Unit 7: Normal Curves

PREREQUISITES

This unit requires an understanding of Unit 3, Histograms, Unit 4, Measures of Center, and Unit 6, Standard Deviation.

ADDITIONAL TOPIC COVERAGE

Additional coverage of normal curves can be found in *The Basic Practice of Statistics*, Chapter 3, The Normal Distributions.

ACTIVITY DESCRIPTION

The purpose of the activity is to familiarize students with the normal density curve. After completing the activity, they should be able to identify the mean and standard deviation of a normal distribution from its normal density curve. Students should also observe that most of the area under a normal density curve falls within three standard deviations of the mean. Therefore, the activity can serve as preparation for Unit 8, Normal Calculations, where students will learn the Empirical rule (or the 68-95-99.7% rule).

THE VIDEO SOLUTIONS

1. It is mound-shaped and symmetric; bell-shaped.

2. Since a normal curve is symmetric, the mean is at the line of symmetry.

3. The normal curve that is low and spread out has a larger standard deviation.

4. The mean arrival time for Year 33 appears to have decreased (shifted to an earlier date) compared to the mean in Year 1.

5. It has decreased from about 10% in Year 1 to 4% in Year 33.

UNIT ACTIVITY SOLUTIONS

1. a. Sample answer: Most of the data from a standard normal distribution should lie in the interval between -3 and 3. (Some students may pick a slightly wider interval. However, whatever interval they give, it should be centered at 0.)

b. Sample answer: Between -3 and 0. (Some students may give a value below -3.)

2. a. The dashed curve is flatter and more spread out than the standard normal density curve. Hence, it has a larger standard deviation than the standard normal distribution, which has a standard deviation of 1. So, the standard deviation will be larger than 1.

b. It looks like it should be around 1.5.

3. a. Normal density curves are symmetric about their mean. The line of symmetry for the dashed curve is at $\mu = 2$.

b. We would expect nearly all the data from this distribution to fall between -1 and 5. That looks to be about 3 standard deviations from the mean of 2.

4. Figure 7.12(a) is centered at $\mu = 15$. Going out three standard deviations on either side of 15 brings you to between 6 and 24, which would mean $\sigma = 3$.

So, Figure 7.12(a) goes with choice (ii).

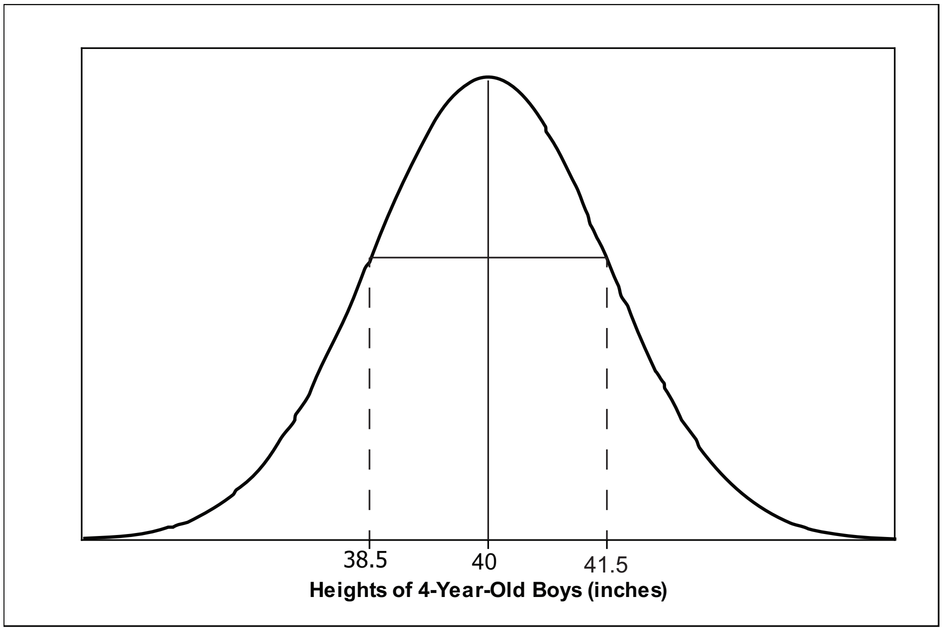
Figure 7.12(b) is centered at $\mu = 15$. Going out three standard deviations on either side of 5 brings you to between -1 and 11, which would mean that $\sigma = 2$. So, Figure 7.12(b) goes with choice (iv).

