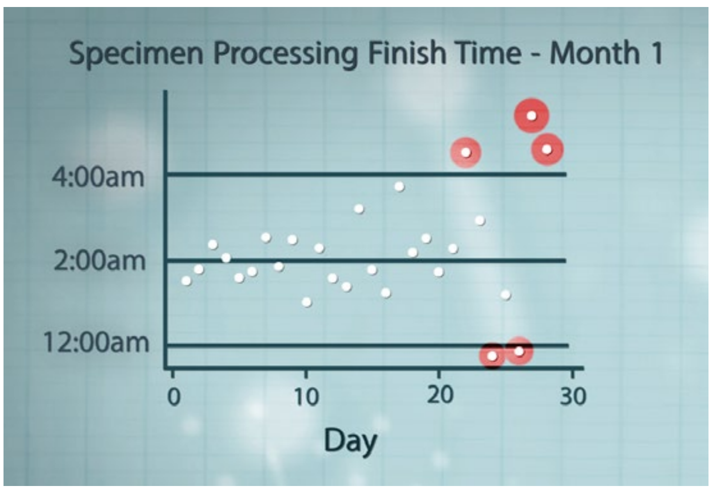

Figure 23.3. Highlighting finishing times beyond the control limits.

There are other indicators that something suspicious might be going on besides points falling outside the control limits. For example, if too many points are on one side of the center line or if a strong pattern emerges (hence, the variability is not random) – then it's time to investigate. Mapping finish times on the control chart helps monitor the process, and alerts techs right away that something has been disturbed. Then they can track down and address the cause immediately.

Another way the control chart helped Quest improve efficiency was by revealing some of the causes of variation in the process, which the team could then address. Quest actually restructured the entire department. It set up pods within the department and changed staffing. These sorts of changes brought the mean finish time much closer to the 2 a.m. target, and the remaining variation clustered more tightly around the center line. The days of wildly erratic finish times were gone thanks to statistical process control.

STUDENT LEARNING OBJECTIVES

A. Understand why statistical process control is used.

B. Be able to distinguish between common cause and special cause variation.

C. Know how to construct a run chart and describe patterns/trends in data over time.

D. Know how to construct an $\bar{x}$ chart and describe the changes in sample means over time.

E. Make decisions based on observed patterns in 7 run charts and $\bar{x}$ charts.

F. Be able to apply decision rules to determine if a process is out of control.

Figure 23.4: Silicon ingots and polished wafers.

Consider the problem of quality control in the manufacturing process of turning ingots of silicon into polished wafers used to make microchips. (See Figure 23.4.) Assume that the manufacturer wants the polished wafers to have consistent thickness with a target thickness of 0.5 millimeters. A sample of 50 polished wafers is selected as a batch is being produced. Table 23.1 contains these data.

0.555	0.543	0.533	0.538	0.533	0.529	0.526	0.522	0.518	0.519
0.516	0.515	0.513	0.515	0.512	0.510	0.508	0.507	0.507	0.507
0.506	0.506	0.506	0.505	0.503	0.502	0.500	0.498	0.499	0.496
0.497	0.493	0.492	0.491	0.487	0.488	0.486	0.485	0.483	0.484
0.482	0.479	0.476	0.476	0.474	0.471	0.471	0.469	0.454	0.447

Table 23.1. Wafer thickness from sample of 50 polished wafers.

In order to gain a sense of the distribution of wafer thickness, a quality control technician constructs the histogram shown in Figure 23.5.

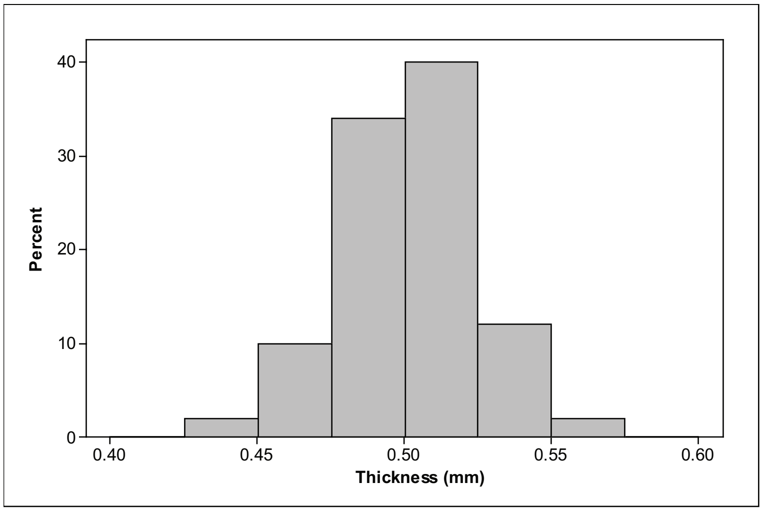

Figure 23.5. Histogram of wafer thickness.

