Workshop 5.
Building on Useful Ideas

One of the most important conditions of the Rutgers' long-term study was that students were invited to work together to solve problems. By sharing and justifying their ideas, students are able to clarify their own thinking. Collaborative work thus becomes the vehicle for advancing each individual student's ideas. Workshop 5 focuses on how teachers can foster thoughtful mathematics through building a learning community.

Part 1—The Changing Role of the Teacher: Elementary Classrooms

Three teachers who participated in the Englewood summer professional development workshop try activities in their elementary school classrooms. The teachers include Michelle, a kindergarten teacher; Melissa, a second-grade teacher; and Blanche, a fourth-grade teacher. We observe Amy, a researcher in the long-term study, as she presents an activity to her fourth-grade students in Colts Necks, New Jersey. Please note the mathematical activities, how these activities are introduced by the teachers, and what strategies their students employ in solving the tasks.

On-Screen Participants: Blanche Young, Englewood Public Schools, Grade 4; Melissa Sharpe, Englewood Public Schools, Grade 2; Michelle Doherty, Englewood Public Schools, Kindergarten; and Dr. Amy Martino, Colts Neck Public Schools.

Part 2—Pascal's Triangle and High School Algebra

As a follow-up to earlier Pizza and Towers tasks, high school math teacher Dr. Gina Kiczek invites her Jersey City, New Jersey, students to solve other counting problems. In another setting, Stephanie, a student in the long-term study, connects the Towers activity to Pascal's Triangle in grade 8, and then the Pizza and Towers problems to Pascal's Triangle in grade 11. Then observe a group of 11th-graders from the long-term study as they investigate the World Series problem, and later link Pascal's Triangle to standard notation in the form of "*n* choose *r*."

On-Screen Participants: Dr. Regina Kiczek, James J. Ferris High School, Jersey City, New Jersey. **Student Participants:** Stephanie, Michael, Romina, Brian, Jeff, and Ankur, Kenilworth Public Schools.

On-Site Activities and Timeline

60 minutes

Getting Ready

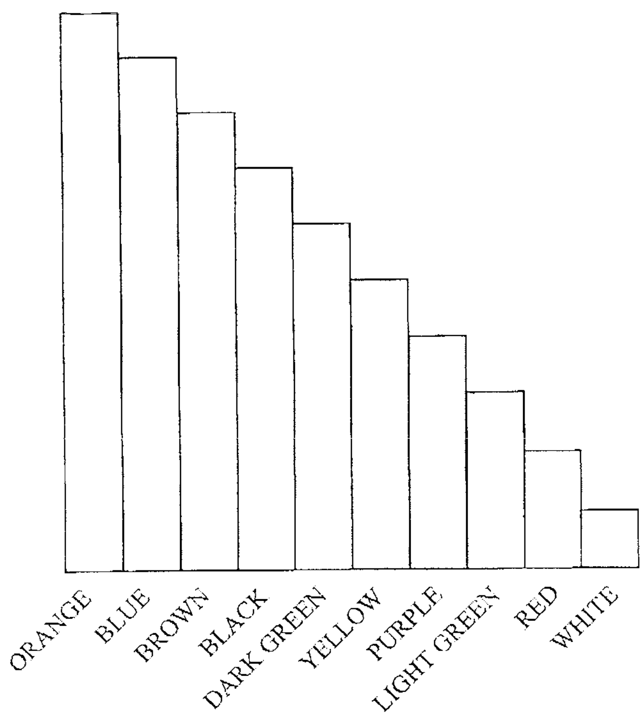
1. The World Series Problem

Your homework for today's workshop was the World Series problem. Share your results in small groups.

Discuss as many of the following as time will allow:

2. The Cuisenaire® Rod Activities

Cuisenaire® Rods are used in the kindergarten and second-grade activities as shown in the video. The traditional set of rods that students use is designed in 1cm increments, starting with white as 1cm. The rod lengths in the set used by the kindergarten teacher and her students are proportionally larger than the traditional set of rods. There are 10 rods in each set; each rod has a permanent color name but has deliberately not been given a permanent number name. For example, the length of the dark green rod might be called "four" in one activity and "one" in another.