Figures 7.12(c) and (d) are centered at $\mu = 4$.

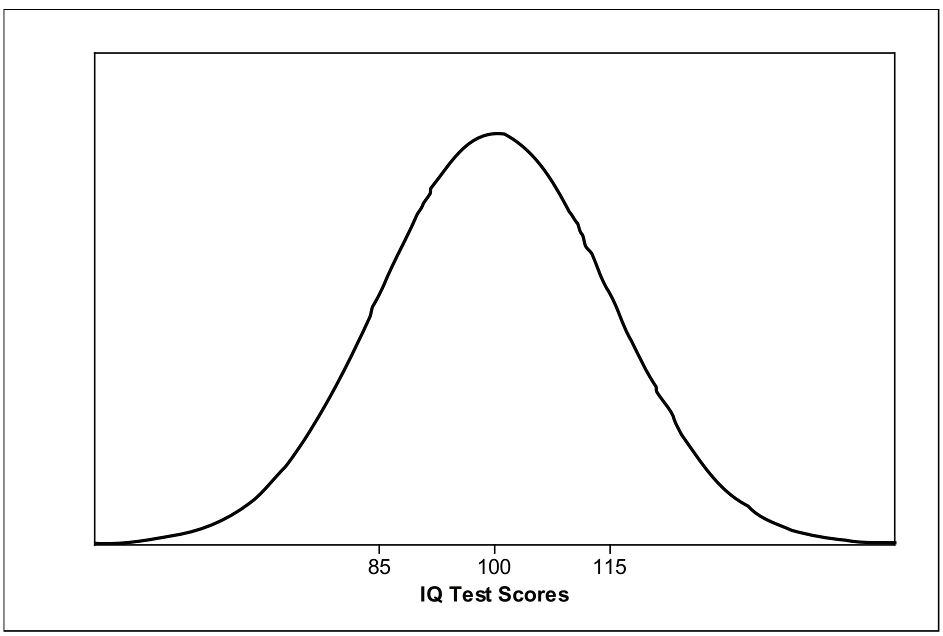
In each case, most of the data from the given distribution will fall within 3 standard deviations from the mean. Data from the distribution in Figure 7.12(d) is more spread out than data from the distribution in Figure 7.12(c). Hence, Figure 7.12(d) goes with choice (i), $\mu = 4$, $\sigma = 1$ and Figure 7.12(c) goes with choice (iii), $\mu = 4$, $\sigma = 0.5$.

EXERCISE SOLUTIONS

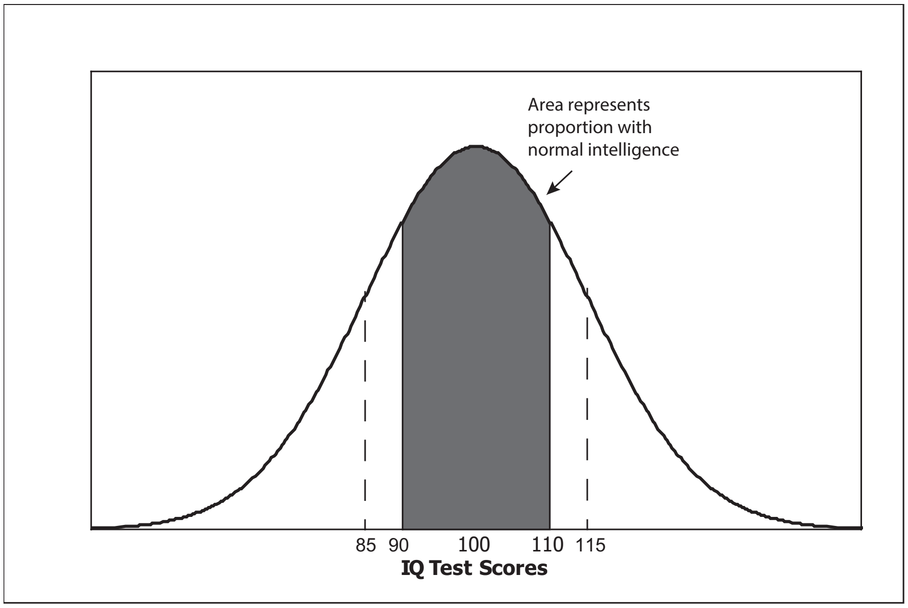
1.