The histogram indicates that distribution of wafer thickness is approximately normal. The sample mean is 0.50064, which is pretty close to the target value. Furthermore, the standard deviation is 0.02227, which is relatively small compared to the mean. The analysis thus far supports the conclusion that the process is in control.

The sample mean and standard deviation together with the histogram provide information on the overall pattern of the sample data. However, there is more to quality control than simply studying the overall pattern. Manufacturers also keep track of the run order, the order in which the data are collected. For the data in Table 23.1, the run order may relate to which part of the ingot – top, middle, or bottom – the wafers came from, or it may relate to the order in which wafers were fed through the grinding and polishing machines. If a process is stable or in control, the order in which data are collected, or the time in which they are processed, should not affect the thickness of polished wafers. One way to check that the production processes of polished wafers are in control is by creating a run chart.

A run chart is a scatterplot of the data versus the run order. To help visualize patterns over time, the dots in the scatterplot are usually connected. Table 23.1 lists the data values in the order they were collected, starting with the first row 0.555, 0.543, . . . , 0.519, followed by the second row, third row, fourth row and ending with 0.447, the last entry in the fifth row. So, the run order for 0.555 is 1, for 0.543 is 2, and so forth until you get to the run order for 0.447, which is 50. Figure 23.6 shows the run chart for the wafer thickness data. A center line has been drawn on the chart at the target thickness of 0.05 millimeters.

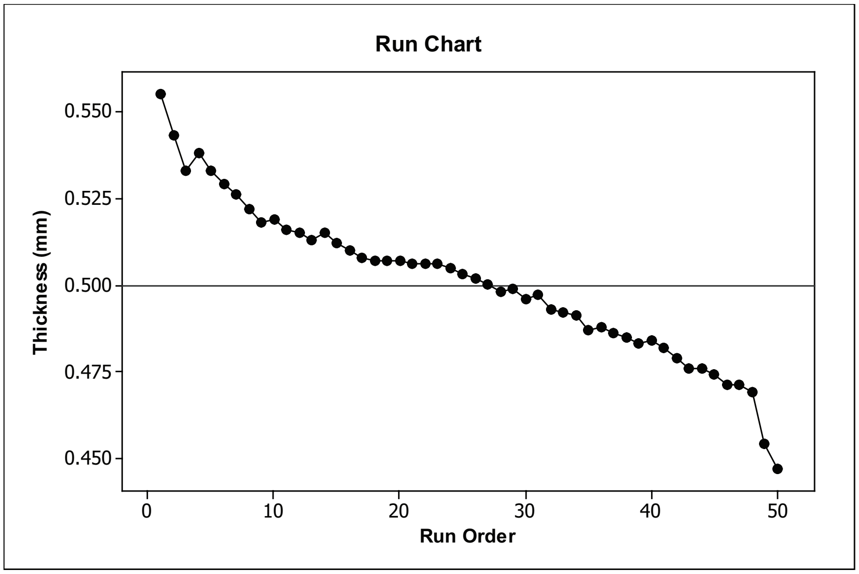

Figure 23.6. Run chart for wafer thickness data from Table 23.1.

Even though the overall pattern of the data gave no indication that there were any problems with the grinding and polishing processes, it is clear from the **run chart** in Figure 23.6 that the thickness of polished wafers is decreasing over time. Processes need to be stopped so that adjustments can be made to the grinding and polishing processes.

The run chart involved plotting individual data values over time (run order). Another approach is to select samples from batches produced over regular time intervals. For example, a quality control plan for the polished wafers might call for routine collection of a sample of n polished wafers from batches produced each hour. The thickness of each wafer in the sample is recorded and the mean thickness, $\bar{x}$, is calculated. The information on mean thickness can be used to determine if the process is out of control at a particular time and to track changes in the process over time.

Suppose when the grinding and polishing processes are in control, the distribution of the individual wafers can be described by a normal distribution with mean μ = 0.5 millimeters and standard deviation $\sigma = 0.02$ millimeters (similar to the data pattern in Figure 23.5). From Unit 22, Sampling Distributions, we know that under this condition the distribution of the hourly sample means, $\bar{x}$, based on samples of size n are normally distributed with the following mean and standard deviation:

$\mu_{\bar{x}} = \mu = 0.05$ millimeters

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.02}{\sqrt{n}}$ millimeters

Assume that the quality control plan calls for taking samples of four polished wafers each hour. In this case, our standard deviation for $\bar{x}$ is:

$\sigma_{\bar{x}} = 0.02/\sqrt{4} = 0.01$ millimeter

Each hour a technician collects samples of four polished wafers, measures their thickness, records the values, and then calculates the sample mean. Suppose that the data in Table 23.2 come from samples collected over an eight-hour period.

