

Annenberg/CPB
Professional Development Workshop Guide

PRIVATE UNIVERSE PROJECT IN MATHEMATICS

A professional development workshop
for K-12 mathematics teachers

**Produced by the Harvard-Smithsonian Center for Astrophysics in collaboration
with the Robert B. Davis Institute for Learning at Rutgers University**

Private Universe Project in Mathematics

is produced by
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Robert B. Davis Institute for Learning at Rutgers University.

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About the Workshop

Overview

This workshop provides an interactive forum for teachers, administrators, and other interested adults to explore issues about teaching and learning mathematics. Central to each session is a 60-minute videotape that offers a sequence of episodes that show children and teachers engaged in authentic mathematical activity and discussion, consistent with state and national standards for teaching and evaluating mathematics. These episodes come from a variety of sources in diverse school communities and across grade levels from pre-kindergarten through grade 12. The episodes and accompanying narratives in each videotape focus on:

1. students and teachers actively engaged in doing mathematics;
2. conditions that encourage meaningful mathematical activity; and
3. implications for learning, teaching, and assessment.

The materials and activities presented in the sessions have been developed in long-term research programs about mathematical thinking that share certain presuppositions about learning and teaching. Key to this perspective is that knowledge and competence develop most effectively in situations where students, frequently working with others, work on challenging problems, discuss various strategies, argue about conflicting ideas, and regularly present justifications for their solutions to each other and to the entire class. The role of the teacher includes selecting and posing the problems, then questioning, listening, and facilitating classroom discourse, usually without direct procedural instruction.

Each videotape contains episodes from a 12-year research study carried out in the Kenilworth, New Jersey public schools through a partnership with the Robert B. Davis Institute for Learning at Rutgers University. This partnership included a classroom-based staff development program in mathematics for teachers and administrators at the K-8 Harding Elementary School. Classroom sessions in which students frequently worked together in small groups on meaningful problem activities were videotaped as a regular part of the project. One classroom of children was followed from first through third grades by regularly videotaping small-group problem-solving sessions, whole class discussions, and individual task-based interviews. From 1991 to the present, with periodic support from two National Science Foundation grants, the research team has continued documenting and studying the thinking of a focus group of these children through grade 12. The videotapes from these later years document the students as they participated in problem-based activities developed by the university researchers in classroom lessons, after-school sessions, a two-week summer institute, and individual and small-group task-based interviews.

In the workshop videotapes, participants will see some of the same children solving mathematical problems at various grade levels over the years. In addition, the activities they engage in have been used in other communities and at other grade levels. In June 2000, the focus group of students in this study graduated from Kenilworth's David Brearley Middle/High School.

Workshop participants will explore particular questions about learning and teaching mathematics based on the shared experience of watching the videotapes. Key questions are:

1. How do children (and adults) learn mathematics?
2. How do children (and adults) learn to communicate about mathematics and to explain and justify solutions to problems?
3. What conditions—environments, activities, and interactions—are most helpful in facilitating this development?

About the Workshop

Overview, cont'd.

4. What does it mean to be a teacher of mathematics?
5. What is the connection between learning and teaching?

Each videotape includes episodes of children engaged in mathematical problem-solving. The goal is for participants to become able to recognize what is mathematical in students' activity by attending very closely to what they do and say. As they observe, study, and discuss what they see on the tape from the perspective of the questions listed above, participants will gain insights about learning and teaching. In preparation, participants need to build their own solutions to the central problems in each tape with assignments given between sessions and during the first hour of each workshop. They are encouraged to select and use appropriate problems with their own students and read further about the learning and teaching of these ideas.

Workshop Descriptions

Workshop 1. Following Children's Ideas in Mathematics

An unprecedented long-term study conducted by researchers at Rutgers University followed the development of mathematical thinking in a randomly selected group of students for 12 years—from first grade through high school—with surprising results. In an overview of the study, we look at some of the conditions that made the students' math achievement possible.