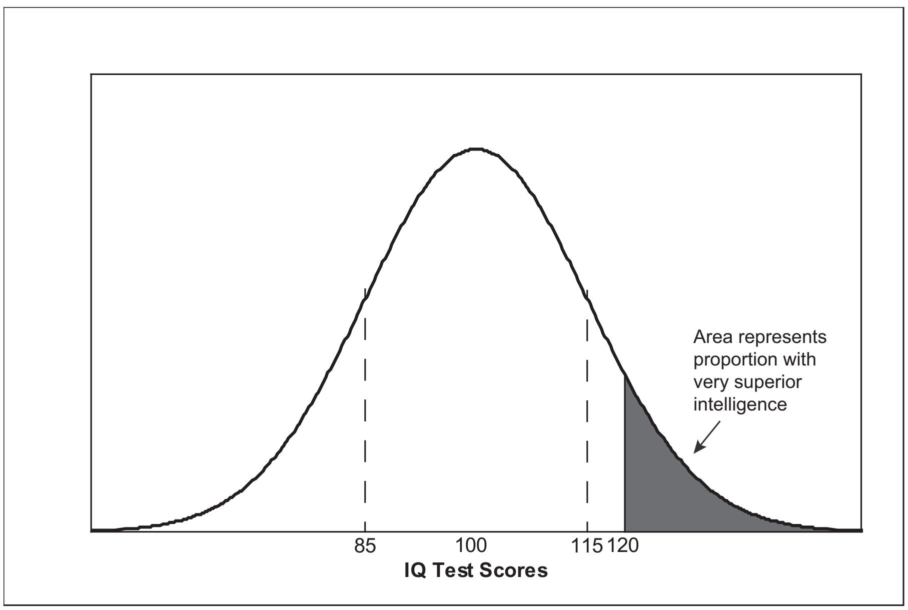


2. a.



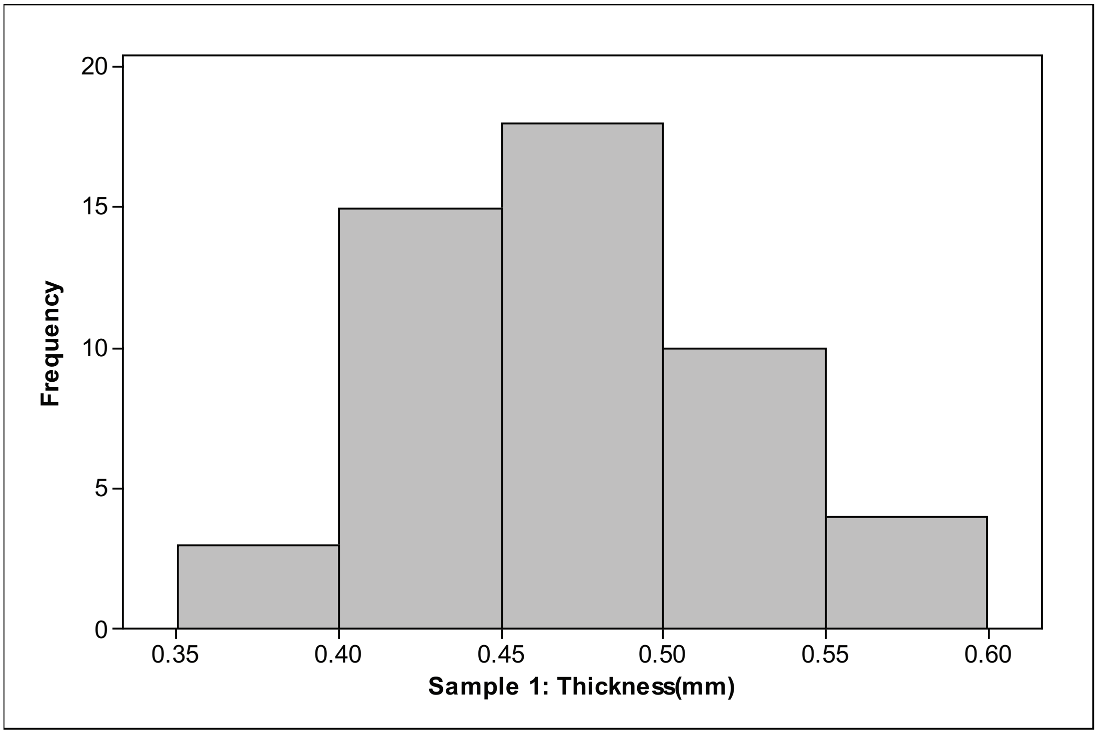
b.