Sample Number	Sample Thickness (mm)	Sample Mean (mm)
1	0.509 0.502 0.521 0.469	0.5003
2	0.504 0.505 0.525 0.468	0.5005
3	0.489 0.506 0.486 0.497	0.4945
4	0.513 0.516 0.482 0.483	0.4985
5	0.552 0.516 0.476 0.472	0.5040
6	0.480 0.484 0.518 0.510	0.4980
7	0.516 0.489 0.513 0.495	0.5032
8	0.508 0.499 0.466 0.480	0.4882

Table 23.2. Data from samples collected each hour.

According to the 68-95-99.7% Rule, if the process is in control, we would expect:

68% of the $\bar{x}$ values to be within the interval 0.5 mm ± 0.1 mm or between 0.49 mm and 0.51 mm.

95% of the $\bar{x}$ values to be within the interval 0.5 mm ± 2(0.1) mm or between 0.48 mm and 0.52 mm.

99.7% of the $\bar{x}$ values to be within the interval 0.5 mm ± 3(0.1) mm or between 0.47 mm and 0.53 mm.

Next, we make an $\bar{x}$ chart, which is a scatterplot of the sample means versus the sample order. We draw a reference line at μ = 0.5 called the center line. We use the values from the 68-95-99.7% Rule to provide additional reference lines in our $\bar{x}$ chart. The lower and upper endpoints on the 99.7% interval are called the lower control limit (LCL) and upper control limit (UCL), respectively. Figure 23.7 shows the completed $\bar{x}$ chart.

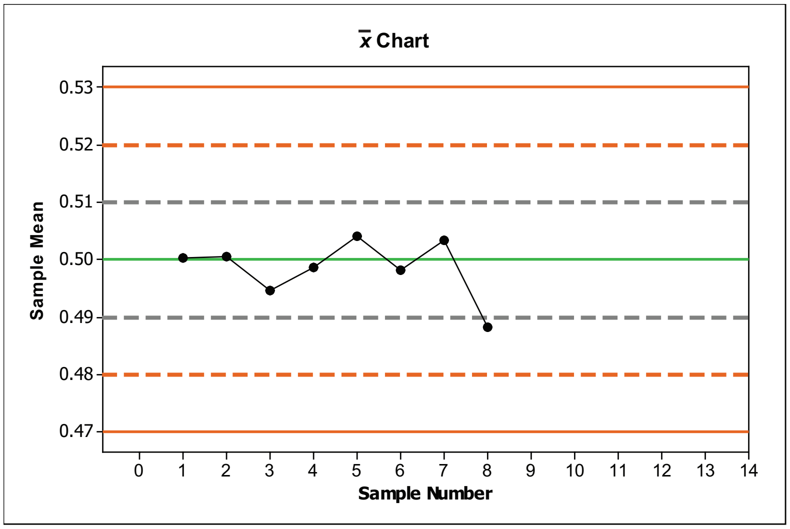

Figure 23.7. $\bar{x}$ *chart for wafer thickness over time.*

The $\bar{x}$ chart in Figure 23.7 does not appear to indicate any problems that warrant stopping the grinding or polishing processes to make adjustments. All of the points except one fall within one $\sigma/\sqrt{n}$ of the mean, in other words, fall between the reference lines corresponding to 0.49 and 0.51. However, as we add additional points, we will need some guidelines – a set of decision rules – that tell us when the process is going out of control. The decision rules below are based on a set of rules developed by the Western Electric Company. Although they are widely used, they are not the only set of decision rules.

Decision Rules:

The following rules identify a process that is becoming unstable or is out of control. If any of the rules apply, then the process should be stopped and adjusted (or the problem fixed) before resuming production.

Rule 1: Any single data point falls below the LCL or above the UCL.

Rule 2: Two of three consecutive points fall beyond the $2\sigma/\sqrt{n}$ limit, on the same side of the center line.

Rule 3: Four out of five consecutive points fall beyond the $\sigma/\sqrt{n}$ limit, on the same side of the center line.

Rule 4: A run of 9 consecutive points (in other words, nine consecutive points on the same side of the center line).

None of the decision rules apply to the control chart in Figure 23.7. Hence, the processes are allowed to continue. Table 23.3 contains data on the next seven hourly samples.

Sample Number	Sample Thickness (mm)	Sample Mean (mm)
9	0.505 0.489 0.499 0.532	0.5063
10	0.534 0.521 0.530 0.511	0.5240
11	0.526 0.514 0.520 0.530	0.5225
12	0.517 0.518 0.511 0.512	0.5145
13	0.506 0.504 0.511 0.511	0.5080
14	0.507 0.499 0.501 0.510	0.5043
15	0.511 0.509 0.512 0.520	0.5130

Table 23.3. Samples from an additional seven hours.

Figure 23.8 shows the updated $\bar{x}$ chart that includes the means from the seven additional samples.