Workshop 2. Are You Convinced?

Proof making is one of the key ideas in mathematics. Looking at teachers and students grappling with the same probability problem, we see how two kinds of proof—proof by cases and proof by induction—naturally grow out of the need to justify and convince others.

Workshop 3. Inventing Notations

We learn how to foster and appreciate students' notations for their richness and creativity. We also look at some of the possibilities that early work in creating notation systems might open up for students as they move on toward algebra.

Workshop 4. Thinking Like a Mathematician

What does a mathematician do? What does it mean to "think like a mathematician"? This program parallels what a mathematician does in real life with the creative thinking of students.

Workshop 5. Building on Useful Ideas

One of the strands of the Rutgers long-term study was to find out how useful ideas spread through a community of learners and evolve over time. Here, the focus is on the teacher's role in fostering thoughtful mathematics.

Workshop 6. Possibilities of Real-Life Problems

Students come up with a surprising array of strategies and representations to build their understanding of a real-life calculus problem—before they have ever taken calculus.

Workshop 7. Next Steps (required for those seeking graduate credit)

Participants will review key ideas presented during the workshops and consider implications for their own teaching.

About the Workshop

Video Clip Descriptions

Workshop 1. Following Children’s Ideas in Mathematics

Part 1—The Youngest Mathematicians

5 min. Mathematics in Free Play?

Prof. Herbert Ginsburg, a psychologist at Columbia University Teachers College, finds that when you examine what children at ages three and four actually do in free play, more than 50 percent of the time they are engaged in mathematical tasks.

5 min. The Beginning of the Rutgers/Kenilworth Long-Term Study

In 1987, Prof. Carolyn Maher, from the Robert B. Davis Institute for Learning at Rutgers University, is invited to the elementary school in Kenilworth, New Jersey, a small town of mostly moderate-income working families. What begins as a professional development project for teachers evolves into a 12-year study—carefully documented on video—of the development of mathematical thinking of a randomly selected group of students.

12 min. Shirts and Pants

Video clips from the research archive show how the students approach a combinations problem. During their first attempt, in the second grade, they come up with a variety of answers. Four months later, in the third grade, they spontaneously arrive at the correct answer.

5 min. Cups, Bowls, and Plates

Later in the third grade, in an extension of the Shirts and Pants problem, one student, Stephanie, shows how she used multiplication to arrive at an answer.

Part 2—From “Towers” to High School

10 min Building Towers Five-High

The Kenilworth students in the fourth grade are seen working on the Towers problem (“How many different towers can you build by selecting from blocks of two colors?”) juxtaposed with clips of the same students, now seniors in high school, reflecting back on the experience.

5 min. Ninth-Grade Geometry

Transferred to a regional school, where the math teacher’s focus was to finish every chapter in the textbook, the students feel they learned very little.

5 min. Romina With Her Parents

Now in her senior year, one of the students in the focus group, Romina, confers with her parents about colleges.

5 min. Romina at School

In her AP Calculus class, Romina solves a problem in an interesting way that surprises her teacher.

About the Workshop

Video Clip Descriptions, cont'd.

Workshop 2. Are You Convinced?

Part 1—Teachers Building Proofs

25 min. Englewood, New Jersey—Teachers Workshop

Englewood, a town with unsatisfactory student test scores, is implementing a long-term project to improve math achievement. As part of a professional development workshop designed in part to give K-8 teachers more confidence in their own mathematical thinking, teachers come up with a wide variety of justifications for their answers to the Towers problem.

Part 2—Students Building Proofs

8 min. Working With Towers

In the third grade, students in the Kenilworth study build towers four-high, and hypothesize about towers three-high. In the fourth grade, they build towers five-high.

20 min. “Gang of Four”

In the fourth grade, a group of four students from the Kenilworth focus group come up with mathematically sound proofs for the number of combinations in the Towers problem—and then generalize their solution to apply to all towers.