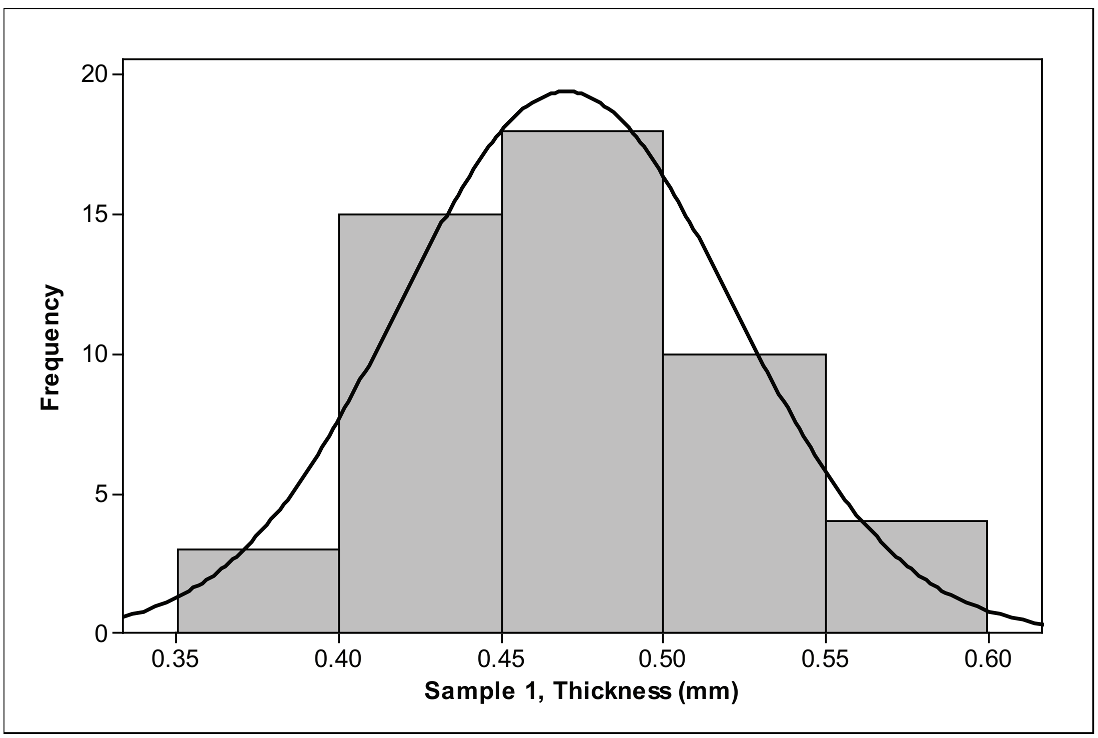


c. The shaded area under the curve over the interval from 90 to 110 appears larger than the shaded area under the curve to the right of 120. This indicates that there is a higher proportion of people with normal (or average) intelligence compared to people with very superior or genius intelligence.

3. a.