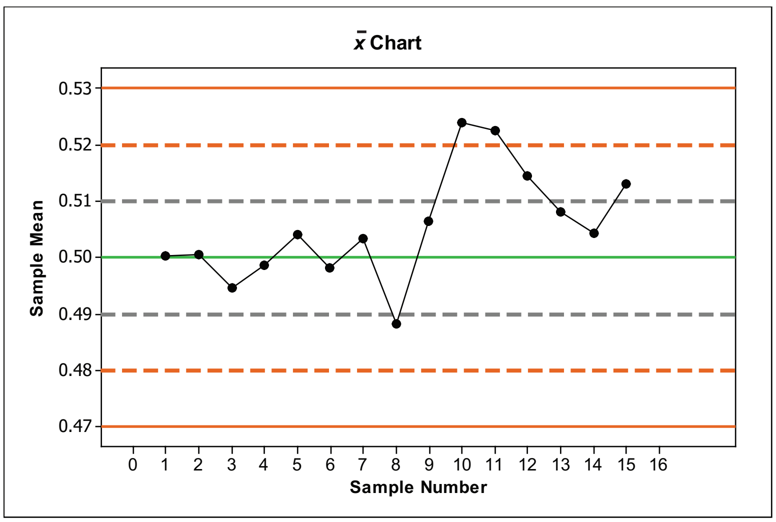

Figure 23.8. Updated $\bar{x}$ chart.

Now, we apply the decision rules. This time, we find that Rule 2 applies. Data points associated with Samples 10 and 11 fall above 0.52 (which, in this case, is above the $2\sigma / \sqrt{n}$ limit). According to Rule 2 the process should be stopped after observing Sample 11's $\bar{x}$-value.

The $\bar{x}$ chart monitors one statistic, the sample mean, over time. The $\bar{x}$ chart is only one type of control chart. As mentioned earlier, the manufacturer is also interested in producing a consistent product. So, instead of tracking the sample mean, the quality control plan could also track the sample standard deviations, or the sample ranges over time. More generally, control charts are scatterplots of sample statistics (or individual data values) versus sample order and are commonly used tools in statistical process control.

Before control charts were popular, there was a tendency to adjust processes whenever a slight change was noted. This led to over-adjustment, which often caused more problems than it solved. In addition, it meant that the process was stopped for adjustment more frequently than was necessary, which was a waste of money. Control charts and decision rules give manufacturers concrete guidelines for deciding when processes need attention.

KEY TERMS

A **process** is a chain of steps that turns inputs into outputs. Every process has variation. **Common cause variation** is the variation due to day-to-day factors that influence the process. **Special cause variation** is the variation due to sudden, unexpected events that affect the process.

When a process is running smoothly, with its variables staying within an acceptable range, the process is **in control**. When the process becomes unstable or its variables are no longer within an acceptable range, the process is **out of control**.

A **run chart** is a scatterplot of the data values versus the order in which these values are collected. The chart displays process performance over time. Patterns and trends can be spotted and then investigated.

Control charts are used to monitor the output of a process. The charts are designed to signal when the process has been disturbed so that it is out of control. Control charts rely on samples taken over regular intervals. Sample statistics (for example, mean, standard deviation, range) are calculated for each sample. A control chart is a scatterplot of a sample statistic (the quality characteristic) versus the sample number. Figure 23.9 shows a generic control chart.

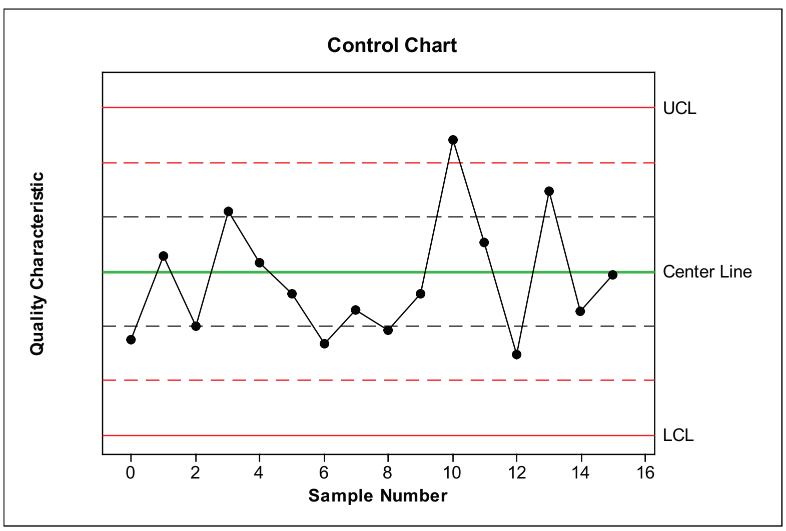

Figure 23.9. Generic control chart.