Cuisenaire® Rods referenced here by permission from ETA/Cuisenaire®, Vernon Hills, Illinois. All rights reserved.

On-Site Activities and Timeline

Getting Ready, cont’d.

2. The Cuisenaire® Rod Activities, cont’d.

“Trains” can be constructed by placing rods together. Trains may be multiples of the same rod, or a mix of different rods. The children construct trains to aid them in finding solutions to the given problems.

> How many different ways can we make dark green? (Kindergarten)
>
> What are all the different ways that we can make a train equal to the length of one magenta rod? (Second Grade) Please note: Cuisenaire® refers to this rod as “purple.”

a. Is this a problem you might use with your students? How do you think your students would solve this problem?

b. An extension to the activities: Can you determine how to find the answer to a similar question for a rod of any length?

3. The Ice Cream Problems

In today’s video, math teacher Gina Kiczek presents the problem below to her high-school students. What mathematical ideas do you believe she wants them to build?

> The new pizza shop in the Heights has been doing a lot of business. The owner thinks that it has been so hot this season that he would like to open up an ice cream shop next door. Because he plans to start out with a small freezer, he decides to initially sell only six flavors: vanilla, chocolate, pistachio, boysenberry, cherry, and butter pecan.
>
> Bowls: The cones that were ordered did not arrive in time for the grand opening so all the ice cream was served in bowls. How many choices for bowls of ice cream does the customer have? Find a way to convince each other that you have accounted for all possibilities.
>
> Cones: The cones were delivered later in the week. The owner soon discovered that people are particular about the order in which the scoops are stacked. “After all,” one customer said, “eating chocolate then vanilla is a different taste than eating vanilla then chocolate.” The owner also discovered rather quickly that he couldn’t stack more than four scoops in a cone. How many choices for ice cream cones does a customer have? Find a way to convince each other that you have accounted for all possibilities.

4. Discuss Pascal’s Triangle

Refer to the Pascal’s Triangle Worksheet on the following pages.

a. Can you model Pascal’s Triangle with block towers?

b. How does the doubling rule work?

c. Can you explain how and/or why the addition rule works?

Pascal's Triangle Worksheet

Patterns provide much of the backbone and motivation in mathematical problem solving. This is true for children as well as for professional mathematicians.

In the video you will see Stephanie's exploration of patterns leading to a comparison between Pascal's Triangle and the Towers Problem.

In the early grades, children were seen on video establishing such patterns to help them determine how many different towers there are and to try to convince themselves and others when they believe they have found all of the possibilities. In the video clips of the children in the early grades you can see several patterns emerging:

note: n = the height of the tower

blue	0	1	2	3	. . .	n
2nd color	n	n-1	n-2	n-3	. . .	0
towers 1-tall	1					1
towers 2-tall	1	2				1
towers 3-tall	1	3	3			1
towers 4-tall	1	4	6	4		1

.
.
.

Sometimes the numbers above are written in triangular arrangement:

```
                    1
                1       1
            1       2       1
        1       3       3       1
    1       4       6       4       1
                    .
                    .
                    .
```

- The mathematics involved in Pascal's triangle forms an important starting point for the branch of mathematics known as combinatorics. Find the pattern represented in the triangle.

The Summing Relationship:
Given that we know a row in Pascal's Triangle, the entries in the next row can be found in the following manner:

The first number in the new row will be 1.

The second number in the new row will be the sum of the 1st and 2nd numbers in the previous row.

The third number in the new row will be the sum of the 2nd and 3rd numbers in the previous row.

.
.
.

And finally the last number in the new row will be 1.

Another relationship among the numbers in Pascal's triangle fits with the children's earlier discovery that as the height of the towers increase by 1 block, the number of different possible towers doubles. If you sum the numbers in any row of Pascal's Triangle, you will observe that those sums double as you progress down the rows.

1 + 1	= 2	= 2^1
1 + 2 + 1	= 2 x 2	= 2^2
1 + 3 + 3 + 1	= 2 x 2 x 2	= 2^3
1 + 4 + 6 + 4 + 1	= 2 x 2 x 2 x 2	= 2^4
etc.		