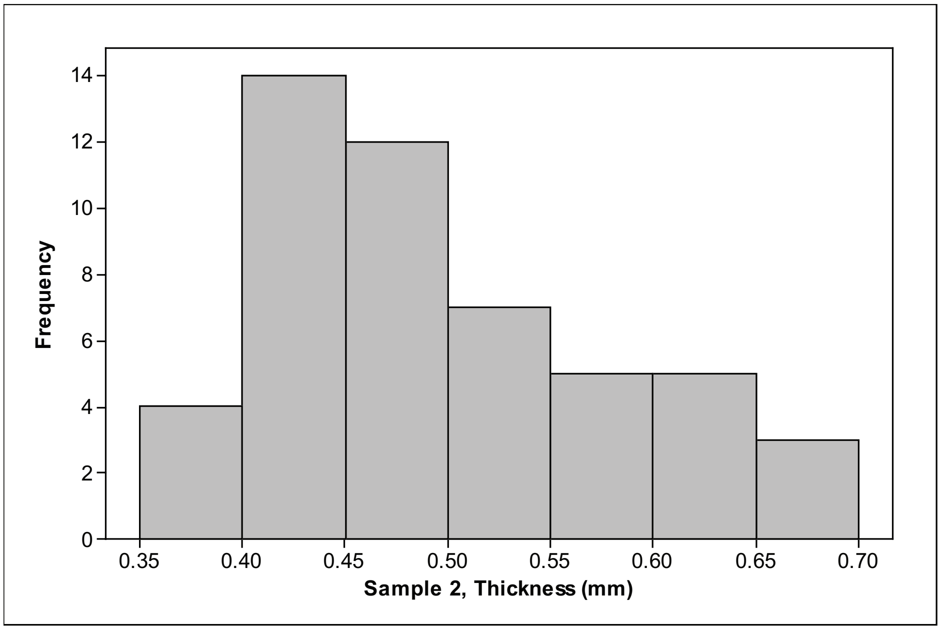


b.

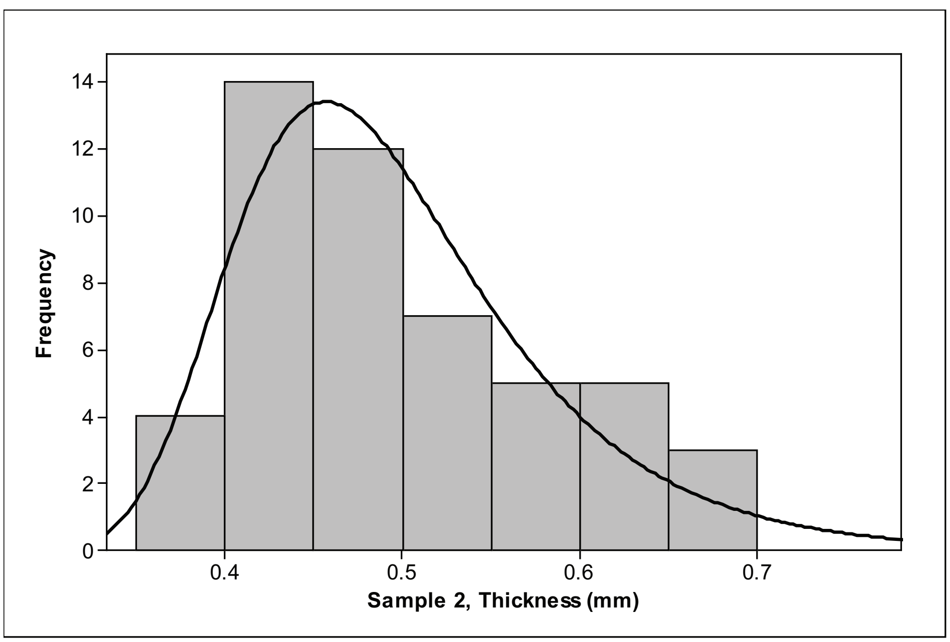


c. The balance point or mean appears to be less than 0.50.

4. a.



b. It does not appear that these data are from a normal distribution. The data appear skewed to the right and so the smoothed curve should also appear skewed to the right. Below is a sample smoothed curve that better captures the shape of this histogram than a normal curve.



c. Sample answer: The balance point appears closer to 0.5 mm with this distribution than for Sample 1 in question 3. The sample mean for Sample 2 is 0.497 mm, which is pretty close to 0.5.