The **center line** on a control chart is generally the target value or the mean of the quality characteristic when the process is in control. The **upper control limit** (UCL) and **lower control limit** (LCL) on a control chart are generally set ±3 $\sigma/\sqrt{n}$ from the center line.

An $\bar{x}$ **chart** is one example of a control chart. It is a scatterplot of the sample means versus the sample number. Scatterplots of sample standard deviations or sample ranges over time are two other types of control charts.

Decision rules consist of a set of rules used to identify when a process is becoming unstable or going out of control. Decision rules help quality control managers decide when to stop the process in order to fix problems or make adjustments.

THE VIDEO

Take out a piece of paper and be ready to write down answers to these questions as you watch the video.

1. What was W. Edwards Deming known for?

2. What is a process, statistically speaking? Give an example.

3. What does it mean for a process to be in control?

4. Why did Quest Diagnostics' lab need a statistical-quality-control intervention?

5. In Quest's control chart, how did they determine where to set the upper and lower control limits?

6. How did Quest respond to what it learned from its control charts? What were the results of these changes?

UNIT ACTIVITY:
YOU'RE IN CONTROL!

For this activity, you will play the role of a semiconductor quality control manager in charge of monitoring the thickness of polished wafers. Open the Control Chart tool from the Interactive Tools menu. You will be working with $\bar{x}$ charts. The activity questions follow the list of steps below.

Step 1: Select a set of values for the mean μ and standard deviation σ. You have three possible choices for each of these parameters.

For now, you will work through the construction of at least two control charts with your selection. In the real world, these values would be determined from past data collected when the process was known to be in control.

Step 2: Since you are in charge of the quality control plan, decide on the sample size *n* you would like to use for monitoring the process. You have three choices: 5, 10, or 20.

Keep in mind the following: The more wafers you sample, the more time it will take, and the more it will cost. On the other hand, with larger samples, results are more precise.

Step 3: Select the Step-By-Step mode.

In this mode, you will get feedback immediately after each decision that you make. If you make a mistake, you will be told to start over and will need to click the "Start Over" button. Once you feel confident about your decisions, you can change to Continuous mode.

Step 4: Calculate the lower control limit to <u>four</u> decimals and enter its value in the box for LCL. Calculate the upper control limit to <u>four</u> decimals and enter its value in the box for UCL. Click the "Change Control Limits" button.

If your calculations are correct, control lines will appear in the $\bar{x}$ chart. In Step-By-Step mode, you will get feedback (see bottom of screen) if you have made a mistake. The feedback will say: Recalculate control limit values. To correct the error, enter new values for LCL and UCL and then click the "Change Control Limits" button.

Step 5: Click on the "Collect Sample Data" button. The data will appear in a column under the heading Thickness (mm) near the top of your screen. To calculate $\bar{x}$, click the "Calculate Mean" button. The mean will appear underneath the column.

Step 6: Click the “Plot Point” button to plot the ordered pair (sample number, mean) on the $\overline{x}$ chart.

Step 7: Make a decision. Your possible decisions are: (1) Continue Process, which means that you have decided the process is in control; or (2) Stop Process, which means that you have decided to shut down the process for adjustments or inspection.

Step 8: Repeat steps 5 – 7 until one of the following three things happens:

(1) You decide to continue and get the following feedback: Process is not in control. It should be stopped immediately. In this case, click the “Start Over” button at the top of the screen.

(2) You decide to continue and get the following feedback: Good decision. In this case, continue constructing the control chart.

(3) After 25 samples, it will be time for routine maintenance even if the process is still in control. At this time you, you can proceed to the next question. Click the “Start Over” button to do so.

1. Work through Steps 1 – 8 using the Control Chart tool. Complete one control chart successfully. Make a sketch of your chart (or do a screen capture and paste the screen capture into a Word document). If the process was stopped before 25 samples were selected, state which of the decision rules applies.

2. Use the same settings as you did for question 1. Rework question 1.

After you have successfully completed two control charts in Step-by-Step mode, you are ready to move on to question 3.

3. Change the settings for μ, σ, and n. Choose Continuous mode. Allow the process to continue until you think it needs to be stopped. After clicking the “Stop Process” button, you will receive feedback.

a. What settings did you choose? What were the values of the upper and lower control limits?

b. Make a sketch of your control chart or save a screen capture of your control chart into a Word document.

c. What feedback did you receive?

d. If your feedback indicated that you made a correct choice to stop the process, state the rule that made you decide it was time to stop the process. If your feedback indicated that you should have stopped the process sooner, state the sample number for when you should have stopped the process and the rule that applies.

4. Select new settings for μ and σ (it is up to you if you also want to change *n*).
Repeat question 3 and make another control chart.