Workshop 3. Inventing Notations

Part 1—Putting It on Paper: Elementary Students Invent Notations

15 min. Pizzas in the Classroom

In Englewood, New Jersey, Blanche Young, who attended the summer workshop, tries out one of the problems with her fourth-grade students. Later, she meets with Arthur Powell to discuss the lesson.

5 min. New Brunswick, New Jersey

In addition to extensive research in Kenilworth, New Jersey, the math researchers from the Robert B. Davis Institute for Learning also visit the urban district of New Brunswick. Here, a group of fifth-graders comes up with a number of useful notations to help solve the Pizza problem.

5 min. Brandon Connects Pizzas and Towers

In Colts Neck, New Jersey, one student, Brandon, explains the correspondence between the Pizza and Towers problems and describes in detail how the same notation can be used for both.

Part 2—Notations Evolve As Students’ Thinking Evolves (and Vice Versa)

25 min. Kenilworth Study: Pizzas

In the fourth grade, the students encounter counting problems where the solutions cannot be built using standard manipulatives. As he invents his own notation systems, one student, Matt, builds on previous work to arrive at a solution for an even more complex problem: Pizzas With Halves.

About the Workshop

Video Clip Descriptions, cont'd.

Workshop 4. Thinking Like a Mathematician

Part 1—Strategies for Solving Problems

10 min. How a Mathematician Approaches Problems

Fern Hunt, a mathematician at the National Institute for Science and Technology, is seen as she collaborates with colleagues to solve difficult technical problems. Using the metaphor of the children's game Towers of Hanoi, she explains her approach to solving problems.

15 min. Towers of Hanoi

In a research session conducted by the late Robert Davis of Rutgers University, sixth-graders from the Kenilworth study put into practice problem-solving strategies that mirror the strategies outlined by Fern Hunt.

Part 2—Encouraging Students To Think

10 min. Provo, Utah—High School Algebra

Presidential award-winning math instructor Janet Walter has inherited a ninth-grade Algebra I class part-way through the school year. How can she overcome their reticence to share ideas, and help them think for themselves?

5 min. Revisiting Problems After Five Years

Kenilworth 10th graders re-examine the same problem they had last seen in the fifth grade—the Pizza problem. One student, Michael, uses the binary number system to his advantage.

10 min. Romina's Proof

Responding to a problem posed by one of the students, Romina, a 10th-grader, invents a proof solution and shares it with the others.

Workshop 5. Building on Useful Ideas

Part 1—The Changing Role of the Teacher: Elementary Classrooms

5 min. Englewood—Kindergarten: Stacking Blocks

In Englewood, New Jersey, a kindergarten teacher from a summer workshop uses blocks as mathematical objects in an addition activity.

5 min. Englewood—Second Grade: Probing Student Thinking

How can a teacher know what an individual student is thinking when there are 24 or more students in the room? In Englewood, a second-grade teacher tries to follow her students' thinking by asking appropriate questions as she moves from table to table.

10 min. Englewood—Fourth Grade: Towers

Fourth-grade teacher Blanche Young attempts the Towers activity for the first time with her students. She feels that their work is valuable, but questions how much time these open-ended activities are taking away from the standard curriculum.

About the Workshop

Video Clip Descriptions, cont'd.

Workshop 5, cont'd.

5 min. “Equations”

In Colts Neck, New Jersey, fourth-grade teacher and former Rutgers researcher Amy Martino finds out that what started as a 15-minute “warm-up” question evolves into an interesting discussion about equations.

Part 2—Pascal’s Triangle and High School Algebra

10 min. Jersey City: Ice Cream Problem

Algebra II teacher Gina Kiczek introduces a problem that helps her students learn the difference between permutations and combinations.

5 min. What Is Pascal’s Triangle?

An overview of the “Arithmetic Triangle”: what it is, its history, and how it is linked to the Towers and Pizza problems.

10 min. World Series Problem

In the 11th grade, the Kenilworth students build on their thinking as young children to tackle a complex—and realistic—probability problem.