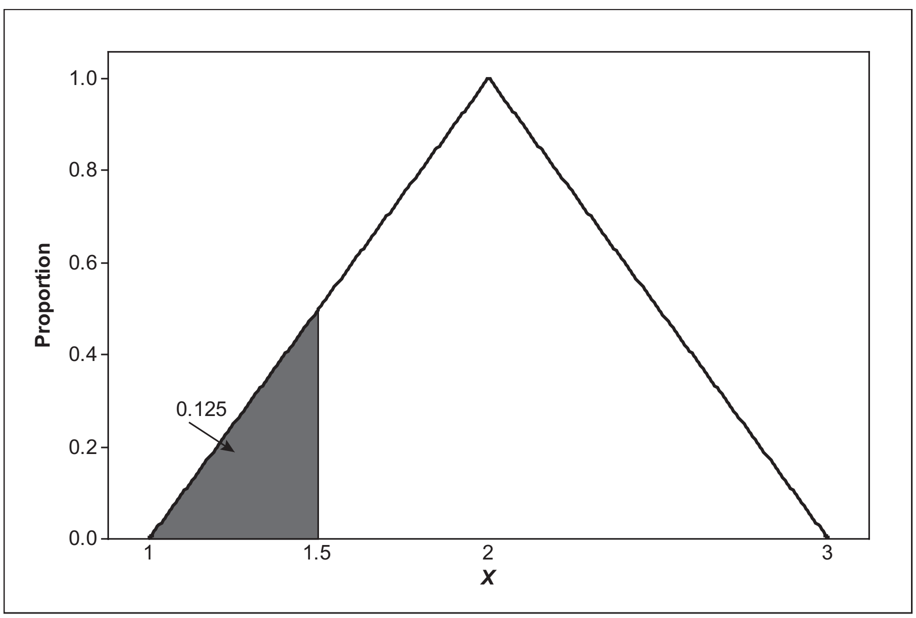
REVIEW QUESTIONS SOLUTIONS

1. a. The dashed curve represents a distribution with the larger mean. For normal curves, the mean is the line of symmetry. The lines of symmetry are around 25 and 30 for the solid curve and dashed curve, respectively.

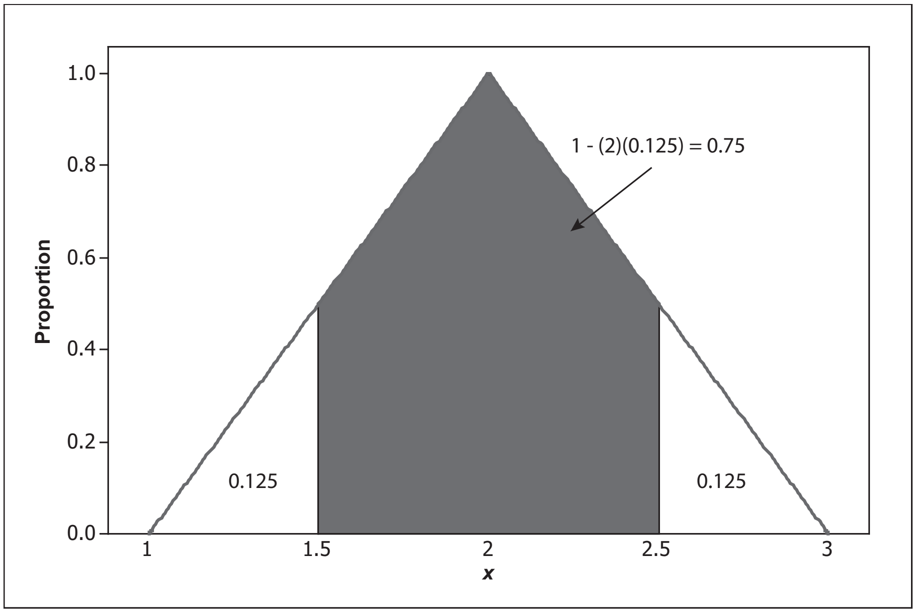
b. The solid curve represents a distribution with the larger standard deviation. It is flatter and its area under the curve is more spread out than for the dashed curve.

2. a. Because this distribution is symmetric, the mean is located at the line of symmetry for the curve. Therefore, $\mu = 2.0$.