The mathematics involved in Pascal's triangle now forms an important starting point for the branch of mathematics known as combinatorics.

Questions:

1. Can you model Pascal's triangle with block towers?

2. How and why does the doubling rule work?

3. Can you explain how and why the addition rule works?

On-Site Activities and Timeline

60 minutes

Watch the Workshop Video

Part 1

On-Screen Math Activities

Trains (Kindergarten)
Students arrange shorter rods end-to-end to match the length of a given longer rod.

Trains (Second Grade)
Students try to find all possible ways to arrange shorter rods end-to-end to match the length of a given rod. They count the number of possibilities and compare results.

Towers (Fourth Grade)
Students try to find out how many different towers four blocks high they can build by selecting from blocks of two colors.

Focus Question

What actions taken by these teachers across the grade levels seem to encourage students to think mathematically? In what way are these effective?

Part 2

On-Screen Math Activities

Ice Cream Problems
Bowls: There are six flavors of ice cream. If the ice cream is served in bowls that can hold up to six scoops, how many different ways can the ice cream be served?

Cones: In a variation of the problem, the ice cream can be served in cones stacked up to four scoops high. Given that the order of stacking matters, how many different cones could be served?

Building Pascal's Triangle
A researcher (Robert Speiser) probes Stephanie's understanding of the relationship of the numbers in row n of Pascal's Triangle to towers n high when choosing from two colors.

World Series Problem
Two evenly matched teams play a series of games in which the first team to win four games wins the series. What is the probability that the series will be decided in 1) four games? 2) five games? 3) six games? or 4) seven games?

"*n* choose *r*"
Students derive the formula for determining the number of ways that a subset of r objects can be selected from a total of n objects.

Focus Question

How do the Pizza problems, Towers problems, and World Series problem relate to Pascal's Triangle?

On-Site Activities and Timeline

30 minutes

Going Further

Please address as many of these questions as possible during your allotted time and consider the remainder as part of your homework.

Episode Box One: Englewood—Rods in Kindergarten

In the fall, a kindergarten teacher who attended the Englewood summer workshop was able to send half of her students to a specialist while she gathered the rest (approximately 12) in a circle. She challenged them to propose ways of stacking shorter rods end-to-end to match a given rod. Each student's proposal was considered by the group.

The children offered the following trains as solutions to the problem "How many different ways can we make dark green"?: 2 light green rods; 1 white rod and 1 yellow rod; 3 red rods; and 6 white rods. Jacob suggested a train of 2 white rods and 1 red rod. After a discussion led by the teacher, Jacob then offered 1 red rod, 2 white rods, and 1 red rod as his model. Other children then suggested a train of 2 white rods and 1 purple rod, and a train of 3 white rods and 1 light green rod.

- What mathematical ideas are the children learning in their comparisons of the rods?

The teacher commented on her enthusiasm for learning mathematics with her students.

- Comment on her views.

Episode Box Two: Englewood—Rods in Second Grade

In a second-grade class, Melissa, a first-year teacher, has been working with trains—short lengths of Cuisenaire® Rods placed end-to-end. She challenges the students to find all the different ways (order doesn't count) of combining shorter rods to match a longer rod of a given length.

We observe different children working in small groups to find possible solutions. One girl has found eight solutions and her partner has found six. The teacher asks, "Why do you think you have them all? How do you know?"

- List some benefits you think this kind of questioning might afford the students as well as the teacher's assessment of student understanding.

Notice that the teacher discusses the use of letters to keep track of the trains the students build.

- What are the advantages and disadvantages of the teacher's suggesting a form of notation for use by the children?

On-Site Activities and Timeline

Going Further, cont'd.

Episode Box Three: Englewood—Towers in Fourth Grade

The voiceover tells us that the teacher introduces the Towers activity by asking her students to make as many towers as they can by selecting from blocks of two colors. This is the first time that this teacher has tried the Towers activity with her students, who will work in pairs or small groups. The teacher states, "Make as many four-tall towers as you can, but when you're done, have as many different arrangements as you can." The activity is listed on the blackboard as "Problem Solving." It is written: Construct as many four-tall towers that are different.

- Have you heard different language in the presentations of this problem? Does it matter how the problem is stated? Discuss.