5 min. n choose r : Moving Toward Standard Notation

In an after-school research session, Mike derives the formula for the general combinatorics problem “compute n choose r .”

Workshop 6. Possibilities of Real-Life Problems

Part 1—The Catwalk: Representing What You Know

25 min. The Catwalk, Part 1: Representing What You Know

In a voluntary two-week summer workshop, 18 high school seniors from Kenilworth and New Brunswick work on a real-life problem based on a sequence of 24 photographs of a cat in motion. The question, “How fast is the cat moving in frame 10 and frame 20?” deals with some of the fundamental ideas of calculus. Students find several ways to represent their growing understanding: visual, symbolic, and kinesthetic.

Part 2—Betting on What You Know

25 min. The Catwalk, Part 2: Betting on What You Know

Continuing the problem to its conclusion, the students use their representations as the basis for reconstructing the cat’s movement.

5 min. The Class of 2000 and Beyond

In Kenilworth, New Jersey, the students in the Kenilworth study are graduating from high school. In a montage of interviews, they reflect back on their involvement with the long-term study, look toward the future, and wonder, “Does the way in which you study math make a difference?”

Workshop Components

On-Site Activities and Timelines

Getting Ready (Site Investigation)

In preparation for watching the video, you will engage in 60 minutes of doing math problems, discussion, and activity—exploring the math for yourself.

Watch the Workshop Video

Watch the 60-minute workshop video, which is divided into two parts. Each part includes a focus question that appears on screen and can be found in this guide. Participants in workshop settings will either view the live broadcast on the Annenberg/CPB Channel or watch a pre-taped video of the program. If you are watching a live broadcast, discuss the focus questions at the end of the broadcast. If you are watching on videotape, stop the tape for a discussion at the end of each part.

Going Further (Site Investigation)

Wrap up the workshop with an additional 30 minutes of investigation through discussion and activity. The Episode Boxes in this guide contain brief descriptions of video clips and related questions for exploring learning and teaching.

For Next Time

Homework Assignments

You will be assigned exercises or activities that tie into the previous workshop or prepare you for the next one.

Reading Assignments

Readings will be assigned to prepare you for the following workshop. They can be found in the Appendix of this guide. Some are also available online at www.learner.org/channel/workshops.pupmath.

Ongoing Activities

You may want to carry on these activities throughout the course of the workshop.

Keep a Journal

Participants are encouraged to keep a reflective journal throughout the workshop, to keep track of reactions to readings and videotapes, to collect and reflect on data, and to record teaching ideas for yourself.

Visit the Web Site: www.learner.org/channel/workshops/pupmath

Go online for additional activities, resources, and discussion opportunities.

Share Ideas on Channel-TalkPUPMath

Participants may subscribe to an email discussion list and communicate with other workshop participants online. To subscribe to Channel-TalkPUPMath, visit: <http://www.learner.org/mailman/listinfo/channel-talkpupmath>.

About the Contributors

Carolyn Maher

Carolyn Maher is a professor of mathematics education in the Graduate School of Education at Rutgers University and the director of the Robert B. Davis Institute for Learning. The Davis Institute has a successful history of long-term commitments to education reform initiatives and works closely with schools and districts in New York and New Jersey. Dr. Maher's longitudinal research, now in its 12th year, focuses on the development of children's mathematical thinking and development of proof. She has given presentations and led workshops for groups of teachers, math educators, and administrators throughout the United States as well as in diverse settings such as Australia, Brazil, Canada, Israel, South Africa, Mozambique, Japan, and China. Dr. Maher is also the editor of *The Journal of Mathematical Behavior* and the director of the regional center at Rutgers University for the New Jersey Statewide Systemic Initiative.