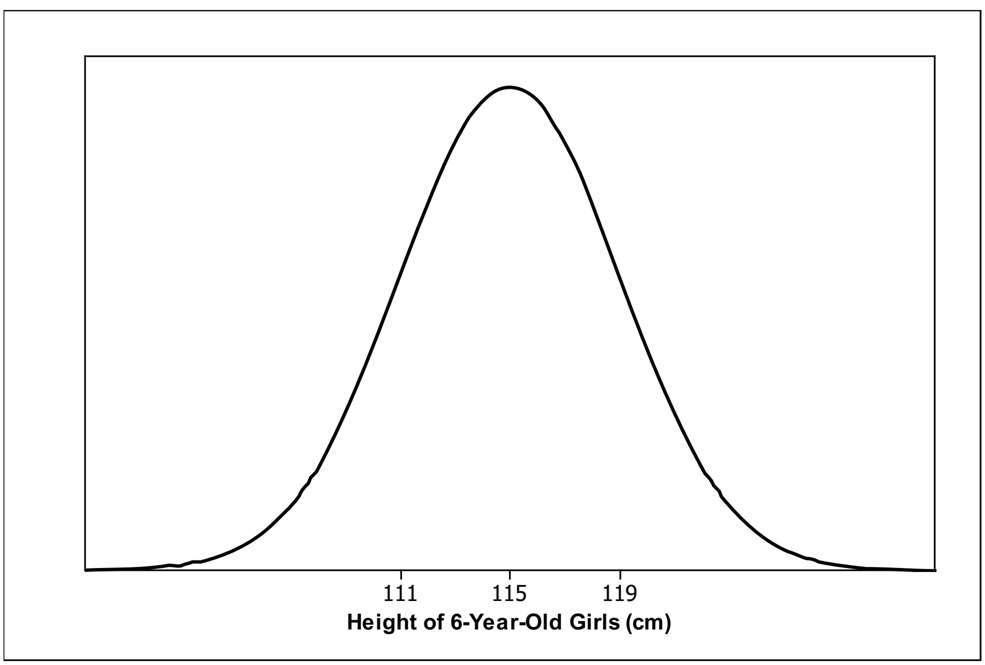
b. The area under the density curve to the left of 1.5 gives the proportion of data that fall below 1.5. This area forms the triangular region shown below. Area of triangle = ½(base)(height). So, in this case, proportion of data less than 1.5 is (1/2)(0.5)(0.5) = 1/8 = 0.125.



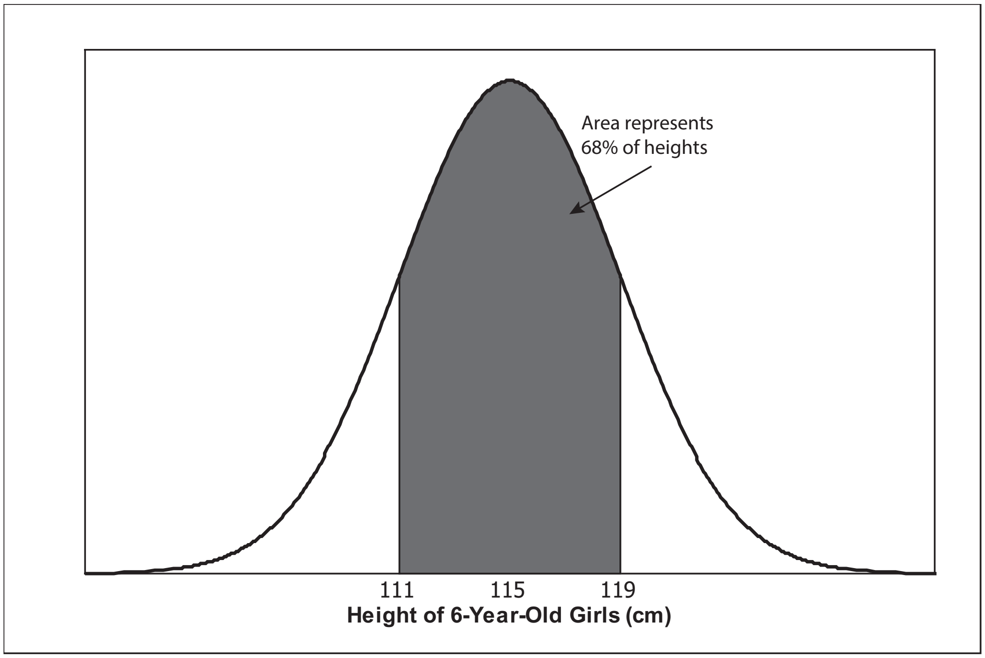
c. Since the density curve is symmetric, the proportion of data that is more than 2.5 is the same as the proportion of data that is less than 1.5. Because the area under a density curve is 1, the area of the region between 1.5 and 2.5 (shown below) is 1 - (2)(0.125) = 0.75.