EXERCISES

1. A manufacturer of electrical resistors makes 100-ohm resistors that have specifications of 100 ± 3 ohms. A quality control inspector collected a sample of 15 electrical resistors and tested their resistance. The results are recorded below:

99	98	101	98	99	101	99	100
100	98	99	102	99	101	100	

Assume that these data are recorded in the order they were collected beginning with the first row 99, . . . 100, followed by the second row 100, . . . 100.

a. Make a run chart for these data. Leave room on the horizontal axis to expand the run orders out to 30. (You will be adding 15 more data points in part (c).) Draw a reference line for the target resistance (100 ohms) and for the tolerance interval (these can serve as control limits).

b. Based on your run chart in (a), is there any evidence that the process is out of control? Support your answer.

c. The quality control inspector continued collecting data on the resistors. Results from an additional 15 data values, in the order values were collected, are recorded below:

100	99	102	99	101	102	101	100
101	102	100	103	101	102	103	

Use these data to complete the run chart in (a) for run orders from 1 – 30.

d. Based on the completed run chart in (c) is there any evidence that the manufacturing process is out of control? Support your answer.

2. Suppose a chemical manufacturer produces a product that is marketed in plastic bottles. The material is toxic, so the bottles must be tightly sealed. The manufacturer of the bottles must produce the bottles and caps within very tight specification limits. Suppose the caps will be acceptable to the chemical manufacturer only if their diameters are between 0.497 and 0.503 inch. When the manufacturing process for the caps is in control, cap diameter can be described by a normal distribution with $\mu = 0.5$ inch and $\sigma = 0.0015$ inch .

a. If the process is in control, what percentage of the bottle caps would have diameters outside the chemical manufacturer's specification limits?

b. The manufacturer of the bottle caps has instituted a quality control program to prevent the production of defective caps. As part of its quality control program, the manufacturer measures the diameters of a random sample of n = 9 bottle caps each hour and then calculates the sample mean diameter. If the process is in control, what is the distribution of the sample mean $\bar{x}$? Be sure to specify both the mean and standard deviation of $\bar{x}$'s distribution.

c. The cap manufacturer has a rule that the process will be stopped and inspected any time the sample mean falls below 0.499 inch or above 0.501 inch. If the process is in control, find the proportion of times it will be stopped during inspection periods.

3. For each of the $\bar{x}$ charts in Figures 23.10 – 23.12, decide whether or not the process is in control. If the process is out of control, state which decision rule applies. Justify your answer. (Note that reference lines at one, two, and three $\sigma/\sqrt{n}$ on either side of the mean have been drawn on the control charts.)

a.

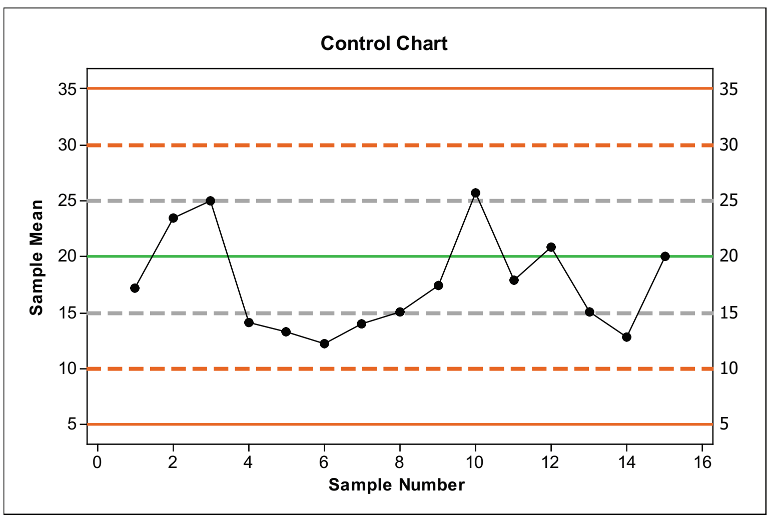

Figure 23.10. Control chart for exercise 3(a).

b.

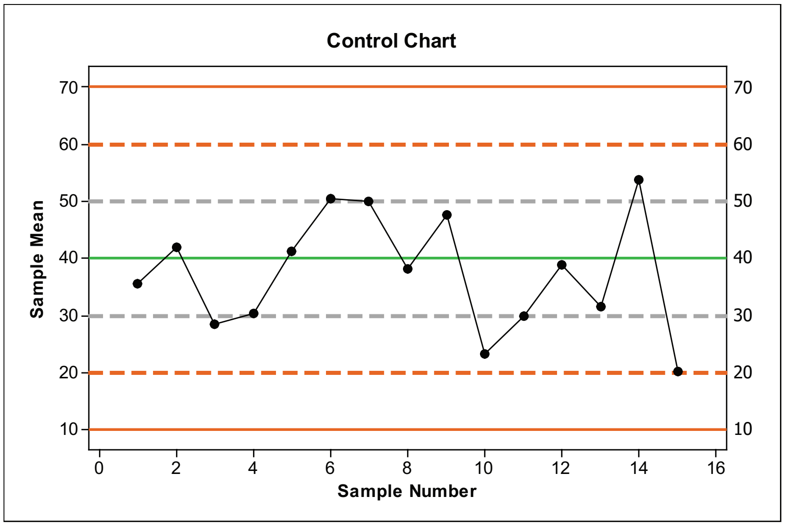

Figure 23.11. Control chart for exercise 3(b).

c.