Alex Griswold

Alex Griswold is a producer and videographer at the Harvard-Smithsonian Center for Astrophysics. Mr. Griswold has been producing educational films and television programs for over 25 years, and has taught video and film production at Harvard University; the Department of Defense Dependents Schools, Madrid, Spain; and the Boston Film/Video Foundation. Since joining the staff of the Center for Astrophysics in 1992, he has specialized in the creation of teacher education materials in mathematics and science, gaining wide experience working with children and teachers in educational settings. His current responsibilities include management of a science visualization laboratory and producing interactive media, video, and film that enhance learning in math and science.

Alice Alston

Alice Alston is a senior mathematics education specialist at the Robert B. Davis Institute for Learning and a visiting associate professor of mathematics education at Rutgers University Graduate School of Education. She co-authored the PACKETS Program for Upper Elementary Mathematics while working at the Educational Testing Service in Princeton, New Jersey. Dr. Alston works extensively with teachers in New Jersey schools, and particularly urban schools. She has additional expertise in standards-based professional development for mathematics educators as a result of experience implementing the NSF Middle Grades Project, Linking Instruction and Assessment. She leads a professional development program in mathematics, science, and literacy in three urban districts in New Jersey. Formerly, Dr. Alston taught middle and high school and was chair of the Middle School Mathematics Department at Princeton Day School.

Emily Dann

Emily Dann is a senior mathematics education specialist at the Robert B. Davis Institute for Learning. She has taught mathematics at the middle school through college level, and mathematics education at both undergraduate and graduate levels. Dr. Dann has worked in both the New York and New Jersey Statewide Systemic Initiatives and provides expertise in implementing standards-aligned, research-based curriculum programs for K-12 mathematics educators. She currently coordinates the Rutgers-Colts Neck Partnership for Implementing a Thinking Curriculum, a professional development project funded by the Exxon Mobil Foundation. Dr. Dann is the associate director of the regional center at Rutgers University for the New Jersey Statewide Systemic Initiative.

About the Contributors

Regina Kiczek

Regina Kiczek is K-8 mathematics supervisor for the Westfield, New Jersey school district. A former high school mathematics teacher with over 25 years of teaching experience, Ms. Kiczek has recently completed her doctorate in mathematics education at Rutgers University. She has been a research team member of the NSF-funded longitudinal study of students' proof making, and has experience planning and implementing professional development programs for K-12 mathematics educators. Her research into the development of probabilistic thinking of students was presented in 2000 at the Ninth International Congress on Mathematics Education, Tokyo, Japan.

Arthur Powell

Arthur Powell teaches mathematics and mathematics education in the Department of Education and Academic Foundations at Rutgers University. For over two decades, Professor Powell has worked with elementary and secondary teachers in the United States, Mozambique, Brazil, and Canada. He is a faculty research scientist at the Robert B. Davis Institute for Learning and currently directs a teacher development project for the district of Englewood, New Jersey, working with teachers in summer workshops and supporting teachers' work with children in their classrooms.

Robert Speiser

Robert Speiser, whose work as a mathematician has been recognized internationally, is currently a professor of mathematics education at Brigham Young University. He has taught elementary school and worked with teachers at all levels. Dr. Speiser leads a K-3 Mathematics Specialist Project which supports the joint efforts of university mathematics educators, a study group of elementary school teachers, and students in a university-led teacher preparation program. His research in education concentrates on the growth of mathematical understanding, especially through explorations of rich tasks in settings which promote reflection and discussion. Dr. Speiser is the editor of *The Journal of Mathematical Behavior*.

Elena Steencken

Elena Steencken is a mathematics education specialist and assistant director of the Robert B. Davis Institute for Learning in the Graduate School of Education at Rutgers University. She is currently completing her doctorate in mathematics education and has worked extensively in preservice elementary and secondary teacher preparation programs at Rutgers University. She has also collaborated with K-12 inservice teachers in various professional development programs in New Jersey and New York. She has recently prepared *Exploring To Build Meaning About Fractions*, a unit booklet produced by the Robert B. Davis Institute for Learning for use by preservice and inservice teachers.