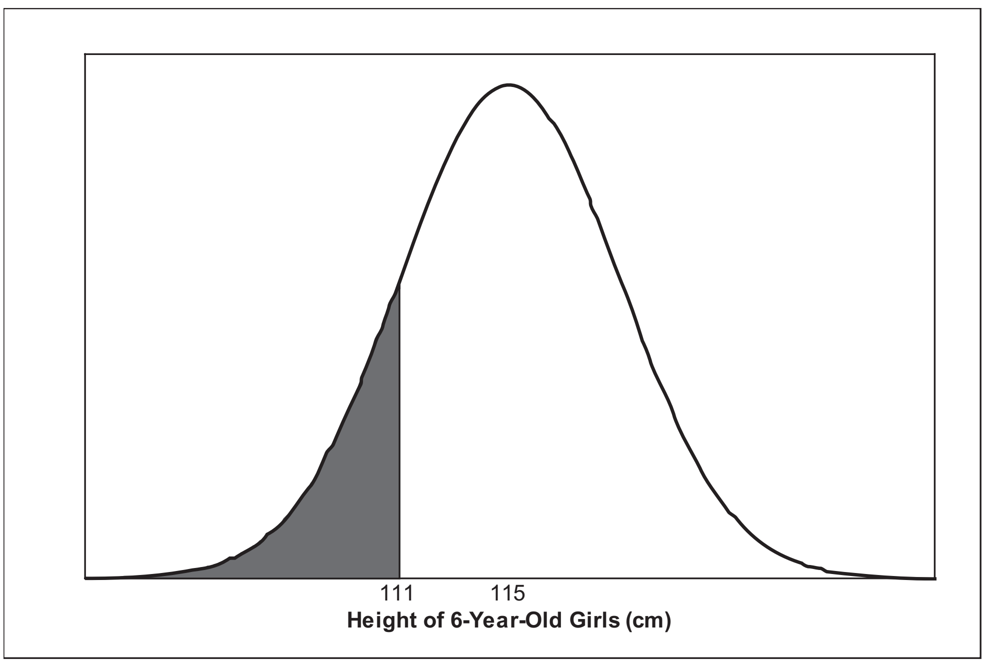
3. a.



b.

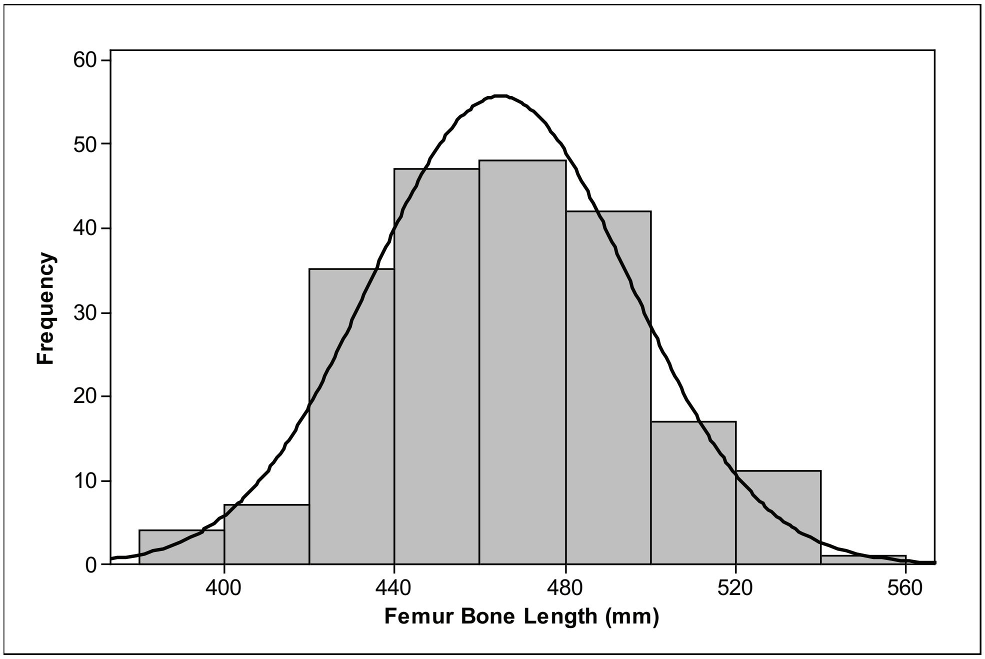


c. (100% - 68%)/2 = 32%/2 = 16%.



4. a. The data do not appear to be from a normal distribution. It looks as though there may be outliers to the left. There is a large peak between 70 and 72.5 and then a steep drop on either side. The data do not have the characteristic bell-shape of normal data.

b. The distribution of femur bone lengths looks like it could be from a normal distribution. The histogram has a symmetric mound shape. Below is a normal curve sketched over the histogram. (Software was used to fit a normal curve to these data.)



c. These data are strongly skewed to the right. Hence, they do not appear to be from a normal distribution.