Episode Box Four: Colts Neck—The Equation Problem in Fourth Grade

Amy, the teacher in this activity, spent several years as a researcher at Rutgers during the Kenilworth study—first as a doctoral student and then as a member of the staff. She has been back in the classroom for approximately six years. A typical day begins when her fourth- graders file into the room and start working quietly on a problem she has placed on the blackboard. Today's problem is to write as many different equations as they can in which one side of the equation is 10.

- What mathematics are the children doing in this activity?

As the children shared their results, one boy, Brian, shared the following: 1000 + 1000 – 2000 + 5000 + 5000 – 10000 + 10. The teacher asked, "Do you think it makes a difference if we go from left to right… or right to left? Do we come up with the same 10?"

- Discuss the teacher's question.

On-Site Activities and Timeline

Going Further, cont'd.

Episode Box Five: Jersey City— The Ice Cream Problems in 10th Grade

Gina is nearing the end of a unit on combinatorics. Her students spent several days doing combinations problems, including Towers and Pizzas, and relating these to Pascal's Triangle. She has now introduced a new task: "How many different bowls of ice cream with up to six scoops can be made by selecting scoops from up to six different flavors?"

- What mathematical ideas do these problems raise? How were students dealing with the idea of order in these problems? Suggest follow-up activities.

Episode Box Six: Kenilworth—A Student Connects Towers to Pascal's Triangle

In an eighth-grade interview, Stephanie drew a series of comparisons between her previous work with Towers and Pascal's Triangle. She showed how each number in a row of Pascal's Triangle represents the number of towers in each subset of towers, organized by the number of blocks of a given color. She also demonstrated the "addition property," showing why each number in a new row (except the second row) of Pascal's Triangle is the sum of the two numbers directly above it.

- How did Stephanie build on her earlier ideas with towers and pizzas to explain how the addition property makes sense?

On-Site Activities and Timeline

Going Further, cont'd.

Episode Box Seven: Kenilworth—The World Series Problem in 11th Grade

Students from the Kenilworth focus group spent a two-hour after-school session working on the World Series probability problem: What are the probabilities for ending a series between two evenly matched teams in four, five, six, or seven games respectively, if the first team to win four games wins the series?

- Describe Jeff's explanation for winning a four-game world series. Why do you think Jeff felt he did not give a convincing argument for winning the series in five games? What did Michael contribute? How was Pascal's Triangle related to Michael's justification? Discuss why you were or were not convinced by the students' solutions. Why do you think Pascal's Triangle keeps coming up?

For Next Time

Homework Assignment

Preparing for Workshop 6

1. Study the Episode Boxes on the previous pages. Reflect on the questions and record your reactions and ideas in your journal.

2. Piece together the two parts of the Catwalk photographs on the following pages and photocopy them onto 11 x 17 paper. This series of 24 photographs shows a cat first walking, then running. The interval between successive frames is .031 seconds. The cat is moving in front of a grid whose lines are five centimeters apart. Some lines are darker than others. Based on these photographs, answer as best you can the following two questions.

a. How fast is the cat moving in frame 10?

b. How fast is the cat moving in frame 20?

These photographs were first published in 1885. They give the only information we have about the cat. Please base your responses on information you can gather from the photos and explain carefully how you arrived at your conclusions.

Reminder: For the next workshop, each participant will need two copies of the Catwalk photographs on 11 x 17 paper and on transparencies; metric rulers (clear plastic ones work best); graph paper; a calculator (graphing calculator if possible); and pens or markers for preparing solutions to the problems. You will also want to have an overhead projector, blank transparencies, and pens for participants to use for sharing solutions.

3. Use one of the tasks from Workshop 5 with your students. Carefully observe their activity and take notes about how they approach the problem, paying particular attention to how their strategies and approaches are similar to and different from the ways that you thought about the problem and to what you observed in the video.

Reading Assignment

The reading assignment can be found in the Appendix of this guide.

Maher, C. A. and Speiser, M. "How Far Can You Go With Block Towers? Stephanie's Intellectual Development." *The Journal of Mathematical Behavior,* 16(2), 125-132, 1997.

Join Here

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cats

Join Here

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Notes