Charles Walter

Charles Walter, a mathematician, is currently a professor of mathematics education at Brigham Young University. His 30-year commitment to mathematics and mathematics education encompasses both the preparation of elementary and secondary teachers and the collaboration with teachers and children in classrooms. He leads a K-3 Mathematics Specialist Project which supports the joint efforts of university mathematics educators, a study group of elementary school teachers, and students in a university-led teacher preparation program. Dr. Walter has designed and conducted NSF-sponsored workshops centered on mathematics and pedagogy in secondary and post-secondary calculus classrooms. His research focuses on how learners, especially children, build and represent mathematical knowledge. He is the assistant editor of *The Journal of Mathematical Behavior*.

Helpful Hints

Successful Site Investigations

Included in the materials for each workshop, you will find detailed instructions for the content of your **Getting Ready** and **Going Further** Site Investigations. The following hints are intended to help you and your colleagues get the most out of these pre- and post-video discussions.

Designate a Facilitator

Each week, one person should be responsible for facilitating the Site Investigations (or you may select two people—one to facilitate Getting Ready, the other to facilitate Going Further). The facilitator does not need to be the Site Leader, nor does the role need to be held by the same person each week. We recommend that participants rotate the role of facilitator on a weekly basis.

Review the Site Investigations and Bring the Necessary Materials

Be sure to read over the Getting Ready and Going Further sections of your materials before arriving at each workshop. The Site Investigations will be the most productive if you and your colleagues come to the workshops prepared for the discussions. The weekly readings and homework assignments also provide for productive and useful workshop discussions. A few of the Site Investigations require special materials. The facilitator should be responsible for bringing these when necessary.

Keep an Eye on the Time

You should keep an eye on the clock so that you are able to get through everything before the workshop video begins. In fact, you may want to set a small alarm clock or kitchen timer before you begin the Getting Ready Site Investigation to ensure that you won't miss the beginning of the video. (Sites that are watching the workshops on videotape will have more flexibility if their Site Investigations run longer than expected.)

Record Your Discussions

We recommend that someone take notes during each Site Investigation, or even better, that you make an audiotape recording the discussions each week. These notes and/or audiotape can serve as "make-up" materials in case anyone misses a workshop.

Share Your Discussions on the Internet

The Site Investigations are merely a starting point. We encourage you to continue your discussions with participants from other sites on the Teacher Talk area of the Web site at www.learner.org/channel/workshops/pupmath and on Channel-TalkPUPMath, the workshop email discussion list.

Materials Needed

Site Investigations

Workshop 1. Following Children's Ideas in Mathematics

Paper, pens, or markers: The group will need paper and pens or markers for preparing solutions to the problems. Especially if you are a large group, you may want to have an overhead projector, blank transparencies, and pens for participants to use when sharing solutions.

Unifix® or other snap cubes: Each participant will need about 100 cubes (50 each of two colors) to complete the homework assignment for Workshop Two. Although not essential, sharing and discussing solutions will be much easier if everyone is using the same two colors. If this is impossible, sets of cubes need to be made up for each participant to use with two colors that can be designated as "light cubes" and "dark cubes" when their solutions are discussed. If Unifix® cubes are not available, use the "cut-out cubes" sheets included in this guide at the end of Workshop 1.

Workshop 2. Are You Convinced?

Unifix® or other snap cubes, or "cut-out cubes" (see above)

Workshop 5. Building on Useful Ideas

Pascal's Triangle Worksheet (see pages 48-49 of this guide).

Workshop 6. Possibilities of Real-Life Problems

Catwalk: Each participant will need at least two copies of the cat photographs on 11 x 17 paper and on transparencies (a copy is included on pages 56-57 of this guide; please piece together the two parts and photocopy onto 11 x 17 paper); metric rulers (clear plastic ones work best); graph paper; a calculator (graphing calculator if possible); and pens or markers for preparing solutions to the problems. You will also want to have an overhead projector, blank transparencies, and pens for sharing solutions.