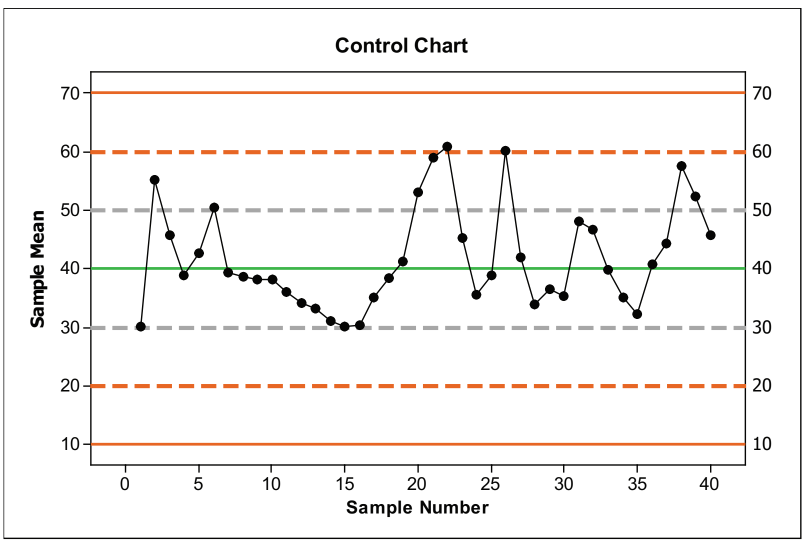

Figure 23.12. Control chart for exercise 3(c).

4. A company produces a liquid which can vary in its pH levels unless the production process is carefully controlled. Quality control technicians routinely monitor the pH of the liquid. When the process is in control, the pH of the liquid varies according to a normal distribution with mean μ = 6.0 and standard deviation $\sigma = 0.9$.

a. The quality control plan calls for collecting samples of size three from batches produced each hour. Using *n* = 3, calculate the lower control limit (LCL) and upper control limit (UCL).

b. Samples collected over a 24-hour time period appear in Table 23.4. Compute the sample means for each of the 24 samples and add the results to a copy of Table 23.4.

Sample	pH level			Sample Mean
1	5.8	6.2	6.0	
2	6.4	6.9	5.3	
3	5.8	5.2	5.5	
4	5.7	6.4	5.0	
5	6.5	5.7	6.7	
6	5.2	5.2	5.8	
7	5.1	5.2	5.6	
8	5.8	6.0	6.2	
9	4.9	5.7	5.6	
10	6.4	6.3	4.4	
11	6.9	5.2	6.2	
12	7.2	6.2	6.7	
13	6.9	7.4	6.1	
14	5.3	6.8	6.2	
15	6.5	6.6	4.9	
16	6.4	6.1	7.0	
17	6.5	6.7	5.4	
18	6.9	6.8	6.7	
19	6.2	7.1	4.7	
20	5.5	6.7	6.7	
21	6.6	5.2	6.8	
22	6.4	6.0	5.9	
23	6.4	4.6	6.7	
24	7.0	6.3	7.4	

Table 23.4. pH samples collected hourly.

c. Make an $\bar{x}$ chart. Add reference lines including lines for the lower and upper control limits.

d. Based on the control chart you drew for (c), decide whether or not the process is in control. If not, state which of the decision rules applies.

REVIEW QUESTIONS

1. A manager keeps track of duplicate e-mail messages he receives, which he views as a waste of his time. His log of the number of duplicate e-mails over 20 consecutive workdays appears in Table 23.5.

Day Number	1	2	3	4	5	6	7	8	9	10
Number of Duplicates	2	1	0	2	12	14	17	15	25	20
Day Number	11	12	13	14	15	16	17	18	19	20
Number of Duplicates	24	27	22	24	26	20	22	5	2	0

Table 23.5. Duplicate e-mail messages per day.

a. Calculate the mean number of duplicate e-mails per day.

b. Draw a run chart of the duplicate e-mail data. Add the mean number of duplicates as a reference centerline.

c. Nine or more consecutive data points on the same side of a center line can signal a special cause variation. Does the run chart from (b) signal a special cause variation?

2. A quality control inspector at a company that manufactures valve linings monitors the mass of the linings. When the process is in control, the mean mass is $\mu = 240.0$ grams and standard deviation $\sigma = 0.4$ gram. The inspector randomly selects a valve liner from batches produced each hour and records its mass. The mass (in grams) of 25 valve liners are displayed in Table 23.6 on the next page.

