Workshop 6.
Possibilities of Real-Life Problems

This session builds on student work from July 1999, part of a two-week Summer Institute for high school students in Kenilworth, New Jersey. The students here include several students seen in prior workshops. Four days toward the end of the Institute, they were devoted to a single task called the Catwalk, in which students are invited to work closely with a set of photos of a moving cat. At Kenilworth, Catwalk was tried for the first time with high school students who had never taken a calculus class.

Part 1—The Catwalk: Representing What You Know

Researchers introduce the cat problem—attempting to measure the instantaneous speed of a cat in photographs taken by Eadweard Muybridge, around 1880. The students pounce on the problem—literally—as they run like a cat along a representation of the cat's journey recreated at a human scale.

On-Screen Participants: Prof. Charles Walter, Brigham Young University; and Prof. Robert Speiser, Brigham Young University. **Student Participants:** Former and present participants in the Rutgers long-term study from Kenilworth, New Jersey and New Brunswick, New Jersey Public Schools, Grade 12.

Part 2—Betting on What You Know

In real-life questions, numerical “answers” need to be interpreted in terms of the problem to be solved. What conclusions can we draw from what we found? Here, to build interpretations that will make sense of the numbers they have calculated, students focus on building and relating different representations of the information the photos make available, and connect these to their own experience. The program ends with the 2000 Kenilworth high school graduation. Students who participated in the Rutgers long-term study ponder: Does the way you study math make a difference?

Student Participants: Students from Kenilworth Public Schools, Grade 12; and New Brunswick Public Schools.

On-Site Activities and Timeline

60 minutes

Getting Ready

1. Catwalk

Last session's homework was to examine the Catwalk photographs and decide:

a. How fast is the cat moving in frame 10?

b. How fast is the cat moving in frame 20?

In small groups, discuss your ideas about the Catwalk. Be prepared to explain to the class (a) what you found, (b) what difficulties you may have encountered, (c) key choices that you made along the way, and (d) remaining questions you may wish to pursue.

Is this a problem you might use with your students? How do you think your students might solve this problem?

60 minutes

Watch the Workshop Video

Part 1

On-Screen Math Activities

Catwalk

Students are given a series of photographs of a cat moving from left to right against a grid background. The photos were taken a little less than 1/30 of a second apart. Students measure the movement of the cat against the grid and try to calculate its instantaneous speed in frames 10 and 20.

Focus Question

How might the personal experience of running help these students to deepen, organize, or clarify their growing understanding of the motion of the cat?

Part 2

On-Screen Math Activities

Catwalk, cont'd.

See description above. Students discuss the level of certainty they have about the accuracy of their answers.

Focus Question

What conditions in the mathematics classroom might be required in order to make mathematics a meaningful subject for all student all the way through high school?

On-Site Activities and Timeline

30 minutes

Going Further

Please address as many of these questions as possible during your allotted time and consider the remainder as part of your final assignments.

Episode Box One: Aquisha Uses Pairs of Transparencies

Romina explains to Chuck how she, Magda, and Aquisha are working frame-by-frame, using transparencies of the cat photos. Aquisha has developed a method of measuring where she places one transparency on top of another. We see Aquisha pass a pair of transparencies, one placed carefully on top of the other, to Magda. We watch as Magda studies these transparencies.

- What, precisely, did Aquisha do with her transparencies? What advantage do you think she gained? What distances did these students measure on the photos? How does distance measured on the photos correspond to the distance traveled by the cat?

Episode Box Two: Shelly

Shelly reports that her group has found three different answers at Frame 10. They don't yet know if what they've done is right.

- What do you think about the situation in Frame 10? On what basis might you say that you are right?

Episode Box Three: Angela and Shelly at the Overhead

Angela (at overhead) and Shelly (foreground) present their findings for Frame 10. They obtain quite different average velocities for Frames 9-10 and Frames 10-11. They average these velocities, and they propose this average as the cat's velocity at Frame 10.

- How do their average velocities (for Frames 9-10, and for Frames 10-11) compare with yours? What do you think about averaging such different results?

- Is the situation at Frame 10 significantly different from the situation in Frame 20?

On-Site Activities and Timeline

Going Further, cont'd.

Episode Box Four: Aquisha's Line Representation

Aquisha goes to the overhead to explain how she found the distances the cat had traveled between frames. She marked these distances as 23 separate line segments, and then assembled these into a composite picture of the cat's movement through all 24 frames.

- How does Aquisha obtain her 23 line segments from the photos?
- How does she put them together, to construct the line representation?
- How might this line representation help us to make sense of the cat's motion at Frame 10? What does this representation help us see?
- How did others in the room respond?

Episode Box Five: Matt Discusses Acceleration

In Part 1 of the video, Mike displayed a graph of position versus time and discussed velocities. Here, with Romina's graph of velocity versus time displayed, Matt discusses acceleration.

- What is similar and what is different between these two discussions? In particular, how are position, velocity, and acceleration being represented?
- How are these representations being used?

Episode Box Six: Romina's Question

Despite his moral and legal objections to betting, Victor just proposed exactly the same velocity for Frame 10 (namely, 145.161 cm/sec) that Angela and Shelly had presented earlier, with exactly the same reasoning. Romina questions Victor's rationale.

- When, in your experience, has averaging made sense? Why?
- In posing her question, Romina stresses the "big jump" between the interval velocities (2/.031 and 7/.031 cm/sec) which Victor and the others averaged. Why might this "jump" in speed be important here?

On-Site Activities and Timeline

Going Further, cont'd.

Episode Box Seven: Matt

Matt gives Mr. Pantozzi a criterion for when he ought to bet. Let's reflect on the students' discussions, over the whole tape, that led up to this criterion.

- How many different ideas contribute necessary evidence for what Matt is suggesting?
- How many different people made important contributions in the process?

Preparing for the Future

Final Assignments

1. Study the Episode Boxes on the previous pages. Reflect on the questions and record your reactions and ideas in your journal.

2. Now study your journal. We have looked at learning, starting with young children, following the story of their growing mathematical understanding over more than 10 years. How might their early building be reflected in the work they did on Catwalk? Is it a direct application of ideas built in the early grades? Is it in the way they reason and demand careful explanation? Is it in the way they rethink, reformulate, and then rebuild what they have done before? Is it in the way they listen to each other?

3. Try the Catwalk, or another real-world task involving motion with your students. Take notes of what the learners do; reflect—in writing—on your own participation; collect all written work; and then reflect alone and with your colleagues about how motion, change, and mathematics might take shape together, in your understanding, in your classroom and your school.

Notes