Hour	Mass	Hour	Mass
1	240.0	14	240.2
2	239.9	15	239.8
3	239.6	16	240.7
4	240.2	17	239.4
5	239.6	18	240.5
6	239.8	19	239.7
7	239.8	20	239.3
8	240.1	21	240.5
9	239.8	22	239.7
10	240.1	23	239.5
11	240.1	24	240.7
12	239.8	25	239.4
13	240.2		

Table 23.6. Mass of valve liners.

a. Make a histogram for mass of valve liners from Table 23.6. For the first class interval, use 239.0 grams to 239.2 grams. Based on the histogram is there any evidence that the manufacturing process is not in control? Explain.

b. Make a run chart for the mass of valve liners. Add a reference center line at μ. Add lower and upper control limits at $\mu \pm 3\sigma$.

c. Does the run chart show any changes in the distribution of valve-liner mass over time? Explain.

3. One process in the production of integrated circuits involves chemical etching of a layer of silicon dioxide until the metal beneath is reached. The company closely monitors the thickness of the silicon dioxide layers because thicker layers require longer etching times. The target thickness is 1 micrometer (μm) and has a standard deviation of 0.06 micrometers (based on past data when the process was in control). The company uses samples of four wafers. An $\bar{x}$ chart based on 40 consecutive samples appears in Figure 23.13 on the next page.

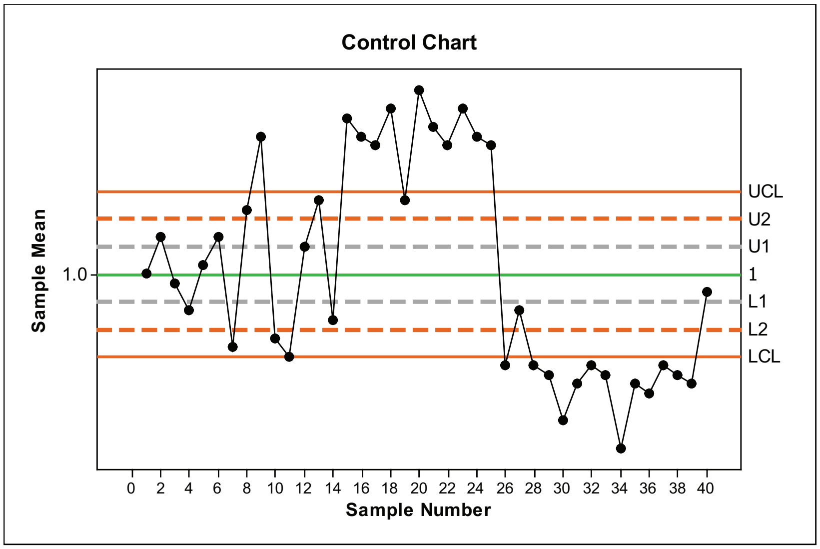

Figure 23.13. Control chart for thickness of silicon dioxide layers.

a. Calculate the appropriate control limits (the values of the reference lines drawn in Figure 23.13). Round the values to two decimals.

b. Decide whether or not the process is in control. If not, explain which decision rule applies and identify the sample number after which the process should be shut down for adjustments.

4. The company referred to in exercise 4 has two plant lines that produce the liquid. Data from the second line appears in Table 23.7. When the process is in control, the pH of the liquid varies according to a normal distribution with mean $\mu = 6.0$ and standard deviation $\sigma = 0.9$. The quality control plan calls for collecting samples of size three from batches produced each hour.

Sample		pH level	
1	7.2	7.4	7.4
2	6.9	6.6	6.5
3	6.2	6.3	6.3
4	6.8	6.4	6.5
5	6.5	6.6	6.7
6	6.8	6.8	6.8
7	6.2	6.3	6.4
8	5.6	5.7	5.9
9	4.9	5.8	5.6
10	6.4	6.0	4.4
11	6.9	5.3	6.2

Continued on the next page...

12	5.5	5.9	5.9
13	5.3	5.1	5.2
14	6.2	6.7	6.5
15	4.9	4.7	4.8
16	6.4	6.1	7.0
17	6.3	5.8	6.0
18	4.9	5.0	5.1
19	5.5	5.7	5.3
20	5.3	5.2	5.4
21	5.8	5.8	5.6
22	5.8	5.6	5.7
23	4.8	4.7	4.6
24	4.8	4.9	4.8

Table 23.7. pH of samples.

a. Calculate the sample means for each of the 24 samples.

b. Construct an $\bar{x}$ chart for the pH samples from the second plant line. Include reference lines marking the center line and one, two, and three $\sigma/\sqrt{n}$ on either side of the center line.

c. Based on the control chart from (b), does the process appear to be in control? If not, which decision rule applies and what appears to be the problem?